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IN
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LONDON 11–13 MAY 1959

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CONTENTS

Page

The Improvement of Process Efficiency

Statistical considerations in process optimisation 1
by G. A. Coutie
The optimal control of chemical and petroleum processes 6
by R. E. Kalman, L. Lapidus, and E. Shapiro
Correlation analysis of a heat exchanger 18
by J. J. Florentin, B. D. Hainsworth, J. B. Reswick, and J. H. Westcott
Superfractionator controllability data obtained by the use of a digital computer 29
by J. M. Keating and D. S. Townend
Discussion of papers presented in the session 38

Process Control Systems

The control of axial flow compressors: the establishment of a control loop to meet unusual performance specifications 49
by J. E. Samson . . .
Ratio flow controller for in-line blending 56
by H. H. Idzerda
Design methods for dual-temperature chemical exchange processes for production of deuterium 60
by C. J. Lyon
A system for the control of volume balance and chemical composition of a process solution and a check on the design of the equipment using an analogue computer 67
by E. Muller and D. C. F. Pratt
Discussion of papers presented in the session 73

Application of On-line Computers

The application of new techniques of control and information handling to rolling mills 83
by W. N. Jenkins
A data processing machine for nuclear power reactor protection 92
by J. L. W. Churchill and R. S. Hopkins
The use of simple computers in electronic process control loops 97
by R. J. Redding
A digital computer for multiple shaft speed regulation 102
by M. W. Sage
Discussion of papers presented in the session 111

Design and Use of Analogue Computers

Analogue computing applied to plant and process economic estimation 121
by D. W. Gillings
Some problems regarding the design of electronic analogue computers for manufacturing process control . 128
by I. M. Witenberg
Use of electric analogue computers for the investigation of the cold rolling process 132
by P. N. Kopai-Gora
Use of electrical analogues in the study of the dynamic behaviour and control of distillation columns . . 135
by J. E. Rijnsdorp and A. Maarleveld
Discussion of papers presented in the session 144

The Use of Computer Techniques in Large and Small Companies

The organisation of a computer aided design department 153
by R. W. H. Sargent
Process development and plant design in large and small companies 158
by S. T. Lunt
Discussion of papers presented in the session 164

Conclusion of the Symposium

The future 171
by Sir Harold Hartley

SESSION 1

THE IMPROVEMENT OF PROCESS EFFICIENCY

Chairman: **Mr. W. K. Hutchison, C.B.E., B.A., B.Sc., M.Inst. Gas E., M.I. Chem. E.**

President of The Institution of Chemical Engineers

STATISTICAL CONSIDERATIONS IN PROCESS OPTIMISATION

By G. A. COUTIE, M.A.*

SUMMARY

Because of the great complexity of many modern chemical processes, methods for improving efficiencies often have to be entirely empirical. Statistical methods are of great value in separating real effects due to deliberate variations in the process conditions from inherent errors that are always present. The paper describes techniques for this, both in the laboratory and on the plant.

In the laboratory it is often possible to approximate the relation of a response (such as cost) to several operating variables (such as temperature, pressure) by a second degree equation; the value of this type of representation is discussed. On the plant scale the technique of "Evolutionary Operation" is more suitable, in which the effects observed as a result of continual, small, deliberate changes in operating conditions are used as a guide to optimum efficiency. Examples of successful applications of these methods are given.

In theoretical, rather than empirical, process studies statistical methods may again be used in assessing the adequacy of a postulated basic mechanism to describe available data, and to evaluate the unknown constants, *e.g.* kinetic rate constants, together with the accuracy of their estimation; the computer is an essential tool in this work.

Introduction

It is largely because of the enormous complexity of many modern chemical processes that attempts to specify optimum operating conditions from theoretical considerations alone are rapidly found to be unsuccessful, and methods for improving efficiencies often have to be entirely empirical. In designing an empirical investigation and drawing conclusions from it, statistical considerations play an important role.

Response Surfaces

In any problem of optimisation, one of the most important decisions to be made is that of the response to be optimised. In simple cases it may be merely a question of maximising the yield of a product, or perhaps of minimising the percentage impurity, but more frequently several such responses may be of importance. Where possible, it is an advantage to express all the various features of process performance, such as raw materials cost, operating costs, yield, and impurity, in the form of a single, composite cost function and to regard this as the response to be optimised. Many difficulties arise in attempts to do this, possibly the greatest one being that of deciding what cost should be associated with the possibly harmful effect of an impurity in the subsequent use of the product by a customer. For this reason it is current practice to specify the most important response, and to optimise this subject to certain restrictions on the values of a number of other responses.

Early applications of statistical methods to chemical laboratory experimentation were largely based upon designing experiments of the "factorial" type introduced by Fisher,[1] in which several factors are studied simultaneously, each at two levels, and observations are carried out on all possible combinations of the factor levels. Such experiments provide information about the average change in response (*e.g.* yield) when a factor (*e.g.* temperature or concentration) is varied by a certain amount, together with the interactions that occur between factors. The information that the experimenter is normally searching for, however, is much more than this; it is in fact the particular combination of factors, within their practicable range, that will give the best response.

Greatly improved methods of experimentation whereby this latter objective may be achieved were introduced by Box and Wilson.[2] Consider, for example, an investigation that is to be carried out on the laboratory scale concerning a chemical reaction for which difficulties are being experienced on the plant. Suppose, for clarity, that only two factors are to be considered, the temperature and the initial concentration of one of the reactants, and that the major response is a measure of cost, subject to satisfactory purity. Figs. 1A and 1B give the possible dependence of the responses cost and purity on the two factors in the region of interest. These are shown in the form of response contours in the same way that height is represented on a map. The response surface analysis consists of approximating each contour system by a

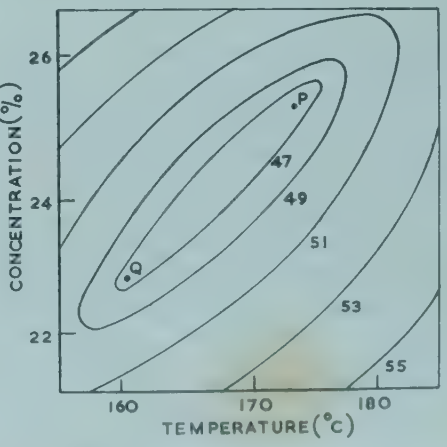

Fig. 1A.—*Contours of constant cost (pence per pound)*

second-degree equation of the form

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* Imperial Chemical Industries Limited, Dyestuffs Division, Hexagon House, Blackley, Manchester, 9.

$$y = b_0 + b_1x_1 + b_2x_2 + b_{11}x_1^2 + b_{22}x_2^2 + b_{12}x_1x_2 \qquad (1)$$

where y represents a response and x_1 and x_2 are suitably-scaled variables representing the temperature and concentration respectively. Methods for choosing the location of the experimental points in order that the constants b in equation (1) may be estimated most accurately have been given by Box.[3] In practice it has been found that an equation of the second-degree is usually adequate to describe the unknown "true" response surface, without recourse to more complicated forms of approximation, although ridge systems such as the one shown in Fig. 1A occur much more frequently than regularly-shaped contours. The detection of ridge systems is of very great practical importance; in Fig. 1A, for example, the experimenter would draw the conclusion that there is not a unique, clearly defined optimum for cost, but that any point along a line from P to Q will give about the same low cost of 47 pence/lb or less, and it is only the further specification that the purity must exceed 89% that indicates that conditions P, corresponding to a temperature of 173° C. and a concentration of 25%, are better than any other.

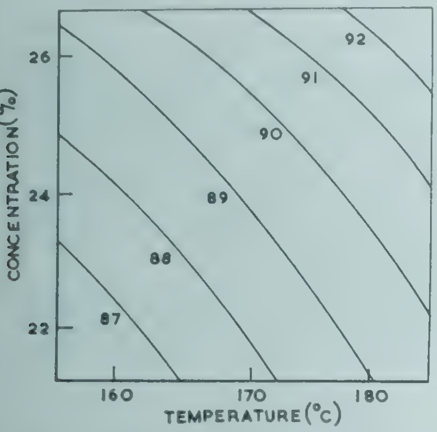


Fig. 1B.—*Contours of constant purity* (%)

Computations

For k variables there are $(k + 1)(k + 2)/2$ constants to be estimated in the second-degree approximating equation, *i.e.* 10 constants for 3 variables, 15 constants for 4 variables, and 21 constants for 5 variables. The method of least squares, which is used for the most efficient estimation of the constants, therefore generates $(k + 1)(k + 2)/2$ linear equations in the same number of unknowns, whose solution gives the required estimates of b_0, b_1, etc. Although for data generated by the most efficient experimental designs[3] these equations will be highly specialised and, therefore, easily solved with only a desk calculating machine, it will often be necessary in the later stages of experimentation to carry out trials at points that are dictated more by the features of the surface that is being explored than by considerations of efficient experimental design, or to combine information from two designs. In such cases the specialised nature of the least squares equations breaks down, and the computer proves invaluable. The necessary calculations[4, 5] may conveniently be summarised in matrix notation, and as most computers have programmes already available for matrix work, the computations present little difficulty.

The best method of interpretation of the fitted equation depends upon the number of variables and the complexity of the surface. For two or three variables the nature of the response surface may be appreciated visually by carrying out a canonical analysis[6] in which the system is referred to a new set of co-ordinate axes parallel to its own axes. For more than three factors, although a similar rotation of axes may be carried out, it is not always easy for a 3-dimensional mind to appreciate the geometrical nature of a higher-dimensional surface, especially when complicated ridge systems exist. A procedure has been given by Hoerl,[7] however, for determining the optimum value of the quadratic equation (1) on any sphere (hypersphere) of radius R centred at the origin of the design. The best value of the response within the experimental region may always be found by this method, and again the calculations consist mainly of matrix operations which may be handled easily on a computer.

Example

The enormous difficulties of attempting to specify optimum operating conditions for a multi-variable system may be demonstrated by a recent laboratory investigation in which the dependence of the yield and purity of a product on five operating variables was sought. The five variables were the concentrations of three of the reactants and the time and temperature of the reaction, and the requirements were that the yield should be as high as possible, and at any rate greater than 78%, subject to the purity exceeding 89%. A combination of previous plant experience and laboratory experimentation mostly varying only one variable at a time had indicated that high yields could only be achieved by sacrificing the purity, and *vice versa*, and the current plant conditions gave in the laboratory, a purity of 88% with a yield of 76%.

Starting from these conditions, if the experimenter makes a change in conditions at random, the chance that he is changing each variable in the right direction only for better response is 1 in $2^5 = 32$, and the chance of obtaining the combination of responses that he is looking for is much smaller. A set of 28 suitably-planned laboratory experiments, however, in the form of a $\frac{1}{2} \times 2^5$ factorial design augmented by 10 points along the axes and two centre points, forming a composite design, enabled five-variable response surfaces to be calculated for both yield and purity. Examination of the canonical forms of these surfaces revealed that there was a single direction, involving simultaneous changes in all five variables, along which almost all the variations in yield and purity occurred. Moreover this direction involved increasing both time and temperature, and at the same time, decreasing the three concentration variables. Defining a new composite variable, which may be called "severity of reaction", corresponding to variation along this direction, the variations in yield and purity could be summarised almost wholly by the graphs of Fig. 2. The most suitable level of severity, corresponding to point A, indicated that a yield of over 80% was possible with a purity of 91%. These responses were confirmed approximately by laboratory trials, and further development is in the form of evolutionary operation on the plant scale.

Theoretical Approach

Although the chemical complexity of many industrial processes rules out anything but a purely empirical approach to optimisation problems, attention may nevertheless be directed sometimes to an attempt at specifying the basic mechanism of the reaction in the search for improvements.

In a recent example, difficulties were experienced in obtaining satisfactory yields in the plant manufacture of a dyestuff. This led to a laboratory study of the kinetics of the reaction involved in the process. After only a comparatively small amount of work it was found that the pH of the system at this stage was critical. This in turn led to plant experimentation in which the effect on yield of varying the pH between the limits 4·5 to 8 was investigated, and an improvement in yield of 5–6% against the former process was achieved. If either plant or laboratory experimentation had been started without knowledge of the kinetic theory many variations in the concentrations of the different reactants might have been tried before it was realised that a single combination of these, corresponding to the pH of the system, was a major yield-determining factor.

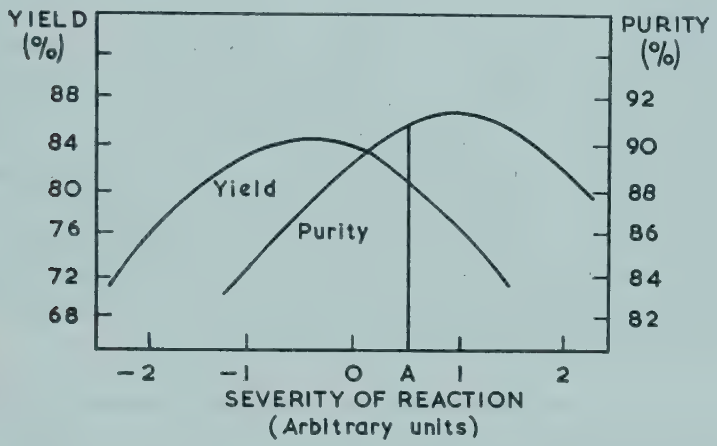


Fig. 2.—*Dependence of yield and purity on "severity of reaction"*

Much of the difficulty associated with pseudo-theoretical investigations arises because of the inadequacy of the available methods for handling them. Given a set of data on a reaction, together with a postulated description of the basic mechanism, there are three important questions that arise:

(*a*) Is the form of the postulated mechanism adequate to account for the available data?

(*b*) If so, what are the best estimates of the unknown parameters in this mechanism?

(*c*) How accurately have these been determined?

In chemical work the mechanism can only rarely be put into explicit form, and more often appears in the form of a set of differential rate equations in which the unknown parameters occur as rate constants, activation energies, etc. Posed in this form, the questions offer a problem in non-linear statistical estimation and statistical methods may be used, in conjunction with a digital computer,[8] in answering them.

Suppose that we have some data on the yield of a reaction under a variety of different conditions and that as a result of the suspected reaction mechanism it is possible to write down a set of differential rate equations summarising the rates of formation or destruction of the various reactants. For a guessed set of parameters, the equations may be solved either explicitly or numerically on a computer to give the yield of the desired product that would be expected in each of the experimental trials if the equations and the parameters were the correct ones. The observed yields will, in general, differ from the computed ones, and the size of this discrepancy provides a measure of the adequacy of the model to describe the data. The discrepancy may be measured by the sum of squares, S^2, between observed and calculated yields taken over all the individual experimental trials. The procedure is then to find those values of the parameters giving a minimum value of S^2. This minimum discrepancy is composed of two parts, one due to the experimental error in the observations on yield and the other due to the inadequacy of the form of the equations—no matter what parameters are chosen—to account for the data. When an independent measure of experimental error is available, for example by the performance of some duplicate trials, a test can clearly be made of whether the postulated mechanism is adequate.

One method for the minimisation of S^2 follows closely the methods adopted in the response surface studies described earlier for the location of optimum response. Values of S^2 now correspond to the response to be minimised, calculations of S^2 take the place of experimental determinations of response, and the values of a set of parameters correspond to the reaction conditions. Calculation of a few values of S^2 from guessed sets of parameters will enable a line of steepest descent to be found, equivalent to estimating only the linear part of equation (1). Repeated applications of this procedure will normally lead to a near-minimum region of S^2, where a second-degree surface may be fitted to enable the location of minimum S^2 to be found, together with the values of the parameters corresponding to this and the accuracy with which they have been determined. The fact that, unlike experimental observations, the calculations of S^2 are error-free enables a quicker minimisation procedure to be adopted based upon the statistical method of regression analysis.

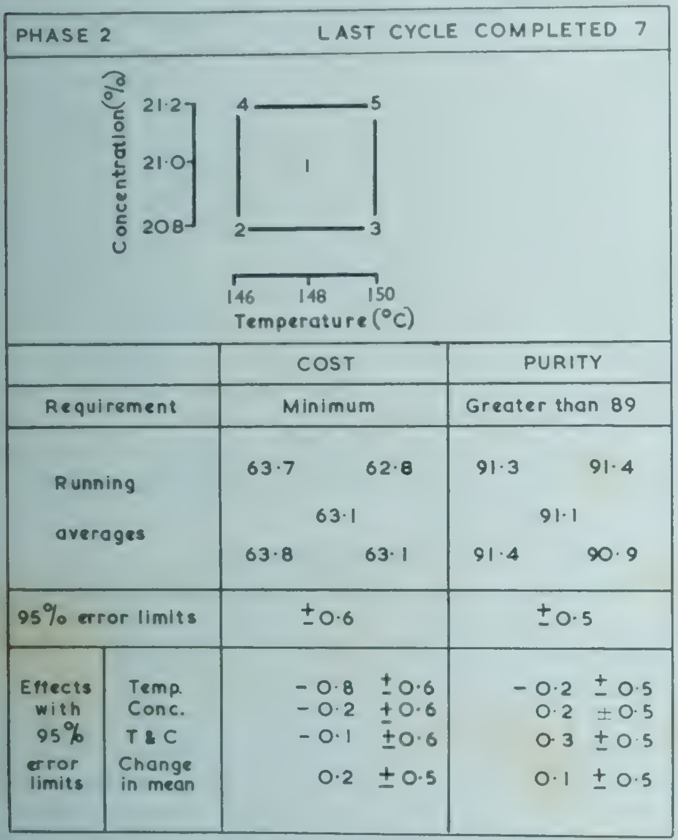
Evolutionary Operation

Methods for process optimisation on the plant scale will normally differ from those used in the laboratory largely because any process variations that are made on the plant must be sufficiently small not to interfere with the smooth running and productivity of the plant. Whereas in the laboratory fairly large variations in process conditions are permissible in order that a response-surface may be fitted; on the plant the fact that only quite small variations are allowable indicates that the variations must usually be continued for a longer time in order that their effect, if any, may be separated from the inherent error of the plant.

Arising out of these considerations, a method for plant-scale process optimisation has been proposed,[9] known as Evolutionary Operation, which has been used with considerable success in a number of chemical plants both in England and, more recently, in America. In this method the plant is not run at a fixed set of operating conditions as is usual in most chemical processes. Instead, systematic small modifications following a carefully planned and continuously repeated cycle are made. Because the modifications are small their effect will probably not be detected in individual cycles, but as the results in successive cycles are averaged evidence of the effect of the modifications is gradually built up. Eventually it is possible to see in what way the method of operation of the plant should be adjusted in order to obtain greater efficiency; this improvement is then incorporated by making the appropriate change and further cycles are started about the newly found conditions.

Example

In practice it has been found that it is best to vary two or three variables at a time when operating in this way, as this allows an easy visual assessment of the situation to be made at any time. The results may be recorded on a chart, such as that shown in Fig. 3, showing the nature and size of the variations being made in the current phase of operation, together with the running averages up to the last cycle completed under each set of operating conditions. Fig. 3 shows the appearance of the information chart at the end of the 7th cycle of the 2nd phase of operation, during which the slightly modified conditions denoted by (2), (3), (4), and (5) were run in cycles with the standard operating procedure (1). At the end of each cycle 95% error limits are computed for each of the averages, these limits becoming narrower as more cycles are performed. Additionally, the average effect of each variable is calculated at the end of each cycle. The effect of changing the temperature in Fig. 3, for example, from 146° C. to 150° C. is obtained by subtracting the average response obtained with the lower temperature from that at the upper temperature. Denoting the running average at a set of operating conditions by the number corresponding to that set, the effects are derived as follows:



		COST	PURITY
Requirement		Minimum	Greater than 89
Running averages		63·7 62·8 63·1 63·8 63·1	91·3 91·4 91·1 91·4 90·9
95% error limits		± 0·6	± 0·5
Effects with 95% error limits	Temp.	− 0·8 ± 0·6	− 0·2 ± 0·5
	Conc.	− 0·2 ± 0·6	0·2 ± 0·5
	T & C	− 0·1 ± 0·6	0·3 ± 0·5
	Change in mean	0·2 ± 0·5	0·1 ± 0·5

Fig. 3.—*Information chart after phase 2, cycle 7*

$$\text{Temperature effect} = \tfrac{1}{2}[(3) + (5) - (2) - (4)]$$

$$\text{Concentration effect} = \tfrac{1}{2}[(4) + (5) - (2) - (3)]$$

$$T \times C \text{ interaction} = \tfrac{1}{2}[(5) + (2) - (3) - (4)]$$

$$\text{Change in mean} = \tfrac{1}{5}[(2) + (3) + (4) + (5) - 4(1)]$$

The final figure, the change in mean, has an important bearing on the cost of running the evolutionary operation scheme. It measures, in fact, the difference between the response that has actually been obtained, and the response that would have been obtained if the evolutionary operation scheme had not been run and the standard operating conditions (1) had been adhered to.

Further procedure

At the end of each cycle one of several types of decision may be taken. The possibilities are:

(*a*) To carry on for another cycle with the same levels of the same variables.

(*b*) To extend the range over which the variables have been tried if little effect has been observed.

(*c*) To make a change in the standard operating conditions and to restart modifications from the new point if a real improvement has been detected.

(*d*) To choose one or more new variables and to start a new phase of the operation.

In the example of Fig. 3 it is possible after the 7th cycle to say, with 95% confidence of being right, that lower costs are to be found with the temperature at 150° C. than with it at 146° C., whereas the purity is not affected much by this difference. The likely conclusion at this stage, therefore, would be to change the standard conditions by increasing the temperature by 2° C. to 150° C., keeping the concentration at 21·0%, and to perform further modifications about this new set of conditions in phase 3 of the evolutionary operation. By continually repeated applications of this method the process conditions are slowly but inevitably nudged to those that are most suitable for the particular plant that is being operated; the plant is, in effect, allowed to produce product plus information—not merely product.

Administration for Evolutionary Operation

Experience in applying evolutionary operation in a chemical works involving many different processes has shown that it is rarely advisable to expect an already-overburdened plant superintendent to cope on his own with even the small amount of extra planning and analysis that are necessary. It has been found worth while and completely satisfactory to train a chemist in simple statistical techniques and to make it his full-time (or part-time, if necessary) task to collaborate with plant superintendents in the design and interpretation of evolutionary operation, and also to maintain liaison with the statisticians so that more advanced techniques may be used where necessary. One of the main objectives to be borne in mind is that evolutionary operation should be simple in design and simple to analyse. If this is so, it will cause the minimum upset in the running of the plant and the calculations may be carried out quickly on a desk calculating machine. Statistical considerations enter primarily into the calculation of confidence limits for the mean response at each set of conditions; quick methods for calculating these have been given by Hunter.[10]

Applications

Evolutionary operation has been applied over the past five years in six of the factories operated by the Dyestuffs Division of Imperial Chemical Industries Ltd., and currently some 40 processes are operating in this way. Almost by definition it will be rare for large percentage increases in efficiency to be gained for processes that have been in operation for many years, and which have had time to approach the best working conditions due to the gradual improvements that come with experience, yet substantial increases have been made on some processes in just these circumstances. For fairly large-scale processes, however, an improvement of only 1% in efficiency may lead to substantial annual savings.

The factors that have entered most frequently into the programmes have been, not surprisingly, temperature, time, and concentration, while the most common responses are yield, purity or, where possible, a composite cost function which takes into account raw material values, operating costs, and yield.

The following examples are representative of the variety of applications that have been made.

DYESTUFF INTERMEDIATE

This concerns a batch process making a dyestuff intermediate in each of ten similar units, the annual output being of the order of 2 million lb. The response of chief interest is the yield, which varies considerably from unit to unit and also between successive batches in the same unit. The size of this variation probably accounts for the fact that plant experimentation, applied spasmodically over a period of many years, and largely for a single unit at a time, had only produced increases in yield very slowly. The item for analysis in evolutionary operation when this was started four years ago was fixed as the average yield over all units for a week, which represented a production of about 30 batches, three from each unit. Initially 12 factors were written down, including 2 times, 4 rates, 2 concentrations, a temperature, and a pressure. Each of these was thought likely to affect the yield. These factors have been varied in phases of 2 or 3 at a time, and there have so far been 10 phases. It was recognised at the start that, owing to the fact that only one observation was obtained per week, each phase of the evolutionary operation could be expected to last for several months, but the potential value of an increase in efficiency was thought sufficient to justify the long-term nature of the scheme. Within the first two years, in fact, an average yield increase over all units of approximately 10% was achieved. During the second two-year period further smaller improvements have been gained, and the previous increase in efficiency has been maintained.

The application of evolutionary operation to this product is typical of many schemes where for one reason or another (*e.g.* large variations in output, intricacies in plant design, etc.), successful experimentation on the plant scale at first sight seemed impossible.

CATALYTIC REACTION

The product in this example is made by a catalytic reaction which is continuous within catalyst runs of about two weeks' duration. The throughput of the plant and factors associated with the catalyst have been varied between catalyst runs, giving only one observation of response per fortnight on each of several similar reaction units. Concurrently with this variation between runs, variations have been made within runs in the ratio of the two reactants and the temperature, measured adequately by the temperature of the hot gas circulating round the catalyst. Conditions are changed each day during a catalyst run, and from samples of the product taken at intervals during the day a measure of efficiency is calculated. For each combination of throughput and catalytic conditions the variations in ratio and temperature have been planned in such a way that contours of the efficiency may be plotted in terms of these two variables. In all cases so far, elliptical contours have been found, enabling the best levels of the ratio and temperature to be specified. Increases in efficiency of the order of 1% have been achieved which, because of the volume and value of production, represent considerable savings.

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THE OPTIMAL CONTROL OF CHEMICAL AND PETROLEUM PROCESSES

By R. E. KALMAN*, L. LAPIDUS† and E. SHAPIRO‡

SUMMARY

This paper outlines the basic concepts of the performance optimisation of chemical reactions by means of a computer control system. A technique of control is described in which the performance of a simulated chemical reactor is continually optimised in real time by means of a digital computer. The optimisation of the dynamic performance of the reactor in terms of a generalised quadratic performance index is accomplished by applying the theory of dynamic programming. This approach is shown to lead to a physically realizable digital computer programme. It is also shown for the type of control problem considered here that the dynamic programming approach leads to a controlled system in which both the static and dynamic optima are obtained by the same digital computer programme.

Introduction

In the last two years there has been increasing interest in the use of high-speed digital computers as controlling elements in the operation of chemical and petroleum processes. The reason for introducing a digital computer into a process control system is the economic gain which may be obtained through precise regulation of the throughput and yield characteristics of the process. For instance, the recent work by Eckman and Lefkowitz[1, 2] shows clearly that for a batch process with empirically determined kinetics the use of computer-control can provide a yield-path for the product which is superior to that which can be achieved under human control. The main advantages of this type of control can probably be realised best in plants and processes currently under development. In such cases the control requirements should be made a part of the over-all design criteria.

There are two principal problems associated with the control of any chemical process.

(1) Measurement of the input and output variables of the process; in other words, measurements of temperature, concentration, product-quality, etc.

(2) Use of the available measurement information so as to control the process in some "optimal way".

This paper is concerned with the second problem. It deals with the basic ideas and with means of implementing them. The chemical engineer may find that he is not accustomed to some of the mathematics employed here. New problems in research and development frequently call for a new language and new methods; the present topic is no exception. It is hoped that this paper will delineate some interesting and profitable areas of investigation as well as suggest some new concepts which may ultimately become a "bread-and-butter" tool of the chemical engineer.

Control Problems

As soon as physical measurements concerning the behaviour of a chemical process are available the problem is to control the process with the help of these measurements. A crude but convenient subdivision of the problems involved in the control operation is the following:

(a) Specification of the performance criteria.

(b) Determination of a suitable model for the process dynamics.

(c) Computation and execution of the control action.

Definitions

Before designing a control system for a process it is necessary to have a clear idea as to the type of performance desired. The "performance index" (or criterion) for the process may take a variety of forms; for instance,

(1) The time necessary to return the process to a chosen equilibrium condition after the process has been disturbed in some manner.[1, 2, 3]

(2) Yield or throughput per unit time.

(3) The cost of energy and material inputs for given product quality or quantity.

We say that the process is under "optimal control" when the performance index is a minimum, [as in cases (1) and (3) above] or a maximum [case (2) above].

To relate the chosen performance index to the design of a control system, relations must be established between product yield and quality and the process parameters which can be used to effect control. In other words, it is necessary to answer questions of the following type:

> "What happens to the process yield, product quality, etc., when a particular process variable used for control (such as catalyst injection, heat input, or agitation) is varied in some known way?"

An important aspect of such a question in connection with control is that the process must be considered from a dynamic rather than a static point of view. A process is said to be dynamic if its behaviour is influenced not only by the present conditions but also by the past history. By contrast, a process is said to be static if it is at all times in a state of equilibrium.

The consideration of processes from the dynamic point of view is a physical necessity. A plant is never a completely static system. In actual operation perturbations about some equilibrium value are inevitable. Moreover, at times it is necessary to change the equilibrium state of the process because of changing performance criteria concerning product quality, throughput, or other items.

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Mathematical models

The common procedure to express the dynamics of a process in quantitative form is to use a mathematical model for the process. The model serves as a means of predicting the behaviour of the process over some interval of time. Qualitatively speaking, the models most commonly used in control system engineering are differential equations (or equivalent items such as transfer functions or the frequency response) which relate the input and output variables of the process. To make the model useful it is necessary to relate it to the physical behaviour of the process. In other words, the various free parameters of the model must be determined in such a way that the dynamic behaviour of the model (which should be easy to compute) closely approximates the experimentally observed dynamic behaviour of the process. The determination of the model parameters from the process measurements is essentially a data-reduction procedure. The computer which performs this function may be regarded as an extension of the basic measuring system.

It should be emphasized that accurate measurement of process dynamics is important in any case where the control problem is not trivial. It is not sufficient to use laboratory data for determining the model; if at all possible the model should be based on measurements taken under actual operating conditions. Moreover, the determination of process dynamics should take into account the effect of the dynamics of the various physical measuring elements in a quantitative way. There are cases where the dynamics of the measuring elements pose more severe problems than the dynamics of the process.

Finally, in any process control system, it is necessary to have a means of using the information provided by the process measurements and the knowledge of the dynamic model of the process to change the control variables in such a way that the desired performance criterion index is minimised (or maximised). Under ideal conditions the process can be made to behave in almost any manner desired. That the ideal cannot be attained is due mainly to inaccuracies and delays in the measurements of the process variables and to inaccuracies in the dynamic model of the process.

These two factors are to some extent interdependent. If the process variables can be measured very accurately and very rapidly it is sufficient to have a model of the process which predicts behaviour reliably only over a very short period of time. On the other hand, if the model supplies accurate predictions of process behaviour over a long period the measurements of the process variables need not be obtained so frequently. Since the dynamic model of the process has to be derived from measurements of the process variables, it is clear that ultimately the precision of control achievable depends only on the accuracy of measurements. However, in many practical cases the information provided by the measurements cannot be fully utilised without extensive computations.

Sketch of a Possible Digital Computer Control System

A possible method of implementing all those requirements for control which are susceptible to being performed by a machine is shown in Fig. 1. The control computer shown may be called an "adaptive" or "self-optimising" system. The name indicates that the system has a certain measure of "learning" ability in evaluating and making use of the process measurements.

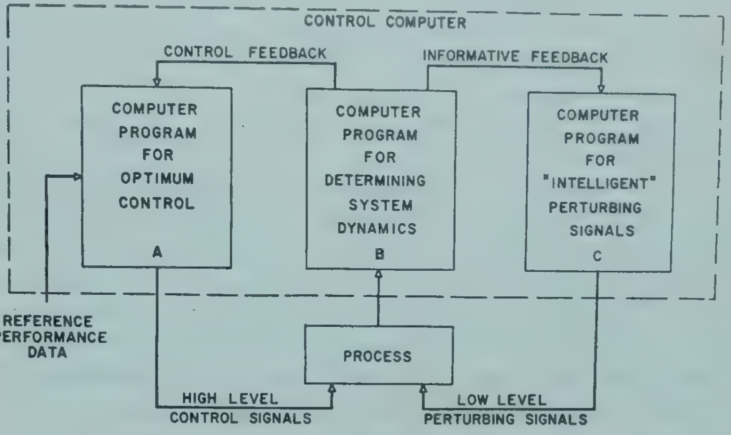
CONTROL COMPUTER
CONTROL FEEDBACK
INFORMATIVE FEEDBACK
COMPUTER PROGRAM FOR OPTIMUM CONTROL
A
COMPUTER PROGRAM FOR DETERMINING SYSTEM DYNAMICS
B
COMPUTER PROGRAM FOR "INTELLIGENT" PERTURBING SIGNALS
C
REFERENCE PERFORMANCE DATA
PROCESS
HIGH LEVEL CONTROL SIGNALS
LOW LEVEL PERTURBING SIGNALS

Fig. 1.—*A possible adaptive digital control system*

The input to the control computer consists of process measurements as well as the performance index. The former are necessary in any control system: the latter may be regarded as representing management decisions dependent on the market situation, availability of raw materials, etc. The performance index represents the means by which over-all "supervision" is exercised over the system.

Function of units

The function of Box A is to compute the desired control variables (in other words, issue commands to the various valve-positioners) using the latest process measurements and taking into consideration the performance index as well as the dynamic model of the process which already has been determined and stored. Box B continually analyses the process measurements to determine an accurate model of the process dynamics. The model, of course, is of such a nature as to facilitate the computations required from Box A.

Since the measurements derived from the normal operation of the process are not always adequate for determining the dynamic characteristics of the process*, Box C is used to generate and inject special calibrating signals into the process. These signals are of such a level and form that they do not affect the normal operation of the process significantly. yet they perturb the process in such a way that useful information can be obtained by means of sophisticated data evaluation techniques. Box C also receives information from Box B which tells it when to change the form of the calibrating signals. This technique makes it possible to achieve a high degree of accuracy in the determination of process dynamics.

Each of the functions outlined can be realised, in principle, by using a single general-purpose digital computer. Each of the boxes represents a certain set of operating instructions for the computer, one set following the other cyclically. The programming and engineering details of this mythical computer are, of course, not simple; however, sufficient theoretical and technological information is available at present to attack the problem.

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* When the process has been very near equilibrium for a long time, it appears to be essentially static to the external observer. In such circumstances the usual measurements contain very little direct information about the dynamic behaviour of the process when it is perturbed from equilibrium.

By constant monitoring of the dynamics of the process the computer system described above will adapt its mode of operation to changes in the process environment (such as the changes in the quality of raw materials, catalyst activity, and heat transfer coefficients), as well as seek out that equilibrium state which yields the optimum performance index. This is an extension of the principle of feedback, since the actions of the computer are ultimately dependent only on the process measurements. As long as the dynamic characteristics of the process do not vary in a very rapid manner the system described will always operate at or near some equilibrium point.

In many situations greatly improved control can be expected from the ability of the control system to "adapt" itself to changes in the process dynamics or performance criteria. In fact, the control systems in current use are designed in such a way that they operate tolerably well under widely varying conditions; but in doing so, they utilise the basic physical measurements very inefficiently. The performance of conventional control systems cannot be appreciably improved without adding the functions of Boxes B and C above, *i.e.*, without making better use of the physical measurements. It should be added that the system proposed here requires only rudimentary data concerning the process dynamics to start with; once the process is under some measure of control, performance will improve continually because of the more and more accurate determination of the dynamic model of the process.

Advantages of digital computer

There are many advantages in using a digital computer for control. In principle, all mathematical operations can be reduced to simple arithmetic steps, and therefore any general-purpose digital computer can be programmed to perform these operations subject to its capacity and speed limitations. The design of the over-all system can be regarded as essentially completed once the programme of operations for the computer has been specified. Once the programme has been proved, the required operations will be repeatedly performed by the computer in exactly the same manner, provided that the computer is reliable. It seems probable that within a few years a large-scale digital computer will be utilised in the control of a number of operations of a refinery. In this way, optimal yields and throughputs commensurate with the over-all refinery operation will be obtained. The computer will also perform all the necessary data logging of temperatures, flow rates, and provide production and quality-control data for use by the refinery administrative and technical staff.

The Formulation of a Reactor Control Problem

The reaction process shown in Fig. 2 will be considered as a typical reactor control problem. (More general types of processes can be handled in a similar manner, but the complexity of the resulting equations would tend to confuse the discussion). Fig. 2 shows a completely-mixed stirred tank reactor with reaction volume V. A single exothermic chemical reaction (or a series of them) is being carried out within the reactor. Inputs to the reactor are the volumetric liquid flowrates $F_1, F_2, \ldots, F_m$, with associated temperatures $T_{01}, T_{02}, \ldots, T_{0m}$. Each of the first $(m-1)$ streams carries one or more reactants and products with concentrations $x_{01}, x_{02}, \ldots, x_{0l}$. The stream F_m represents the cooling water flow rate. The symbols $x_1, x_2, \ldots, x_{n-1}$ denote the concentrations of reactants and products within the reactor itself. Since perfect mixing is assumed, $x_1, x_2, \ldots, x_{n-1}$ are also the reactor effluent concentrations. The effluent temperature is T, and the outgoing cooling-water temperature is T_m. For uniform symbolism, T will be denoted as x_n. By virtue of a liquid-level controller which maintains a fixed volume in the reactor, the total effluent liquid stream is given by

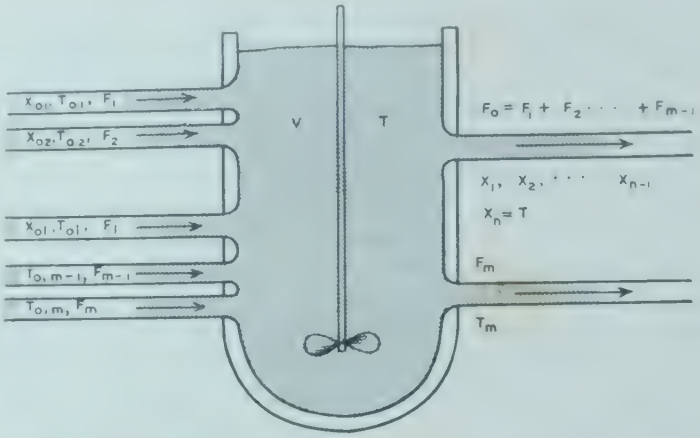

Fig. 2.—*Schematic diagram of a continuous perfectly-stirred reactor with exothermic chemical reaction*

$$F_0 = F_1 + F_2 + \ldots + F_{m-1}.$$

The desired result of the operation of this reactor is to obtain certain concentrations of reactants and products in the effluent stream. Control of the system is achieved by varying the influent streams in some manner. The reaction kinetics of the system must be considered to determine the effect on the effluent stream concentrations of changing, say, the flowrate, F_1.

Model of reaction

For simplicity, assume a second order reaction in the stirred reactor. This may be represented by

$$\mathrm{A} + \mathrm{B} \xrightarrow{k_{AB}} \mathrm{C} \quad . \quad . \quad . \quad (1)$$

In the nomenclature of Fig. 2, $x_A = x_1$, $x_B = x_2$, and $x_C = x_3$. If the concentrations x_A, x_B, x_C of the various components are small, the rate of increase of the concentration of C is given by the Arrhenius equation:

$$\mathrm{d}x_C/\mathrm{d}t = k_{AB}(T)\, x_A x_B \quad . \quad . \quad . \quad (2)$$

The reaction rate coefficient in equation (2) is given by:

$$k_{AB}(T) = \alpha_{AB} \exp\left(-E_{AB}/RT\right) \quad . \quad . \quad (3)$$

where α_{AB} is a constant, and E_{AB} is the activation energy of the reaction.* The rates of decrease of the concentration of reactants A and B are of the same form.

The assumptions which lead to the particular form of equation (2) are not always true, but in general the reaction rate is still a function of the temperature and concentrations. Thus, equation (2) could be replaced by:

$$\mathrm{d}x_C/\mathrm{d}t = k_{AB}(x_A, x_B, x_C, T) \quad . \quad . \quad (4)$$

where k_{AB} is some scalar function of the four variables indicated.

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* *Symbols have the meanings given to them on p. 16.*

With the reactor operating in a transient manner, material balances on component A, B, and C yield

$$\frac{dx_A}{dt} = \frac{F_A}{V} x_{0A} - \frac{F_0}{V} x_A - k_{AB} x_A x_B \quad . \quad (5)$$

$$\frac{dx_B}{dt} = \frac{F_B}{V} x_{0B} - \frac{F_0}{V} x_B - k_{AB} x_A x_B \quad . \quad (6)$$

$$\frac{dx_C}{dt} = \frac{F_C}{V} x_{0C} - \frac{F_0}{V} x_C + k_{AB} x_A x_B \quad . \quad (7)$$

where t is the time variable. It has been assumed in the derivation of these equations that each influent stream carries only one component.

A heat balance on the reactor yields

$$\frac{dT}{dt} = \frac{F_A}{V} T_{0A} + \frac{F_B}{V} T_{0B} - \frac{F_0}{V} T - k_{AB} x_A x_B \frac{\Delta H_{AB}}{\rho c} - \frac{U}{V \rho c} (T - T_{ave}) \quad . \quad (8)$$

In the derivation of the latter equation it has been assumed that the densities of all the influent and effluent streams are the same and also that the specific heats are the same. The dynamics of the process may now be said to be determined by the set of ordinary nonlinear differential equations (5), (6), (7), and (8).

Extending these concepts to a generalised reaction system, the following set of nonlinear, time-dependent differential equations can be written:

$$\frac{dx_i}{dt} = \mathrm{f}_i(x_1, x_2, \ldots, x_n) + \mathrm{g}_i(F_1, F_2, \ldots, F_m; x_{01}, x_{02}, \ldots, x_{0l}) \quad . \quad (9)$$

where one of the $x_i = T$.

The terms in g_i are seen to represent the effect of external influences or forcing functions. It is known that (under slight mathematical restrictions) the system represented by equation (9) has a unique solution for all values of $t \geqslant t_0$, given the functions $\mathrm{g}_i(t)$ for all values of $t \geqslant t_0$ and given the initial conditions, $x_1(t_0), \ldots, x_n(t_0)$. While this set of equations was derived for a particular process, they represent, in a more general sense, the most common form of the model used to represent dynamic physical systems.[4]

Vector-matrix notation

Let **x** be a vector [$(n \times 1)$ matrix] with components $x_1, x_2, \ldots, x_n$. Similarly **F** and $\mathbf{x}_0$ are defined as $(m \times 1)$ and $(l \times 1)$ matrixes respectively. **f** is a vector function of n arguments with components $f_1, f_2, \ldots, f_n$ and **g** is a vector function of n components and $(l + m)$ arguments. In terms of this notation, equation (9) can be written as:[5]

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}) + \mathbf{g}[\mathbf{F}(t); \mathbf{x}_0(t)] \quad . \quad . \quad (10)$$

The vector **x** is the "state of the system" and the components of **x** are the "state variables". If the reactor inputs $\mathbf{F}(t)$ and $\mathbf{x}_0(t)$ are specified for all time $t \geqslant t_0$, then the knowledge of $\mathbf{x}(t_0)$ supplies the initial conditions from which the solutions of differential equation (10) can be uniquely determined for all future values of the time. Thus the state is a fundamental mathematical concept in describing the reactor dynamics.

The state is also a physical concept. The temperature and various concentrations can be physically measured (at least in principle); thus the state at time t_0 may be regarded as the information necessary to determine the properties of the material inside the reactor at time t_0.

Those particular arguments

$$F_1, F_2, \ \ldots \ , F_m; \ x_{01}, x_{02}, \ \ldots \ , x_{0l}$$

of the function **g** which can be changed at will are the "control variables" of the process.

Solution of model equation

If equation (10) is solved by any means whatsoever, the solution can be written in the form

$$\varphi_i[t; \ x_1(t_0). \ x_2(t_0), \ \ldots \ , x_n(t_0); \ \mathrm{g}_1(u), \mathrm{g}_2(u), \ \ldots \ , \mathrm{g}_n(u), \ t_0 \leqslant u \leqslant t] \quad (11)$$
$$i = 1, 2, \ \ldots \ , n.$$

The φ_i are functions which, when substituted in place of the x_i, satisfy both sides of equation (9). Let t take on a specific value, say t_1, and using vector notation:

or

$$\mathbf{x}(t_1) = \boldsymbol{\varphi}[t_1; \ \mathbf{x}(t_0); \ \mathbf{g}(u), \ t_0 \leqslant u < t_1] \quad . \quad (12)$$

Equation (12) represents the state of the system at time $t = t_1$ which has evolved from the given state at time $t = t_0$ under the influence of specific external influences. The functions φ_i are called the "state transition functions" of the dynamic system and specify how the state of the system evolves from one instant of time to the next. While equations (11) and (12) represent the state transitions over a finite interval of time, the original set of differential equations (10) may be regarded as specifying the state transitions over an infinitesimal interval of time.

There is obviously much to be gained by short-circuiting the process of having to solve the differential equations and starting directly with the transition functions. This may be done by defining the system by means of a set of difference equations of the form

$$\mathbf{x}(t_0 + \tau) = \boldsymbol{\varphi}[t_0 + \tau; \ \mathbf{x}(t_0); \ \mathbf{g}[\mathbf{F}(t_0), \mathbf{x}(t_0)] \quad . \quad (13)$$

In other words, a dynamic system may be described by restricting attention to the values of the state variables at certain discrete points in time (at $t = t_0$, $t = t_0 + \tau$, $t = t_0 + 2\tau$, . . .) and evaluating the state transitions corresponding to the time interval of length τ, the "sampling period", during which the external influences $\mathrm{g}_i(t)$ are assumed to be approximately constant. This step is important in the use of digital computers in control and computation applications because a digital computer can only accept or deliver data at discrete instants of time.[3, 6, 7]

Linearisation of Process Dynamics

In the steady state all the derivative terms become zero, *i.e.*, the state is not moving in any direction. Equation (8) may be written in the form,

$$\frac{F_0}{V} T^* - \frac{F_A}{V} T_{0A} - \frac{F_B}{V} T_{0B} = k_{AB} (T^*) \, x_A{}^* x_B{}^* \, \frac{\Delta H_{AB}}{\rho c} \quad . \quad (14)$$

The asterisk is used to denote a steady-state value.

The left side of the equation represents the heat removed from the system and the right side the heat liberated by the chemical reaction. For constant values of the parameters, the left side is a linear function of T^* and the right side a sigmoidal or s-shape function of T^*. This is shown in Fig. 3 which can be obtained by choosing a value for T^*; $k_{AB}(T^*)$ is immediately defined, and by setting equations (5), (6), and (7) equal to 0, $x_A{}^*$ and $x_B{}^*$ can be evaluated. Both sides of equation (14) can then be calculated and plotted; this is repeated for a suitable range of T^*. The intersection of the two curves determines the actual equilibrium states. As shown, there are three equilibrium states each of which satisfy the system of steady-state equations. The low temperature point is characterised by very low conversion and the high temperature point by high conversion in the chemical reaction. As a rule, these two points are stable points of operation since the system will return to its original position if it is perturbed slightly. The centre point, however, is usually unstable; the system will tend to move over to one of the two stable points if the temperature of the system is perturbed slightly. Amundson and Aris[8, 9] and van Heerden[10] illustrate these points in more detail.

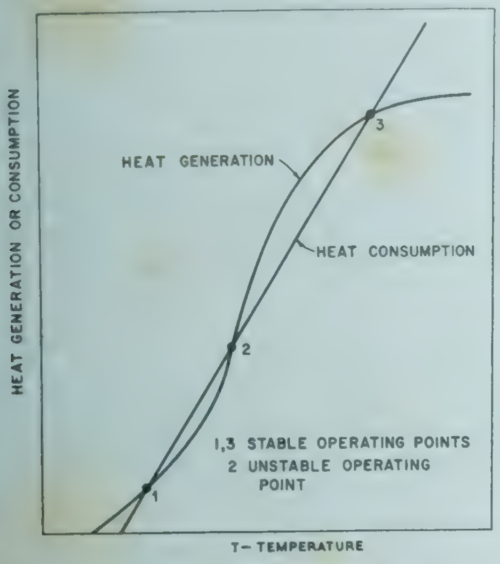

Fig. 3.—*Schematic graph of heat generation and heat consumption against the temperature as given by equation* (1)

In order to predict the response of the system to large perturbations, non-linear dynamic equations must be used, but the present state of knowledge of the stability of non-linear systems is such that an analysis can only be carried out in the most simple cases. In practice "linearised models" which hold only for the case of small perturbations ("local stability") must almost invariably be used. Once an operating point has been chosen, the linearised model can be used to investigate local stability as well as to predict the dynamics near the operating point.[4, 5]

By definition, the equilibrium states $\mathbf{x}^*$ of the reactor are given by

$$\frac{d\mathbf{x}^*}{dt} = \mathbf{f}(\mathbf{x}^*) + \mathbf{g}(\mathbf{F}^*;\ \mathbf{x}_0{}^*) = 0 \qquad (15)$$

with the starred quantity indicating equilibrium or steady-state values. The non-linear dynamic equations may be linearised about the equilibrium states $\mathbf{x}^*$ and the values of the corresponding flow rates $\mathbf{F}^*$ by first letting:[4, 5]

$$\mathbf{x} = \mathbf{x}^* + \mathbf{a} \qquad (16a)$$

and

$$\mathbf{F} = \mathbf{F}^* + \mathbf{b} \qquad (16b)$$

$\mathbf{a}$ and $\mathbf{b}$ represent deviations from the equilibrium values and if these are sufficiently small, equation (10) leads (by expanding into a multi-dimensional Taylor series) to the linear matrix differential equation

$$\frac{d\mathbf{a}}{dt} = \mathbf{M}\mathbf{a} + \mathbf{N}\mathbf{b} \qquad (17)$$

$\mathbf{M}$ is a constant $(n \times n)$ matrix and $\mathbf{N}$ is a constant $(n \times m)$ matrix.[5]

The elements of these matrices are determined from equation (15) by means of equations (18) and (19):[4, 5]

$$m_{ij} = \frac{\partial \mathrm{f}_i}{\partial x_j} \bigg|\ \mathbf{x} = \mathbf{x}^*,\ \mathbf{F} = \mathbf{F}^* \qquad i, j = 1, 2, \ldots, n \qquad (18)$$

$$n_{ij} = \frac{\partial \mathrm{g}_i}{\partial F_j} \bigg|\ \mathbf{x} = \mathbf{x}^*,\ \mathbf{F} = \mathbf{F}^* \qquad \begin{matrix} i = 1, 2, \ldots, n \\ j = 1, 2, \ldots, m \end{matrix} \qquad (19)$$

The time-domain solution of the differential equation (4) has the form:

$$\mathbf{a}(t) = \mathbf{\Phi}(t - t_0)\,\mathbf{a}(t_0) + \int_{t_0}^{t} \mathbf{\Phi}(t - \tau)\,\mathbf{N}\mathbf{b}(\tau)\,d\tau \qquad (20)$$

The matrix $\mathbf{\Phi}(t)$, called the "transition matrix," is given by:

$$\mathbf{\Phi}(t) = \exp(\mathbf{M}t) = \sum_0^\infty \frac{\mathbf{M}^i t^i}{i!} \qquad (21)$$

[See Appendix II for a discussion of equations (20) and (21).]

Specification of the Optimal Control Problem

The problem of optimal control of a chemical reactor is best stated in two parts:[5]

(*a*) Select a set of desired values $x_1{}^d, \ldots, x_n{}^d$ of the concentrations in the output stream on the basis of economic or management considerations.

(*b*) Given the actual concentrations at time t_0, bring these concentrations rapidly as close as possible to the desired concentrations. If at any time $t_1 > t_0$ the desired values of the concentrations are changed, the procedure is repeated.

Model of the process

The states of the reactor may be regarded as points in an n-dimensional Euclidean space (the state variables at time $t > t_0$ being coordinates of a point) called the "state space". Suppose r components of the state vector are specified as desired values, the remaining $(n - r)$ components being arbitrary. This step may be regarded as essentially a management decision relating to the question of how one should try to operate the reaction process, and is equivalent to part (*a*) of the reactor control problem. The set of states for which the operation of the reactor meets the management requirement is clearly an $(n - r)$ dimensional hyperplane and is defined as the "optimal hyperplane" for the process. If the state of the reactor at any instant of time does not lie in this hyperplane, one can measure the degree of unsatisfactory performance by the "distance" of that state from the optimal hyperplane. The definition of the distance function is arbitrary and depends on a management estimate as to what types of deviations from the desired values are more harmful than others. One possible definition of the distance function is

$$\beta[\mathbf{x}^d - \mathbf{x}] = \left[\sum_{i=1}^{r} (x_i{}^d - x_i)^2\right]^{\frac{1}{2}} \qquad (r < n) \quad . \quad (22)$$

More generally, if $\mathbf{Q}$ is any positive semi-definite matrix, β can be defined by the quadratic form,

$$\beta^2[\mathbf{x}^d - \mathbf{x}] = (\mathbf{x}^d - \mathbf{x})' \, \mathbf{Q}(\mathbf{x}^d - \mathbf{x}) \quad . \quad . \quad (23)$$

where the prime is used to denote the transpose of a vector or matrix.

Static optimisation

"Static optimisation" of the process is the selection of a constant value $\mathbf{F}^\circ$ of the control vector such that, at equilibrium, the actual state lies as close as possible to the optimal hyperplane. Since the equilibrium states $\mathbf{x}^*$ are given by equation (14), the statically optimal control vector $\mathbf{F}^\circ$ and equilibrium state $\mathbf{x}^{*\circ}$ are determined by solving the minimisation problem

$$\underset{\mathbf{F}}{\mathrm{Min}} \; \beta\,[\mathbf{x}^d - \mathbf{x}^*] \quad . \quad . \quad . \quad (24)$$

From equation (5), $\mathbf{x}^*$ can be expressed as an explicit function of $\mathbf{F}$ to find the optimal control vector $\mathbf{F}^\circ$ from equation (24). This leads to great analytic difficulties when $\mathbf{f}$ and $\mathbf{g}$ in equation (10) are non-linear functions. However, even in cases where the static optimisation problem can be solved, it does not provide a complete answer to the basic problem for several reasons:

(1) Static optimisation does not indicate how the control variables should be manipulated to bring an arbitrary state as close as possible (in terms of the arbitrarily adopted distance function) to the desired state.

(2) The equilibrium state closest to the hyperplane of optimal states may not be stable.

(3) The values of the control variables computed by static optimisation will not remain optimal when some of the process parameters (concentrations in the input flows, ambient temperature, etc.) change.

In other words, static optimisation does not incorporate the principle of feedback.

Dynamic Optimisation

A "performance index" for the reaction under dynamic conditions may be defined as:

$$\mathscr{P}[\mathbf{x}(t_0)] = \int_{t_0}^{\infty} \beta^2[\mathbf{x}^d - \mathbf{x}(t)] \exp\,[\alpha(t - t_0)]\,\mathrm{d}t \quad . \quad (25)$$

where α is a real non-negative constant and $\mathbf{x}^d$ is the desired value of $\mathbf{x}$. The phrase, " . . . bring the actual concentrations rapidly as close as possible to the desired concentrations . . . " is now precisely defined to mean that the control variables $\mathbf{F}(t)$ are to be chosen, as functions of time, in such a way as to minimise the performance index equation (25) for any initial state $\mathbf{x}(t_0)$. This is "dynamic optimisation." The definition of $\mathscr{P}$ and, in particular, the value of α in equation (25) are arbitrary and depend on management estimates just as the definition of $\mathbf{x}^d$ and β.

Static optimisation is a special case of dynamic optimisation as may be seen by setting $\mathbf{x}(t_0) = \mathbf{x}^{*\circ}$ in equation (25). In fact, dynamic optimisation may lead to the result that instead of trying to maintain the control variables at constant (equilibrium) values, it is better to vary the control variables continuously, for instance, in a periodic fashion. In such a case, dynamic optimisation will lead to a smaller value of $\mathscr{P}(\mathbf{x}^{*\circ})$ than static optimisation.

To carry out dynamic optimisation, a particular vector function $\mathbf{F}^\circ(t)$ must be found, (defined for all $t \geqslant t_0$) among the set of all such functions for which the integral expression (25) assumes its minimum. This is in general a difficult problem in the calculus of variations and, for all practical purposes, cannot be solved by conventional analytic methods.

To keep the mathematical problems in tractable forms, it will be assumed that the control variables have constant values over a "sampling" interval τ. Thus, instead of minimising $\mathscr{P}$ with respect to all possible functions $\mathbf{F}(t)$, the minimisation is to be performed with respect to all possible sequences of constant vectors[5, 11]

$$\mathbf{F}(t_0), \mathbf{F}(t_0 + \tau), \mathbf{F}(t_0 + 2\tau), \; . \; . \; . \; , \mathbf{F}(t_0 + k\tau), \; . \; . \; . \quad (26)$$

For simplicity the integral in equation (25) is replaced by a sum:

$$\mathscr{P}[\mathbf{x}(t_0)] = \sum_{k=1}^{\infty} \beta^2[\mathbf{x}^d - \mathbf{x}(t_0 + k\tau)]\,\lambda^k \quad . \quad (27)$$

where $\lambda = \exp\,(\alpha\tau) \geqslant 1$.

From equation (13) it can be seen that there is a large number of possible state transformations $\mathbf{x}(t_0) \rightarrow \mathbf{x}(t_0 + \tau)$, depending on the choice of $\mathbf{F}(t_0)$, assuming that $\mathbf{x}_0$ is constant. Thus, the minimisation of $\mathscr{P}$ may be regarded as an infinite step decision procedure, the optimal choice of $\mathbf{F}(t_0 + k\tau)$ at the kth step in general depending both on the preceding and succeeding steps. At first sight, it would appear that to obtain the optimal sequence of control vectors,

$$\begin{aligned} &\mathbf{F}^\circ(t_0). \; \mathbf{F}^\circ(t_0 + \tau), \\ &\qquad \mathbf{F}^\circ(t_0 + 2\tau), \; . \; . \; . \; , \mathbf{F}^\circ(t_0 + k\tau), \; . \; . \; . \end{aligned} \quad (28)$$

expression (27) must be minimised simultaneously with respect to all the terms of the sequence (26). This is impossible. Fortunately, at this point a decisive simplification can be achieved by making use of the following intuitively obvious, but powerful, observation due to Bellman.[6, 12]

Principle of optimality

An optimal sequence (28) of control variables has the property that whatever the initial state $\mathbf{x}(t_0)$ and the initial choice $\mathbf{F}^\circ(t_0)$ of control vector, the remaining terms $\mathbf{F}^\circ(t_0 + \tau)$, $\mathbf{F}^\circ(t_0 + 2\tau)$, . . . of the sequence must constitute an optimal sequence with regard to the state $\mathbf{x}(t_0 + \tau)$ resulting from the choice of $\mathbf{F}^\circ(t_0)$

Application of principle of optimality

Various expressions can be obtained for the theoretical study and practical determination of the optimal control sequence using the principle of optimality. (Methods derived from the principle of optimality are known by the generic name of dynamic programming.)

Equation (27) can be written in the form:

$$\begin{aligned} \mathscr{P}[\mathbf{x}(t_0)] = \lambda\beta^2[\mathbf{x}^d - \mathbf{x}(t_0 + \tau)] & \\ + \sum_{k=2}^{\infty} \beta^2[\mathbf{x}^d - \mathbf{x}(t_0 + k\tau)]\,\lambda^k & \\ \lambda\{\beta^2[\mathbf{x}^d - \mathbf{x}(t_0 + \tau)] + \mathscr{P}[\mathbf{x}(t_0 + \tau)]\} & \quad . \quad (29) \end{aligned}$$

Equation (27) has been written in the form of a sum of two terms, where the second term depends only on the steps after t_0 and therefore can be optimised independently of the first step.

Let $\mathscr{P}^{\circ}[\mathbf{x}(t_0)]$ be the value of the performance index when the optimal sequence (28) is used. Substituting equation (29) and invoking the Principle of Optimality, a functional equation for $\mathscr{P}^{\circ}$ is obtained.

$$\mathscr{P}^{\circ}[\mathbf{x}(t_0)] = \underset{\mathbf{F}(t_0)}{\text{Min}}\ \lambda\{\beta^2[\mathbf{x}^d - \mathbf{x}(t_0+\tau)] + \mathscr{P}^{\circ}[\mathbf{x}(t_0+\tau)]\} \quad . \quad (30)$$

The right hand side of equation (30) is a function of $\mathbf{F}(t_0)$ and $\mathbf{x}(t_0)$ through equation (13). The solution of the functional equation (30) determines $\mathbf{F}^{\circ}(t_0)$ as some function of $\mathbf{x}(t_0)$ which may be denoted by

$$\mathbf{F}^{\circ}(t_0) = \mathbf{h}[\mathbf{x}(t_0)] \quad . \quad . \quad . \quad (31)$$

At time $(t_0+\tau)$ precisely the same decision problem is encountered as at time t_0. This indicates that $\mathscr{P}^{\circ}[\mathbf{x}(t_0+\tau)]$ also satisfies the functional equation (30) and therefore $\mathbf{F}^{\circ}(t_0+\tau)$ is the same function of the state at time $(t_0+\tau)$ as $\mathbf{F}^{\circ}(t_0)$ was at time t_0.

Thus: If the performance of a dynamic system governed by equation (13) is optimal in the sense that the performance index equation (27) is a minimum, then the sequence of optimal control variables is obtained by observing the state of the system at times

$$t_0,\ (t_0+\tau),\ (t_0+2\tau),\ \ldots\ ,\ \ldots\ (t_0+k\tau),\ \ldots$$

and computing the optimal control variables at each sampling instant by means of the equation

$$\mathbf{F}^{\circ}(t_0+k\tau) = \mathbf{h}[\mathbf{x}(t_0+k\tau)] \quad . \quad . \quad (32)$$

$\mathbf{h}$ being determined by solving equation (30).

Equation (32) shows that the feedback principle is included in the framework of dynamic optimisation. Note that the entire future evolution of the dynamic system, including the values of the optimal control variables at each sampling instant, could be predicted in principle by means of equations (13) and (32). However, because of the inaccurate knowledge of the state transition function, unknown disturbances acting on the process, and random effects such as turbulence, etc., the prediction based on equation (13) will be less correct as the prediction interval increases. By remeasuring the state of the system at the sampling instance (assuming that τ has been chosen small enough so that the one-step prediction based on equation (13) is sufficiently accurate), the prediction errors are corrected so that the control variables assume very nearly their optimal values at all times. This mode of operation defines a feedback control system. As is well known, feedback will also tend to minimise the sensitivity of $\mathbf{h}$ to variations in $\mathbf{\Phi}$.

Iterative solution of minimisation problem

To solve the functional equation (30), it is convenient to use an iterative procedure. To derive the iteration scheme, the performance index of equation (27) is replaced by:

$$\mathscr{P}_N[\mathbf{x}(t_0)] = \sum_{k=1}^{N} \beta^2[\mathbf{x}^d - \mathbf{x}(t_0+k\tau)]\,\lambda^k \quad . . \quad (33)$$

In other words, the original infinite-step decision process is converted into a finite-step decision process, where N is the number of steps. By proceeding exactly as in the derivation of equation (30), it can be shown that the successive optimal performance indexes $\mathscr{P}^{\circ}_N$ are connected by the functional recurrence relations:

$$\mathscr{P}^{\circ}_1[\mathbf{x}(t_0)] = \underset{\mathbf{F}(t_0)}{\text{Min}}\ \lambda\beta^2[\mathbf{x}^d - \mathbf{x}(t_0+\tau)]$$

$$\mathscr{P}^{\circ}_{N+1}[\mathbf{x}(t_0)] = \underset{\mathbf{F}(t_0)}{\text{Min}}\ \lambda\{\beta^2[\mathbf{x}^d - \mathbf{x}(t_0+\tau)] + \mathscr{P}^{\circ}_N[\mathbf{x}(t_0+\tau)]\} \quad . \quad (34)$$

In each stage of the iteration of equation (34), the optimal control signal $\mathbf{F}^{\circ}(t_0)$ is determined as some function $\mathbf{h}_N$ of $\mathbf{x}(t_0)$. As $N \to \infty$, it can be shown under mild restrictions that $\mathscr{P}^{\circ}_N$ converges to $\mathscr{P}^{\circ}$, and $\mathbf{h}_N$ converges to $\mathbf{h}$.

Computation of a Sequence of Optimal Control Vectors for a Linear Process

The ease of carrying out the iterations of equation (34) is determined largely by the complexity of the dynamics of the reaction and by the limits imposed on the control variables. To illustrate these computations concretely, consider the very special (but practically important) linear case where the reaction dynamics are governed by an ordinary linear differential equation with constant coefficients and there are no amplitude constraints on the control variables.

When the control (or forcing) term $\mathbf{b}(t)$ is constant during the intervals between sampling instants, equation (20) takes the simple form:

$$\mathbf{a}(t_0+\tau) = \mathbf{\Phi}(\tau)\,\mathbf{a}(t_0) + \mathbf{\Delta}(\tau)\,\mathbf{b}(t_0) \quad . \quad . \quad (35)$$

where

$$\mathbf{\Delta}(\tau) = \int_0^{\tau} \mathbf{\Phi}(\tau-\sigma)\,\mathbf{N}\,d\sigma \quad . \quad . \quad . \quad (36)$$

Equation (35) is the explicit form of equation (13) in the linear constant-coefficient case.

Explicit solution of minimisation problem

If β is given by equation (23) and $\mathscr{P}$ by equation (27) it can be shown by induction that the optimal performance index has the form:

$$\mathscr{P}^{\circ}_N[\mathbf{a}(t_0)] = \mathbf{a}'(t_0)\,\mathbf{P}_N\,\mathbf{a}(t_0) - 2\mathbf{a}'(t_0)\,\mathbf{R}_N\,\mathbf{a}^d + \mathbf{a}^d\,\mathbf{S}_N\,\mathbf{a}^d \qquad (N \geqslant 0) \quad . \quad (37)$$

with $\mathbf{P}_N$, $\mathbf{R}_N$, $\mathbf{S}_N$ being $(n \times n)$, $(n \times m)$, and $(m \times m)$matrices respectively, and $\mathbf{P}_0 = \mathbf{R}_0 = \mathbf{S}_0 = 0$

For simplicity the arguments of $\mathbf{\Phi}(\tau)$ and $\mathbf{\Delta}(\tau)$ are dropped. Using equations (34) and (35) the derivative of the scalar $\mathscr{P}_{N+1}[\mathbf{a}(t_0)]$ with respect to the vector $\mathbf{b}(t_0)$ can be calculated. This is a vector [with components $\partial\mathscr{P}_{N+1}/\partial b_i(t_0)$], which is given by:

$$\partial\mathscr{P}_{N+1}/\partial\mathbf{b}(t_0) = -2\{\mathbf{\Delta}(\mathbf{R}_N+\mathbf{Q})\,\mathbf{a}^d - (\mathbf{P}_N+\mathbf{Q})[\mathbf{\Phi}\mathbf{a}(t_0) + \mathbf{\Delta}\mathbf{b}(t_0)]\} \quad . \quad (38)$$
$$(N \geqslant 0)$$

$\mathscr{P}_{N+1}[\mathbf{a}(t_0)]$ is evidently a quadratic function of each of the incremental control variables $b_i(t_0)$. It follows that $\mathscr{P}_{N+1}$ has a single extreme value [which may be a minimum or a maximum, depending on the value of $\mathbf{a}(t_0)$] at that value of $\mathbf{b}(t_0)$ which makes the right-hand side of equation (38) zero. It can be shown that the extreme value is a minimum for every $\mathbf{a}(t_0)$. Hence, the optimal value of $\mathbf{b}(t_0)$ is found by setting equation (38) equal to zero which yields, at each step of the iteration, the following expressions for $\mathbf{b}^{\circ}(t_0)$ and $\mathscr{P}^{\circ}_{N+1}$:

$$\mathbf{b}^{\circ}(t_0) = \mathbf{A}_N\,\mathbf{a}^d - \mathbf{B}_N\,\mathbf{a}(t_0) \quad . \quad . \quad (39)$$

where

$$\left.\begin{aligned}
\mathbf{A}_N &= [\mathbf{\Delta}'(\mathbf{P}_N+\mathbf{Q})\mathbf{\Delta}]^{-1}\,\mathbf{\Delta}'(\mathbf{P}_N+\mathbf{Q})\,\mathbf{\Phi} \\
\mathbf{B}_N &= [\mathbf{\Delta}'(\mathbf{P}_N+\mathbf{Q})\mathbf{\Delta}]^{-1}\,\mathbf{\Delta}'(\mathbf{R}_N+\mathbf{Q}) \\
\mathbf{P}_{N+1} &= \boldsymbol{\lambda}(\mathbf{\Phi}-\mathbf{\Delta}\mathbf{A}_N)'\,(\mathbf{P}_N+\mathbf{Q})\,\mathbf{\Phi} \\
\mathbf{R}_{N+1} &= \boldsymbol{\lambda}(\mathbf{\Phi}-\mathbf{\Delta}\mathbf{A}_N)'\,(\mathbf{R}_N+\mathbf{Q}) \\
\mathbf{S}_{N+1} &= \boldsymbol{\lambda}[\mathbf{S}_N-\mathbf{B}'_N\mathbf{\Delta}'\,(\mathbf{P}_N+\mathbf{Q})\,\mathbf{\Delta}\mathbf{B}_N+\mathbf{Q}]
\end{aligned}\right\} . \qquad (40)$$

The iterations indicated by equations (40) can be readily performed on a digital computer. $\mathbf{S}_N$ need not be computed if only the optimal control vectors are of interest. In the limit $N \to \infty$, and, under mild restrictions, all quantities in equations (40), except $\mathbf{S}_N$, may be shown to converge. The iteration is performed by starting with $\mathbf{P}_0$, $\mathbf{R}_0$, and $\mathbf{S}_0$ and calculating $\mathbf{A}_0$ and $\mathbf{B}_0$. From equation (40) $\mathbf{P}_1$, $\mathbf{R}_1$, and $\mathbf{S}_1$ are calculated, followed by $\mathbf{A}_1$, $\mathbf{B}_1$. This cycle is continued until convergence.

Equation (39) indicates that, in the linear case, the optimal control variables are linear functions of the actual and desired states of the reactor.

Since the control variables are linear functions of the state variables, it follows that under closed-loop control the reactor is a linear dynamic system. The only possible type of limiting behaviour in such systems as $t \to \infty$ is for the state $\mathbf{x}(t)$ to converge to an equilibrium state $\mathbf{x}^*$. Since dynamic optimisation includes static optimisation, it follows at once that, in the linear case, the states of a dynamically optimised system tend asymptotically to the same equilibrium state $\mathbf{x}^{*\circ}$ which is obtained under static optimisation.

Numerical Example

The steady state characteristics of the chemical reaction process chosen for this example will lead to the form of Fig. 3.

The problem

A stable equilibrium process is disturbed by perturbing one of the inputs. The process must then be returned to its original stable condition so as to minimise a performance index of the type of equation (27). This is to be accomplished by varying one of the process flowrates in a manner determined by the digital computer.

Solution

A first-order non-isothermal reaction, $A \xrightarrow{k} B$, takes place in the stirred reactor of Fig. 2. In terms of the previous nomenclature the dynamic equations become

$$\frac{dx_1}{dt} = \frac{F_1}{V}\,x_{01} - \frac{F_0}{V}\,x_1 - kx_1 \quad . \quad . \quad . \quad . \quad . \qquad (41)$$

$$\frac{dT}{dt} = \frac{F_1}{V}\,T_{01} - \frac{F_0}{V}\,T - kx_1\,\frac{\Delta H}{\rho c} - \frac{U}{V\rho c}\,(T - T') \quad . \qquad (42)$$

where

$$k = a_A \exp\,[-E/RT] \quad . \quad . \quad . \qquad (43)$$

It has been assumed that one stream containing component A and a cooling water stream are the only inputs to the reactor. F_0 is thus equal to F_1.

Equation (42) can be rearranged by using a heat balance around the cooling coil. This leads to

$$T_{ave} = \frac{UT + 2\,F_2\,\rho c\,T_{ce}}{U + 2\,F_2\,\rho c}$$

where T_{ce} = cooling water inlet temperature, F_2 = cooling water flow rate and, ρ and c are the properties of the cooling water assumed identical to the reaction stream. Calling $K = 2\,\rho c/U$, equation (42) becomes:

$$\frac{dT}{dt} = \frac{F_1}{V}\,T_{01} - \frac{F_0}{V}\,T - kx_1\,\frac{\Delta H}{\rho c} - \frac{UKF_2}{V\,\rho c(1 + KF_2)}\,(T - T_{ce}) \quad . \qquad (44)$$

As Amundson has already performed some calculations on a system of this type, the same numerical values can be used.[13] These are given in Table I. The steady-state equilibrium points of Fig. 3 are given in Table II.

TABLE I.—*Numerical Values Used in Calculation.*

$k = 7{\cdot}86 \times 10^{12} \exp\,[-\,22\,500/RT]$

$T_{ave} = 305°\ \mathrm{K}$

$T_{ce} = 287°\ \mathrm{K}$

$x_{01} = 0{\cdot}005$ mole/litre

$T_{01} = 300°\ \mathrm{K}$

$-\Delta H = 10\,000$ cal/mole

$U = 1{\cdot}356$ cal/°C

$V = 2000$

$F_1 = 10\ \mathrm{cm^3/s}$

$F_0 = 10\ \mathrm{cm^3/s}$

$c = 1$ cal/°C

$\rho = 1\ \mathrm{g/cm^3}$

$KF_2 = 1{\cdot}47$

TABLE II.—*Equilibrium or Steady-State Points.*

T^*	x_1^*	k at T^*
306	$4{\cdot}42 \times 10^{-3}$	$6{\cdot}6 \times 10^{-4}$
330	$1{\cdot}67 \times 10^{-3}$	$1{\cdot}00 \times 10^{-2}$
330·8	$1{\cdot}57 \times 10^{-3}$	$1{\cdot}09 \times 10^{-2}$

Linearised Dynamic Equations

The dynamic programming procedure will be applied to the linearised form of the nonlinear dynamic equations (41) and (44). Following the procedure described earlier these linearised equations may be written as the matrix equation

$$\frac{d\mathbf{a}}{dt} = \mathbf{Ma} + \mathbf{Nb} \quad . \quad . \quad . \qquad (45)$$

where **M** is now a (2 × 2) matrix, **N**, a 2-element vector, **a**, a 2-element vector and **b**, a 1-element vector. Specifically,

$$\mathbf{M} = \begin{bmatrix} -\left[\dfrac{F_0}{V} + k^*\right] & \dfrac{-\,k^*E}{RT^{*2}} \\[2ex] -\dfrac{\Delta H}{\rho c}\,k^* & -\left[\dfrac{F_0}{V} + \dfrac{\Delta H}{\rho c}\,\dfrac{k^*E\,x_1^*}{RT^{*2}} + \dfrac{UKF_2^*}{V\,\rho c(1 + KF_2^*)}\right] \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} 0 \\[2ex] -\dfrac{(T^* - T_{ce})\,UK}{V\,\rho c(1 + KF_2^*)^2} \end{bmatrix}$$

$$\mathbf{a} = \begin{bmatrix} a \\ d \end{bmatrix}$$

$$\mathbf{b} = [\,e\,]$$

where

$$x_1 = x_1^* + a$$
$$T = T^* + d$$
$$F_2 = F_2^* + e$$

a, d, and e represent the perturbations around the mean values x_1^*, T^*, and F_2^*.

Equation (45) is a non-homogeneous, first-order, matrix differential equation with constant coefficients. The solution is:

$$\mathbf{a}(t) = \mathbf{\Phi}(t - t_0)\,\mathbf{a}(t_0) + \int_{t_0}^{t} \mathbf{\Phi}(t - \mu)\,\mathbf{N}\mathbf{b}(\mu)\,\mathrm{d}\mu \quad . \quad (46)$$

where the first term is the homogeneous solution and the second term the particular solution. The matrix $\mathbf{\Phi}(t)$ is the transition matrix for the dynamic system of equation (45) and is given by equation (21). Since $\mathbf{b}(t)$ will be assumed constant between sampling instants (this being merely a matter of convenience), equation (46) can be written in the simpler form

$$\mathbf{a}(t_0 + \tau) = \mathbf{\Phi}(\tau)\,\mathbf{a}(t_0) + \mathbf{\Delta}(\tau)\,\mathbf{b}(t_0) \quad . \quad (47)$$

where $\mathbf{\Delta}(\tau)$ is given by equation (36). Equation (47) is the basic difference equation for the linearised process.

Using the data of the numerical solution discussed earlier, the **M** and **N** matrices have the following numerical elements:

$$\mathbf{M} = \begin{bmatrix} -0{\cdot}0159 & -1{\cdot}77 \times 10^{-6} \\ +109 & +0{\cdot}0123 \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} 0 \\ -0{\cdot}715 \times 10^{-2} \end{bmatrix}$$

In addition, the use of equation (20) with 20 terms in the Taylor series determines the values of the transition matrix, $\mathbf{\Phi}(\tau)$, and of $\mathbf{\Delta}(\tau)$.

$$\mathbf{\Phi}(\tau) = \begin{bmatrix} 1{\cdot}152\,607\,7 & 0{\cdot}176\,456 \\ -0{\cdot}101\,686\,2 & 0{\cdot}881\,410\,5 \end{bmatrix}$$

$$\mathbf{\Delta}(\tau) = \begin{bmatrix} 0{\cdot}877\,221\,4 \\ 9{\cdot}410\,221\,7 \end{bmatrix}$$

The iterative calculations discussed in connection with equation (34) then lead to:

$$\mathrm{e}(\mu + k\tau) = a(k\tau) + d(k\tau) \qquad 0 < \mu < \tau \quad . \quad (48)$$

In this oversimplified example, the digital computer would calculate $\mathrm{e}(\mu + k\tau)$ through equation (48) at

$$t = 0,\ \tau,\ 2\tau,\ \ldots\ k\tau,\ \ldots$$

[an analogue computer could also be used to compute equation (48)]. With this definition of the control variable, the response of the system starting with a known set of initial conditions was determined using an IBM 701 digital computer. One set of initial conditions was chosen:

$$a(o) = 0{\cdot}000\,57 \text{ and } d(o) = 290{\cdot}8$$

Results of the calculations

Some of the results of these calculations are shown in Figs. 4 and 5. Actually the measure of the deviation from the equilibrium value corresponding to the highest temperature point of Table II is shown on these graphs. Thus, in Fig. 4 the system is disturbed by $-0{\cdot}001\,00$ mole/litre. It is obvious that the larger value of the weighting factor λ leads to a faster response of the system towards the equilibrium state. These calculations require about 2–5 seconds of total time on the IBM 701. Additional, more elaborate calculations of this type may be found in recent papers of Kalman and Koepcke.[5, 6]

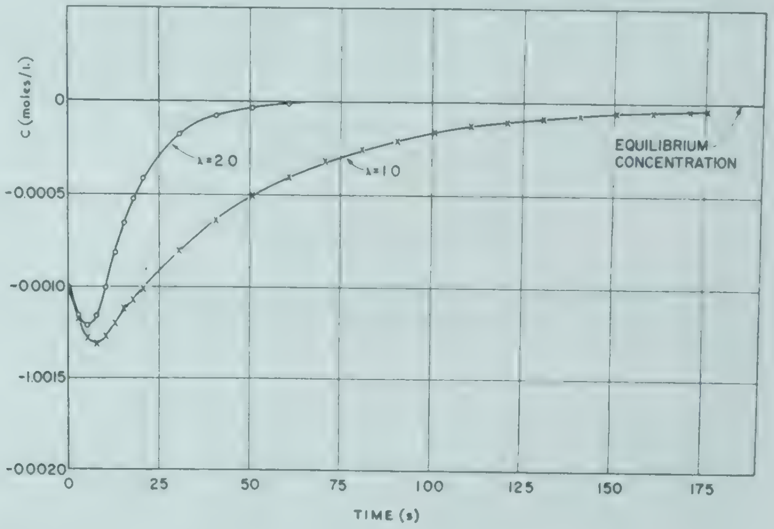

Fig. 4.—A graph of the return of the system concentration to the equilibrium point with two weighting factors, $\lambda = 1$ and $\lambda = 2$. Initial disturbed condition is a(o)=0·000 57 mole/litre and d(o)=290·8° K

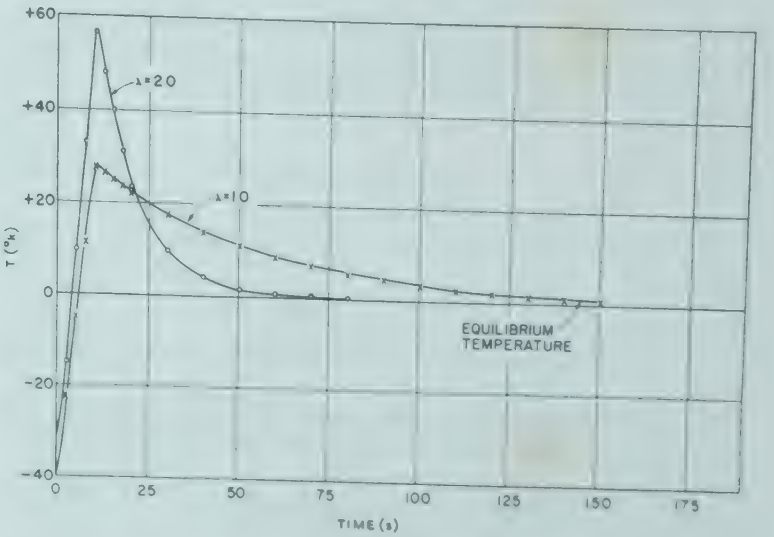

Fig. 5.—A graph of the return of the system temperature to the equilibrium point with two weighting factors, $\lambda = 1$ and $\lambda = 2$. Initial disturbed condition is a(o)=0·000 57 mole/litre and d(o)=290·8° K

Appendices

Matrix-vector operations and quadratic forms

A vector is a set of *n* numbers (*n*-tuple) which is usually written in the form of the array

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \quad . \quad . \quad . \quad (A.1)$$

A matrix is a rectangular array of numbers.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & & \dots & \\ & & \dots & \\ & & \dots & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \quad . \quad (A.2)$$

As shown the matrix has *m* rows and *n* columns. Each element of the matrix **A** is located at the "intersection" of the row and a column. The elements of **A** are denoted by a_{ij}; the first subscript refers to the row and the second to the column in which the element is located. A vector can always be regarded as a matrix with *n* rows and 1 column. The $(n \times n)$ unit matrix **I** is a matrix of *n* rows and columns such that all elements are zero except on the main diagonal; *i.e.*, $a_{11}, a_{22}, \dots, a_{nn}$ are all equal to unity. The transpose **A′** of a matrix **A** is given by interchanging the rows and columns of **A**:

$$\mathbf{A}' = \begin{bmatrix} a_{11} & a_{21} & \dots & a_{m1} \\ a_{12} & a_{22} & \dots & a_{m2} \\ & & \dots & \\ & & \dots & \\ & & \dots & \\ a_{1n} & a_{2n} & \dots & a_{mn} \end{bmatrix} \quad . \quad (A.3)$$

The transpose **x′** of a vector is seen to be a matrix (or vector) with one row and *n* columns.

The product of an $(m \times n)$ matrix **A** by an $(n \times q)$ matrix **B** is an $(m \times q)$ matrix **C** whose elements are given by

$$c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj} \qquad \begin{array}{l} i = 1, 2, \quad \dots \quad , m \\ j = 1, 2, \quad \dots \quad , q \end{array} \quad . \quad (A.4)$$

This product is defined only if the number of columns of **A** equals the number of rows of **B**, and the order of multiplication must be observed since, in general, $\mathbf{AB} \neq \mathbf{BA}$.

The product of a transposed vector with another vector is a (1×1) matrix, *i.e.*, a scalar given by:

$$\mathbf{x}'\mathbf{y} = \sum_{i=1}^{n} x_i y_i = \mathbf{y}'\mathbf{x} \quad . \quad . \quad (A.5)$$

This is called the scalar product of **x** and **y**.

The inverse, $\mathbf{A}^{-1}$, of a matrix is defined by:

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \quad . \quad . \quad (A.6)$$

The inverse exists if and only if $\det\mathbf{A} \neq 0$ where $\det\mathbf{A}$ is the determinant formed from the array (A.2).

For our present purposes the usefulness of matrices is that they afford a convenient shorthand notation for dealing with systems of linear algebraic equations. Thus the system:

$$\begin{array}{llll} a_{11}x_1 + a_{12}x_2 + & \dots & + a_{1n}x_n & = b_1 \\ a_{21}x_1 + a_{22}x_2 + & \dots & + a_{2n}x_n & = b_2 \\ \dots & \dots & \dots & \\ a_{m1}x_1 + a_{m2}x_2 + & \dots & + a_{mn}x_n & = b_n \end{array} \quad . \quad (A.7)$$

can be written concisely as:

$$\mathbf{Ax} = \mathbf{b} \quad . \quad . \quad . \quad (A.8)$$

using the previous definitions.

A quadratic form associated with an $(n \times n)$ matrix **A** is a scalar function of the vector **x** given by:

$$Q(\mathbf{x}, \mathbf{A}) = \mathbf{x}'\,\mathbf{A}\mathbf{x} = \sum_{i,j=1}^{n} x_i a_{ij} x_j \quad . \quad (A.9)$$

In equation (A.9), **A** may be replaced by its symmetric part, $\mathbf{A}_s = \frac{1}{2}(\mathbf{A} + \mathbf{A}')$, without changing the value of $Q(\mathbf{x})$. When the summation indicated by equation (A.9) is written out:

$$Q(\mathbf{x}, \mathbf{A}) = a_{11}x^2 + a_{12}x_1x_2 + \quad \dots \quad + a_{nn}x_n^2$$

it is seen that $Q(\mathbf{x})$ is the weighted sum of all possible cross-products of the components of the vector **x**.

The quadratic form is positive semi definite if $Q(\mathbf{x}, \mathbf{A}) \geqslant 0$ for all **x**. If, in addition, $Q(\mathbf{x}, \mathbf{A}) = 0$ implies $\mathbf{x} = 0$, then the quadratic form is positive definite. For example:

$$Q(\mathbf{x}, \mathbf{I}) = \mathbf{x}'\mathbf{x} = \sum_{i=1}^{n} x_i^2 \quad . \quad . \quad (A.10)$$

is positive definite. Evidently equation (A.10) is always positive, and zero only when each component of **x** is zero. One says, with a slight abuse of language, that the matrix **A** is positive (semi) definite if $Q(\mathbf{x}, \mathbf{A})$ is positive (semi) definite.

The matrix **A′A** is always positive semi definite; it is definite if $\det\mathbf{A} \neq 0$. In general, **A** is positive definite (semi-definite) if all the principal minors of **A** are positive (non-negative).

Solution of the linear vector differential equation with constant coefficients

Consider the vector differential equation

$$\frac{d\mathbf{x}}{dt} = \mathbf{Fx} + \mathbf{Dm}(t) \quad . \quad . \quad . \quad (A.11)$$

where **F** is a constant $(n \times n)$ matrix, **D** is a constant $(n \times r)$ matrix, **x** is an *n*-vector (state vector) and $\mathbf{m}(t)$ is an *r*-vector (forcing function). Now consider equation (A.11) with the forcing terms $\mathbf{m}(t)$ identically zero, *i.e.*, the homogeneous equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{Fx} \quad . \quad . \quad . \quad (A.12)$$

When **x** is a scalar quantity then **F** is also a scalar and equation (A.12) can be integrated directly to yield:

$$x(t) = [\exp F(t - t_0)] \,.\, x(t_0) \quad . \quad . \quad (A.13)$$

This equation shows explicitly how the state of the system represented by equation (A.12) at any time t depends on the initial state $x(t_0)$.

The form of the solution, equation (A.13), remains valid even when the state is a vector and $\mathbf{F}$ is a matrix. To show this, it is first necessary to define the exponential function of the matrix $\mathbf{F}t$. This is done by means of the Taylor series:

$$\exp(\mathbf{F}t) = \sum_{i=0}^{\infty} (\mathbf{F}t)^i/i! \qquad . \qquad . \qquad (A.14)$$

Differentiating this function, we obtain:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\exp(\mathbf{F}t) &= \lim_{h\to 0}\frac{1}{h}\{\exp[\mathbf{F}(t+h)] - \exp(\mathbf{F}t)\}\\ &= \lim_{h\to 0}\frac{1}{h}\exp(\mathbf{F}t)[\exp(\mathbf{F}h) - 1]\\ &= \exp(\mathbf{F}t)\,.\,\mathbf{F} = \mathbf{F}\,.\,\exp(\mathbf{F}t) \qquad . \qquad (A.15)\end{aligned}$$

This shows that:

$$\mathbf{x}(t) = [\exp\mathbf{F}(t-t_0)]\,.\,\mathbf{x}(t_0) \qquad . \qquad . \qquad (A.16)$$

is a solution of equation (A.12). The authors call

$$\mathbf{\Phi}(t-t_0) = \exp\mathbf{F}(t-t_0) \qquad . \qquad . \qquad (A.17)$$

the transition matrix of the system represented by equation (A. 12).

Note: When $\mathbf{F}$ is a function of time, the *scalar* equation (A.12) can be integrated directly to yield

$$x(t) = [\exp\int_{t_0}^{t}\mathrm{F}(t)\,\mathrm{d}t]\,x(t_0)$$

When equation (A.12) is a vector equation, the concept of the transition matrix still exists, *i.e.*, the solutions of equation (A.12) can be written in the form:

$$\mathbf{x}(t) = \mathbf{\Phi}(t;\ t_0)\,\mathbf{x}(t_0)$$

But now in general

$$\mathbf{\Phi}(t;\ t_0) \neq \exp\int_{t_0}^{t}\mathbf{F}(t)\,\mathrm{d}t$$

unless $\mathbf{F}(t)$ commutes with

$$\int_{t_0}^{t}\mathbf{F}(t)\,\mathrm{d}t.$$

From the definitions, equations (A.15)–(A.17), the following important properties of the transition matrix may be obtained immediately:

$$\left.\begin{aligned}\mathbf{\Phi}^{-1}(t) &= \mathbf{\Phi}(-t)\\ \mathbf{\Phi}(0) &= I\\ \mathbf{\Phi}(t_1)\,\mathbf{\Phi}(t_2) &= \mathbf{\Phi}(t_1+t_2)\end{aligned}\right\} \qquad . \qquad . \qquad (A.18)$$

Substitute equation (A.16) into the original differential equation, with $t = 0$. Then

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{\Phi}(t)\,\mathbf{x}(0)\\ &= \dot{\mathbf{\Phi}}(t)\,\mathbf{x}(0)\\ &= \mathbf{F}\mathbf{x}(t)\\ &= \mathbf{F}\mathbf{\Phi}(t)\,\mathbf{x}(0)\end{aligned}$$

Since this holds for any initial state $\mathbf{x}(0)$, one may conclude that

$$\mathbf{\Phi}(t) = \mathbf{F}\mathbf{\Phi}(t) \qquad . \qquad . \qquad . \qquad (A.19)$$

In other words, the transition matrix satisfies the differential equation (A.12). Using equation (A.18), equation (A.12) can be written in the form:

$$\mathbf{F} = \dot{\mathbf{\Phi}}(t)\,\mathbf{\Phi}^{-1}(t) = \dot{\mathbf{\Phi}}(t)\,\mathbf{\Phi}(-t) \qquad . \qquad (A.20)$$

Differentiating $\dot{\mathbf{\Phi}}(t)\,\mathbf{\Phi}(-t) = \mathbf{I}$, shows that, for any value of t:

$$\mathbf{F} = \dot{\mathbf{\Phi}}(t)\,\mathbf{\Phi}(-t) = -\mathbf{\Phi}(t)\,\dot{\mathbf{\Phi}}(-t) \qquad . \qquad (A.21)$$

The solution of the nonhomogeneous equation (A.11) can be obtained as follows. By equation (A.21):

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{D}\mathbf{m} = -\mathbf{\Phi}(t)\,\dot{\mathbf{\Phi}}(-t)\,\mathbf{x} + \mathbf{D}\mathbf{m}(t)$$

Multiplying by $\mathbf{\Phi}(-t)$ and transposing:

$$\mathbf{\Phi}(-t)\,\dot{\mathbf{x}} + \dot{\mathbf{\Phi}}(-t)\,\mathbf{x} = \frac{\mathrm{d}}{\mathrm{d}t}[\mathbf{\Phi}(-t)\,\mathbf{x}] = \mathbf{\Phi}(-t)\,\mathbf{D}\mathbf{m}(t)$$

This equation can be integrated directly to yield:

$$\mathbf{\Phi}(-t)\,\mathbf{x}(t) = \int_{t_0}^{t}\mathbf{\Phi}(-\mu)\,\mathbf{D}\mathbf{m}(\mu)\,\mathrm{d}\mu + \mathbf{\Phi}(-t_0)\,\mathbf{x}(t_0)$$

Hence, using equation (A.18):

$$\mathbf{x}(t) = \mathbf{\Phi}(t-t_0)\,\mathbf{x}(t_0) + \int_{t_0}^{t}\mathbf{\Phi}(t-\mu)\,\mathbf{D}\mathbf{m}(\mu)\,\mathrm{d}\mu \quad . \quad (A.22)$$

This is the generalisation of the well-known convolution integral formula.

An important aspect of the derivation given here is that the solutions can be obtained without first solving the characteristic equation $\det[\mathbf{F} - \lambda\mathbf{I}] = 0$, which is the case, for instance, with the Laplace-transform method. It is sufficient to compute the transition matrix, which can be done numerically to any desired degree of accuracy by means of the infinite series of equation (A.14). For large-scale systems, this is perhaps the only practical numerical method of obtaining such solutions.

Symbols Used

A, B = reactants.
c = heat capacity.
E_{AB} = activation energy of reaction.
F_0 = total effluent volumetric liquid flow rate.
$F_1, F_2, \ldots, F_m$ = inputs to reactor (volumetric liquid flow rates).
F_m = cooling water flow rate.
$\mathbf{g}$ = a vector function which represents the effect of external influences or forcing functions; those particular arguments $F_1, F_2, \ldots, F_m;\ x_{01}, x_{02}, \ldots, x_{0i}$ of the function $\mathbf{g}$ which can be changed at will are the control variables of the process.
ΔH_{AB} = heat of reaction.
$k_{AB}(T)$ = reaction rate coefficient.
m = number of control variables.
n = number of state variables.
$\mathbf{M}$ = constant $n \times n$ matrix.
$\mathbf{N}$ = constant $n \times m$ matrix.

$\mathscr{P}$ = performance index.

$$\left.\begin{array}{ll}\mathbf{P}_N & = n \times n \text{ matrix}\\ \mathbf{R}_N & = n \times m \text{ matrix}\\ \mathbf{S}_N & = m \times m \text{ matrix}\end{array}\right\} \text{matrices which appear in dynamic programming formulation of the optimal control problem.}$$

$\mathbf{Q}$ = positive semi-definite matrix.

R = gas constant.

T_m = outgoing cooling water temperature.

t = time; t_0 = initial time.

$T_{01}, T_{02}, \ldots T_{0m}$ = temperatures associated with flow rates $F_1, F_2, \ldots, F_m$.

$T = x_n$ = effluent temperature.

T_{ave} = average temperature of cooling water.

U = heat transfer coefficient for the cooling coil.

V = reaction volume of stirred tank reactor.

$\mathbf{x}$ = vector denoting the state of the system; the components of $\mathbf{x}$ are called the state variables.

$x_{01}, x_{02}, \ldots, x_{0l}$ = concentrations of input streams.

x_n^d = desired setting of the nth state variable.

α_{ab} = a constant.

β = distance function in state space.

$'$ = transpose of the matrix (or vector) immediately to the left.

λ = exp $(\alpha\tau)$, where α is a constant.

$\mathbf{\Delta}(\tau)$ = matrix denoting effect of a change in the setting of the control variables on the state of the system.

ρ = density of effluent solution.

τ = sampling period (time interval between sampling instants).

$\boldsymbol{\varphi}$ = infinitesimal state-transition function; the components of $\boldsymbol{\varphi}$ are denoted by ϕ_i.

$\mathbf{\Phi}(t)$ = transition matrix.

$(\ )^*$ = steady state value or equilibrium state.

$(\ )^\circ$ = optimal value.

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CORRELATION ANALYSIS OF A HEAT EXCHANGER

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SUMMARY

Direct measurements of the transfer functions between liquid flow, steam supply, and output temperature in a simple heat exchanger are difficult to interpret since turbulent flow gives large random fluctuations in the readings. However, a statistical reduction of the readings makes a straightforward determination of the transfer functions possible. Statistical analysis requires randomly fluctuating inputs so that most measurements may be made under normal running conditions.

A statistical analysis of typical heat exchanger records is given in the paper. It indicates two ways of reducing the readings. In the first, correlation functions are used which on further calculation yield the time response; in the second a spectral analysis of the data leads to the frequency response of the unit.

Introduction

In order to design or improve the automatic control of a physical system it is necessary to know the transfer functions relating the input to the output quantities. In many cases the random variations of these input and output quantities, monitored during normal operation, prove to be the most convenient source of data. These variations form statistical time series, and the mathematical relations connecting such series have been investigated in many other applications, including economics, sociology, and meteorology.

When the input and output time series have simple forms, such as a sine wave, direct measurements on the waveforms yield quantities which are simply related to the transfer function, *e.g.* gain as the ratio of two amplitudes. However, with a random series it is more useful to consider the statistical parameters of the series to be linked by the transfer function. Thus, the first step in calculating the transfer function is a statistical reduction of the data. If the statistical parameters obtained are independent of time, the series are stationary and the theory is much simplified. Practical steady-state operation usually produces stationary series, and only this condition is considered here.

For engineering purposes the most convenient form of mathematical relations are those given by Wiener.[1] Wiener formulates his results in terms of the statistical correlation functions, and these can be related by the impulse weighting functions or frequency response functions which are already in engineering use. Unfortunately, his work does not cover all the needs of a practical problem. Practical problems involve considerations at three levels:

(*a*) Stochastic theory, that is the mathematical relations between quantities measured in ideal experiments of infinite duration.

(*b*) Sampling theory, that is the relations between quantities measured in practical experiments of finite duration. This involves the evaluation of statistical error limits on all derived parameters.

(*c*) Organisation of the measurement and computation. The amount of computation required is considerable, and an automatic computer is virtually essential.

Previous papers in the control engineering field have concentrated on point (*a*) and very few actual applications have been reported. This paper describes in detail the evaluation of the two transfer functions which characterise a simple heat exchanger. The signal level used in the measurements is substantially lower than any previously reported.

The practical significance of the mathematical constraints under which the stochastic theory is developed is pointed out. Serious deficiencies in the statistical sampling theory are noted, and an *ad hoc* procedure for overcoming one of them, is given. Two equivalent sets of parameters were completely worked out, the impulse weighting function in the time domain, and the frequency and phase response in the frequency domain. The merits of each method are discussed and it is shown that misleading results can be obtained by an unwise choice of method.

Stochastic Theory

In effect, the statistical method of analysis is the calculation of the correlation between input and output time series. The correlation can be calculated in either time or frequency terms, leading to the impulse weighting function or the frequency response respectively. Difficulty arises when there is a third unknown time series adding to, and corrupting, the output. This occurs in many practical problems. It is then easier to discover a relation between the two time series if some additional mathematical criterion is supplied to define the solution. The model used in this problem is similar to that used by Wiener in determining a predicting filter which would give a "least squares" prediction error. If Wiener's mathematical derivation is followed, then the transfer function determined is optimum in the same least squares sense.

Time domain theory

For non-feedback systems the following model is assumed. A stationary time series $x(t)$ is passed into a linear filter, with impulse weighting function $h(t)$. Somewhere, a corrupting noise series is added which contributes $n(t)$ to the output

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y(t). Thus the output is given by

$$y(t) = n(t) + \int_0^\infty h(u) \, . \, x(t-u) \, . \, du \qquad . \qquad (1)$$

In equation (1), x(t) and y(t) are known but n(t) and h(t) are not. In order to solve, it is assumed that the best form of h(t) is the one which makes the mean square value of n(t) a minimum, subject to h(t) being physically realisable, *i.e.*

$$h(t) = 0 \text{ for } t < 0$$

This assumption leads to an integral equation for h(t)

$$\varphi_{xy}(T) = \int_0^\infty h(u) \, \varphi_{xx}(T-u) \, du \qquad T \geqslant 0 \qquad . \qquad (2)$$

φ_{xx} is the auto-correlation function defined by

$$\varphi_{xx}(T) = \frac{\text{average } [x(t) - \mu_x][x(t+T) - \mu_x]}{\sigma^2_x} \qquad . \qquad (3)$$

Similarly

$$\varphi_{xy}(T) = \frac{\text{average } [x(t) - \mu_x][y(t+T) - \mu_y]}{\sigma_x \, \sigma_y} \qquad . \qquad (4)$$

The constraint condition $T \geqslant 0$ in equation (2) should be noted as this is essential in obtaining numerical solutions from experimental records.

An integral equation similar to equation (2) can be derived more directly from equation (1) by multiplying by $x(t + T)$ and then averaging (using the method of Goodman and Reswick).[2] The noise term, n(t), disappears because it is assumed to be uncorrelated with the input x(t). However, the constraint condition, $T \geqslant 0$, no longer appears, and a value of h(t) must be found which satisfied the equation for all values of T.

The second method will only work properly with perfectly accurate correlation functions, *i.e.* those derived from experiments of infinite duration, whereas the first method gives an optimum answer under true experimental conditions, and is clearly preferred. In addition it is directly helpful in the method used for solving for h(t) described below.

These formulae cannot be applied directly in a feedback system because the loop returns each signal upon itself. This causes the unwanted disturbances to correlate with themselves at the output, and produce a spurious component in the cross correlation between input and output. However, Goodman and Reswick[2] have shown that the spurious and main part of the cross correlation occur in different ranges of T. This makes it possible to attempt a separation of the effects. For a single loop system defined by Fig. 1 the equation is:

$$\varphi_{ec}(T) = -\int_0^\infty b(u) \, \varphi_{nn}(T+u) du + \int_0^\infty h(u) \, \varphi_{ee}(T-u) \, du \qquad . \qquad (5)$$

where b(t) is the impulse weighting function relating n(t) to e(t).

In this problem e(t), (tc) are known, but n(t), h(t) are not. It will be seen that the first integral involves $(T + u)$ and the second $(T - u)$. Thus the first acts mainly in the region of negative values of T, and the second in the region of positive values. With this separation it is possible to guess a form for $\varphi_{nn}(t)$ and h(t) which will be consistent over the whole range of $\varphi_{ec}(T)$, and, hence, is a good solution for h(t). In the problem considered here, open loop measurements gave a good approximation to $\varphi_{nn}(T)$, so simplifying the application of equation (5).

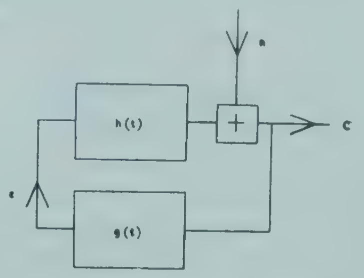

Fig. 1.—*Feed-back system*

Equation (5) does not incorporate least squares minimisation as an inherent feature, and it would be advantageous if equations for a feedback system could be derived which would incorporate this feature.

Frequency domain theory

Wiener has shown that the above equations can be expressed in terms of a Fourier analysis of the data, that is, in frequency terms.

Equation (2) then becomes:

$$\Phi_{xy}(\omega) = H(\omega) \, \Phi_{xx}(\omega) \qquad . \qquad . \qquad (6)$$

where

$$\Phi_{xx}(\omega) = \int_{-\infty}^{\infty} e^{-j\omega t} \, \varphi_{xx}(t) \, dt \qquad . \qquad . \qquad (7)$$

is the power density spectrum of the series x(t),
H(ω) is the frequency response of the transfer h(t), and
$\Phi_{xy}(\omega)$ is similarly defined:

The constraint condition appears as a restriction on the location of the poles of H(p), where p is the Laplace variable. In application to feedback systems, no separation of effects is possible along the frequency axis, so the relation equivalent to equation (5) is not used. It is necessary to subtract out the effects of the noise before applying a Fourier transformation to the cross-correlation function.

Sampling Theory

The quantities used in the above equations are those which would be measured in ideal experiments of infinite duration. Quantities computed from experiments of finite duration will be subject to sampling error which may be significant even for observations of long duration. In addition, it does not follow that the stochastic formulae above will give the best approximation to the required transfer function.

At the present time statisticians have only derived a sampling theory for the estimation of the parameters of a single time series. Thus, the error limits for the auto-correlation function and the power density spectrum are known, but the errors in parameters derived from two times series, under reasonably general conditions, are not known. However, Bartlett has given the error in the cross-correlation function for normal series.[3]

A summary of the difficulties has been given by Quenouille.[4] Statisticians have often studied problems in economics where the relations between time series could be of a very general type and have found the difficulties correspondingly considerable. In engineering problems the relations would be of a much more restricted type and it seems reasonable to suppose that more definite results could be obtained.

To the engineer, the biggest lack is that it is impossible to say what length of record must be analysed in order to obtain a specified accuracy in the resultant transfer function.

In this paper error margins are quoted where possible.

The Equipment Studied

The equipment studied was a simple heat exchanger* of a type in common use. Water, entering at a nominal steady flow, is heated by a steam supply. Unavoidable turbulence in the water and steam supplies causes continuous random fluctuations in the temperature. Hitherto, investigators have used step, impulse, or sine wave disturbances to find the transfer function.[5, 6] Because of the irregular fluctuations the applied disturbances needed to be large if they were to be measured accurately.

The transfer of the random fluctuations themselves was studied in the present work. The measurements were made during normal operation so that the transfer function derived is exactly that encountered in practice.

Description of the equipment

Fig. 2 shows the working arrangement of the exchanger. Cold water enters at 20 gal/min from the town mains and is heated in the exchanger by a steam supply maintained at 40 lb/in² g. The steam passes into the exchanger through a valve which is actuated by a controller. This controller monitors the effluent temperature which is regulated at 150° F.

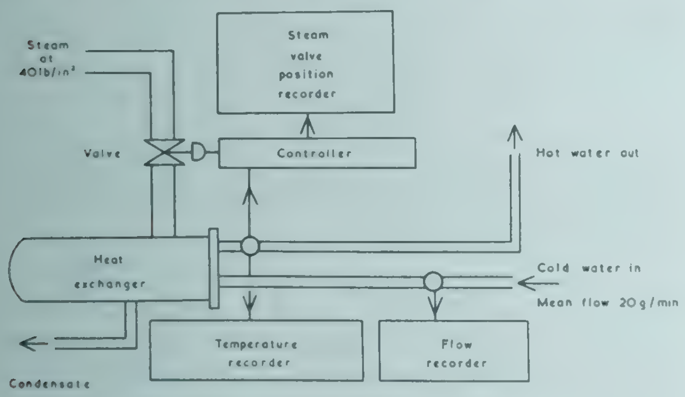

Fig. 2.—*General arrangement of heat exchanger*

The exchanger is of a conventional shell-and-tube design. The shell is 4½ft long and 8 in. in diameter, with one separator plate but no baffles. The water flows in four passes through twenty-two 18-gauge copper U tubes, each ¾ in. O.D. providing 33·5 ft² of heat transfer surface.

Water flowrate, the position of the steam control-valve, and the outlet temperature were measured and recorded.

The water flowrate was measured by an orifice plate and a differential pressure transmitter in the cold water line. The output air-pressure signal was converted to an electrical signal by a specially sensitive strain gauge pressure transducer.

The valve-position was measured by a strain gauge attached to a flexure strip which was fixed between the valve stem and valve-motor frame. The strain-gauge resistance was measured by a standard Foxboro Dynalog instrument.

The temperature of the effluent hot water was measured in the line about 6 in. from the exchanger outlet by a bare bulb type 12A Foxboro temperature transmitter. This is a gas-filled system, consisting of a ⅜ in. O.D. stainless steel bulb, connected by 5 ft of fine-bore capillary tubing to a tiny bellows. The motion of the bellows is opposed through a "force balance" mechanism by a pneumatic pressure of 3–15 lb/in.² in a larger bellows. This air pressure was transduced and recorded in the same way as in the flow measurement.

All recordings were made on commercial pen recorders. Fig. 3 shows a typical record.

Functional model

Figs. 4A and 4B show the assumed theoretical model of the system. All quantities are to be taken as deviations from the nominal steady values, as it is assumed here that only these small fluctuations can be considered to be linearly related.

To some extent this model is an arbitrary simplification. For example, the effects of fluctuations in the water flow are actually distributed throughout the exchanger and not added at one defined point. This model is considered to be a reasonable compromise between mathematical simplicity and engineering reality.

Measurements

Measurements were made in two stages under nominally steady operating conditions. Firstly the controller was disconnected and 10 minute records taken of water flowrate and of the water outlet temperature. From these records the water flow rate/temperature dynamics could be calculated. Secondly with the controller operating, 10 minute records

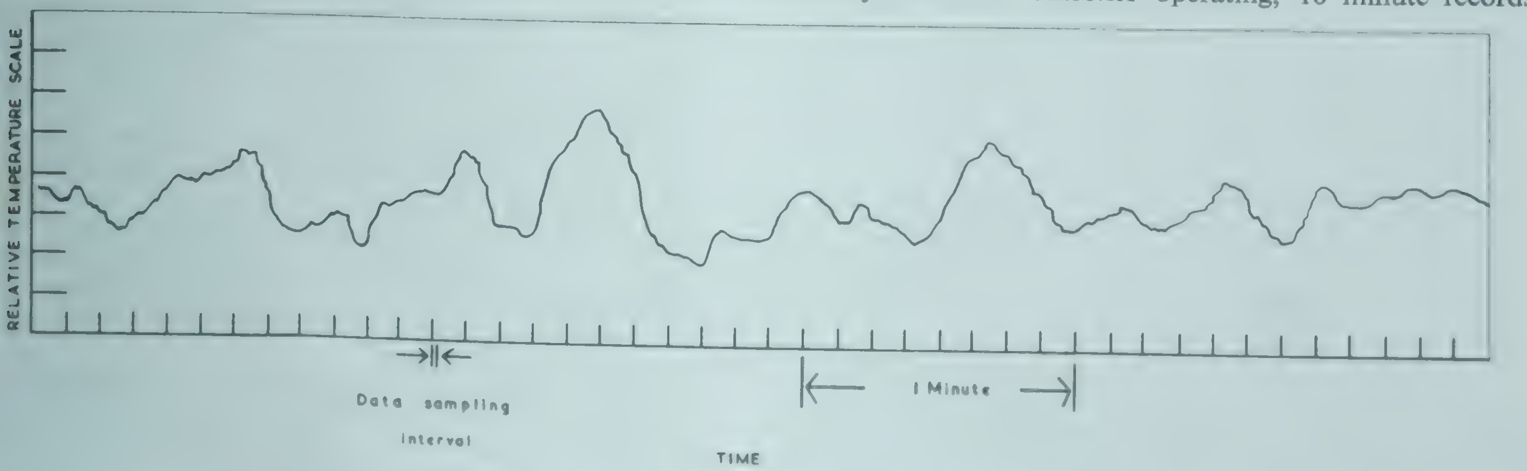

Fig. 3.—*Portion of a typical temperature record*

were taken of the steam-valve position and of the outlet temperature. From these, the controller and valve position/temperature dynamics could be calculated.

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* *Installed in the Research Laboratory of the Foxboro Co., Foxboro, Mass.*

It was found that the R.M.S. value of the flowrate fluctuations was about 0·1 gal/min and the outlet temperature 0·5° F. The order of magnitude of these is much lower than the applied disturbances previously used.

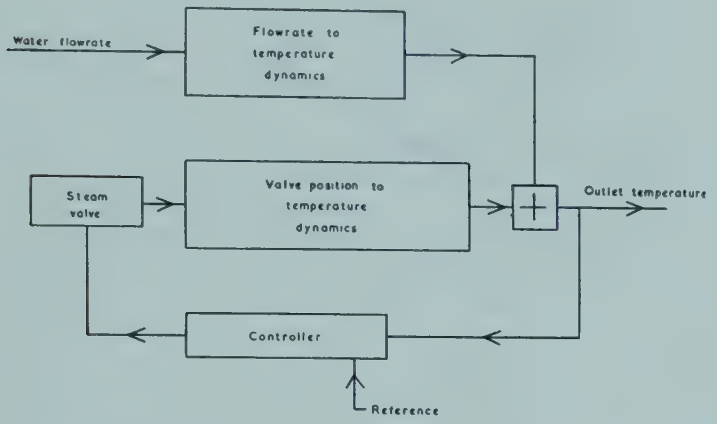
Water flowrate
Flowrate to temperature dynamics
Steam valve
Valve position to temperature dynamics
Outlet temperature
Controller
Reference

Fig. 4A.—*Functional model of exchanger*

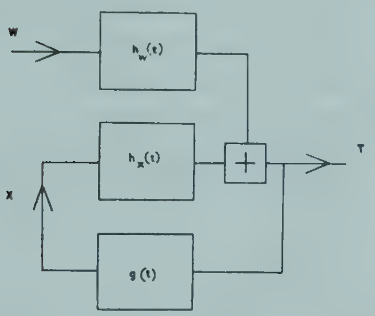

Fig. 4B.—*Theoretical model of exchanger*

Computation of Results

Most of the computation was done by a digital machine. It was therefore necessary to convert the pen records into a form suitable for computer input. The records were read at intervals of 1¼ second, yielding 480 readings from each record. These were punched on to computer cards. Reading was done visually: fortunately there was a check on accuracy in that some auto-correlations had previously been computed on an analogue machine directly from the records. A comparison showed that no serious human error occurred.

The size of the sampling interval is justified by the spectral results obtained later.

Time domain calculation

Equation (2) was first used with the open loop results to calculate the water-flow/temperature weighting function. The correlation functions were computed using the discrete series form for equations (3) and (4)

$$\varphi_{xx} = \frac{\sum_{i=1}^{n-k} (x_i - \mu_{x1})(x_{i+k} - \mu_{x2})}{\left[\sum_{i=1}^{n-k} (x_i - \mu_{x1})^2 \sum_{i=1}^{n-k} (x_{i+k} - \mu_{x2})^2\right]^{\frac{1}{2}}} \quad . \quad (8)$$

and similarly for φ_{xy}. Notice that as a refinement, separate means have been calculated for each section of the lagged series*

Figs. 5 and 6 shows the resulting auto- and cross-correlations out to 40 seconds lag. In order to observe their general characteristics they were actually calculated to 100 seconds lag. Over 40 000 multiplications and additions were required for each auto-correlation function and twice this number for each cross-correlation function. The errors in these functions are discussed below.

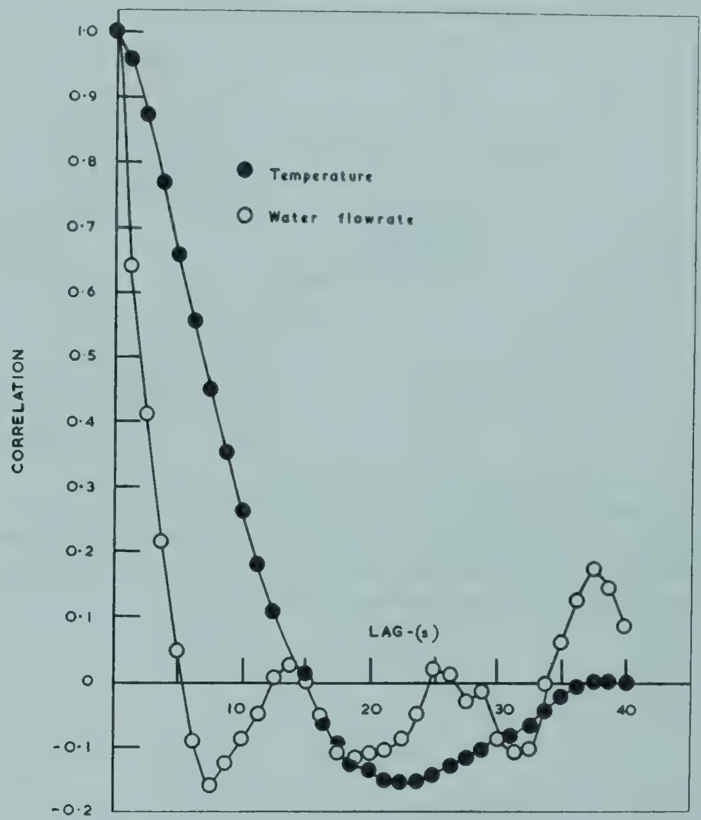

Fig. 5.—*Auto-correlations. Open loop results*

Equation (2) was then solved by a numerical version of the graphical method proposed by Reswick.[7] The auto-correlation was shifted along the positive T axis, multiplied by a constant, and subtracted from the cross-correlation, ordinate by ordinate. This was repeated until the cross-correlation was reduced to zero in the region of $T \geqslant 0$. The value of the shift of T and of the multiplier had to be estimated at each stage, but in practice, because of the triangular peaked shape of the correlation functions, consistent solutions can be obtained quite easily. After each subtraction a single well-defined peak was left, and this was removed at the next stage; this process was continued until the residue was less than 2 or 3% of the original cross-correlation. The calculations were done numerically to three decimal places. Fig. 7 illustrates this.

The successive multipliers give the ordinates of the impulse weighting function which is shown in Fig. 8; an integrated step response is also shown as this is more readlily intelligible.

The closed loop results were used to find both the controller and steam-valve/temperature dynamics. This is possible because equation (2) applies to the controller, which has a temperature input and valve position output, and equation (5) applies to the steam-valve/temperature dynamics, which have a valve-position input and a temperature output. In each case the same correlation function is used, but the positive and negative directions of T are interchanged.

No extra noise is assumed to enter the system in solving for the transfer function of the controller, so that the result should be as accurate as the instrumentation allows. The

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* *The programme for calculating equation (8) on the DEUCE high speed electronic computer was prepared and tested by Mr. R. H. Tizard.*

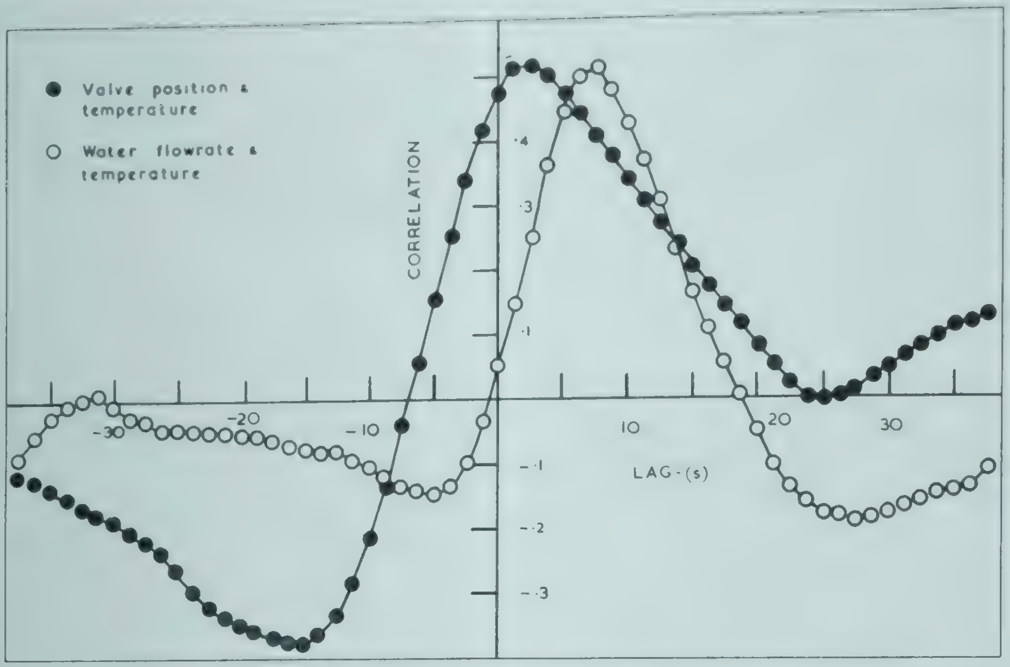


Fig. 6.—*Cross-correlations. Open and closed loop results*

auto- and cross-correlations on closed loop are shown in Figs. 6, 9, and 10. Figs. 9 and 10 also show some check calculations which are discussed later. The impulse and step responses of the controller are shown in Fig. 11.

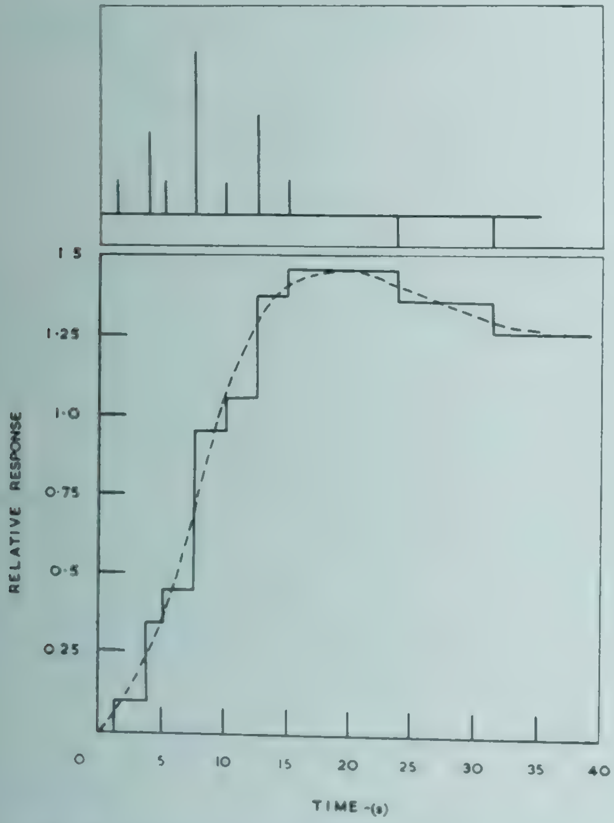


Fig. 8.—*Flowrate-to-temperature dynamics: Impulse and step responses*

In applying equation (5) it is necessary to know φ_{nn}; this is actually the outlet temperature auto correlation found in the open loop tests, because then the fluctuations in temperature represents the sum of all disturbances in the system. It is also necessary to postulate a form for $b(t)$; the circumstances of this problem made it unexpectedly easy because of the large time delays involved. An approximate initial solution for $h_x(t)$ gave a pure delay of 15 seconds with a Laplace transform $0{\cdot}9\,e^{-15p}$. Now, in transform,

$G(p)$ = Controller transfer function
$B(p)$ = Temperature disturbances to valve-position transfer
$H_x(p)$ = Valve-position/temperature transfer

Then

$$B(p) = \frac{G(p)}{1 + H_x(p)\,G(p)}$$
$$= G(p)\{1 - [H_x(p)\,G(p)] + [H_x(p)\,G(p)]^2 + [\quad \ldots \quad]\} \quad . \qquad (9)$$

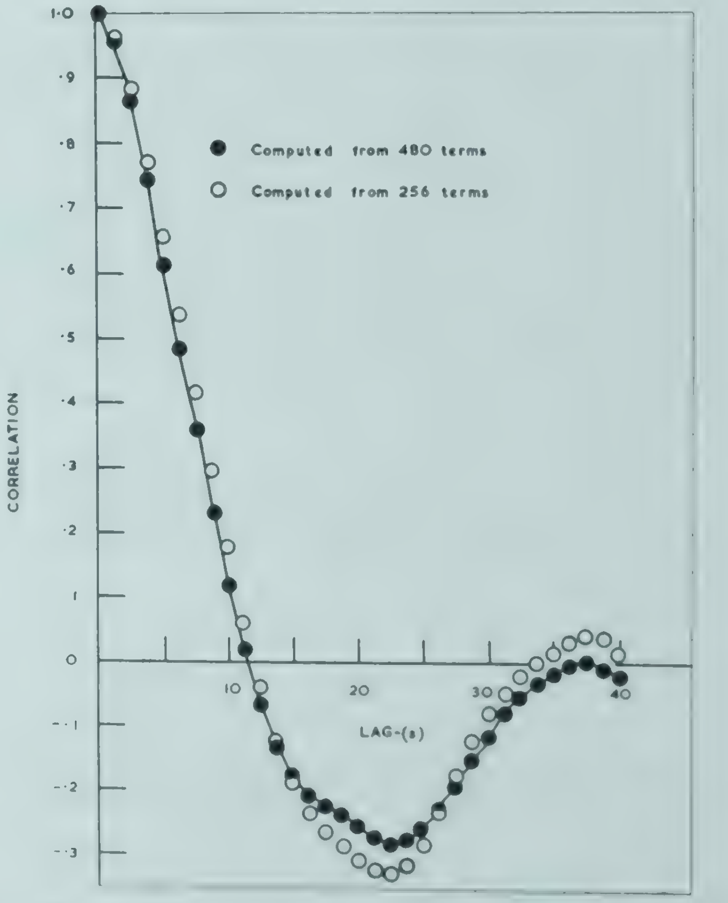
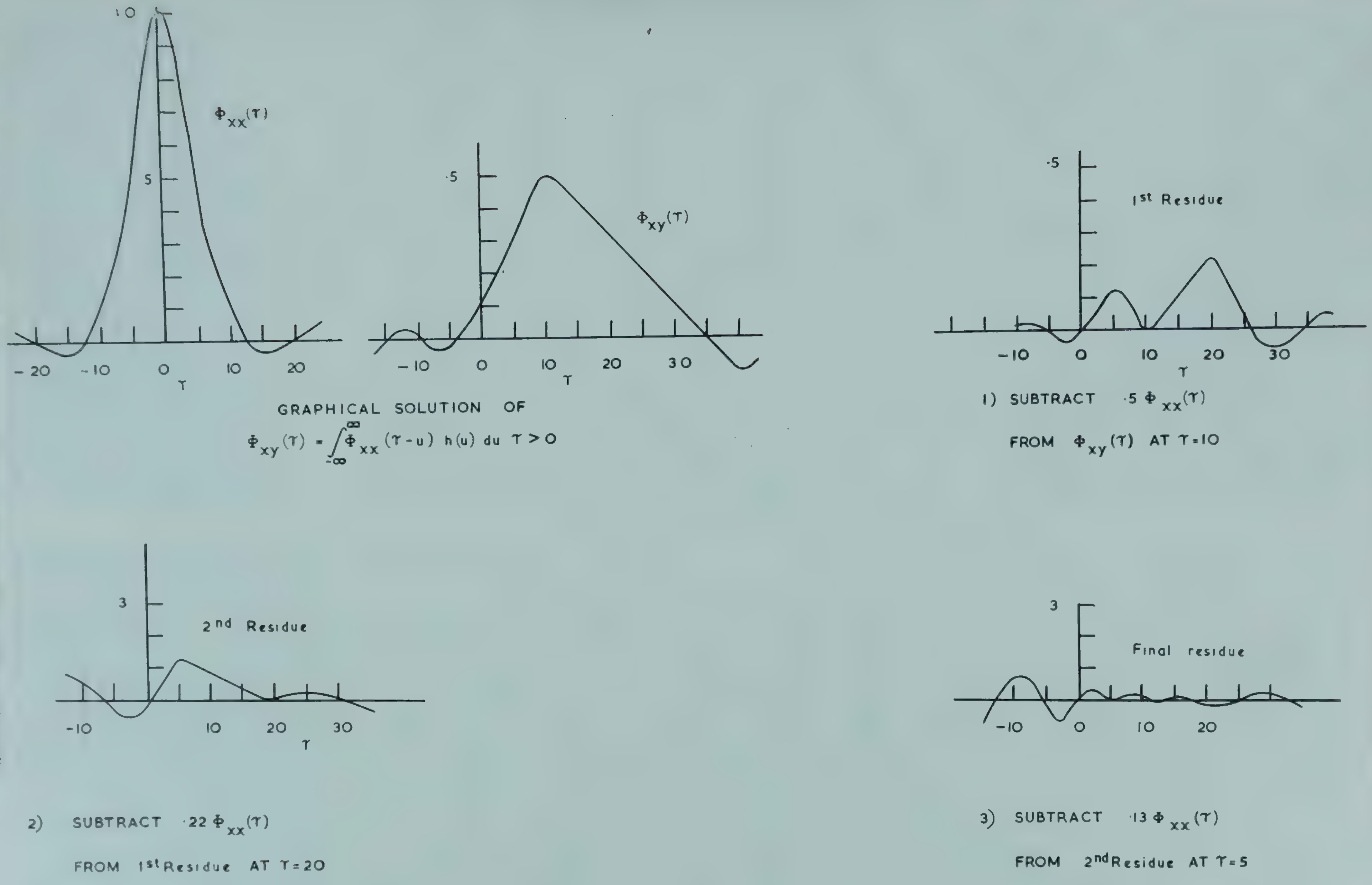


Fig. 9.—*Auto-correlation. Closed loop outlet temperature*



SOLUTION $H(p) = \cdot 13 e^{-5p} + \cdot 5 e^{-10p} + \cdot 22 e^{-20p}$

Fig. 7.—*Illustration of graphical method of solving convolution equation*

and

$$G(p) = (0{\cdot}5\, e^{2{\cdot}5p} + 0{\cdot}1\, e^{12{\cdot}5p} + 0{\cdot}2\, e^{18{\cdot}25p} + 0{\cdot}1 e^{30p})$$

So that

$$B(p) = G(p) + \text{terms of delay more than } 20 \text{ seconds} \quad . \quad (10)$$

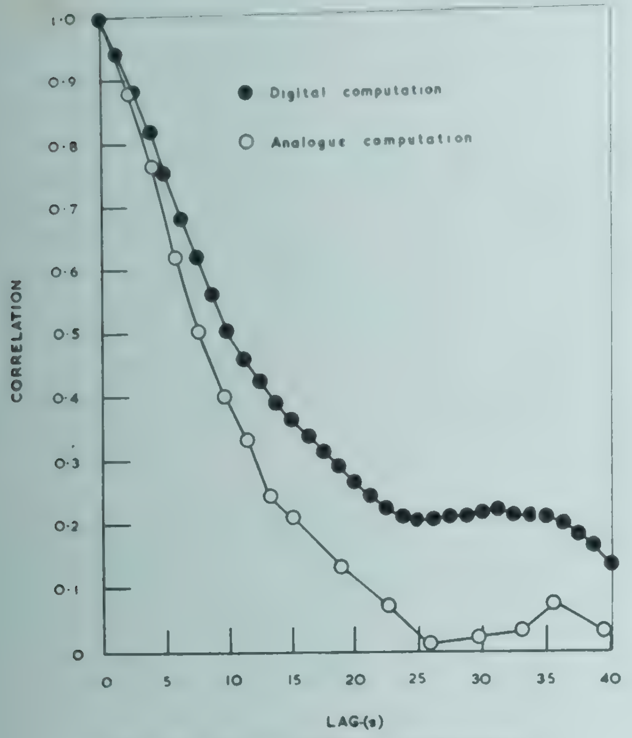


Fig. 10.—*Auto-correlation. Valve-position*

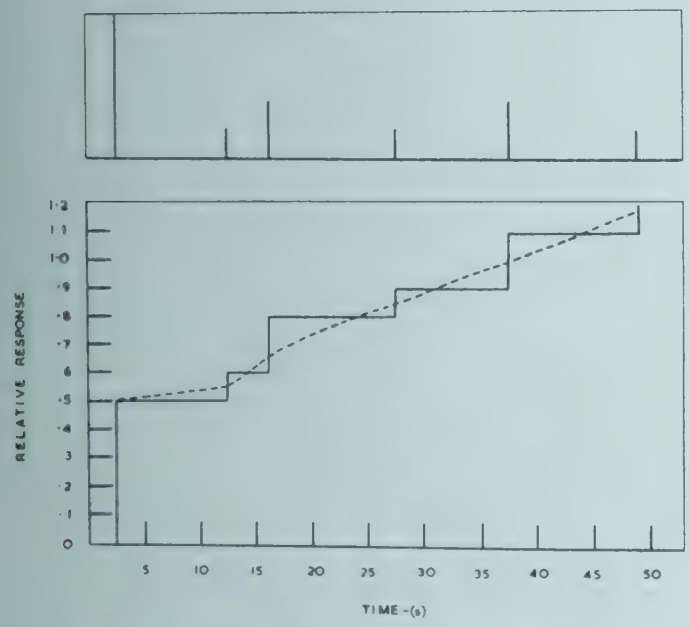


Fig. 11.—*Controller impulse and step response*

Inspection of φ_{nn} shows that it has become very small at 20 seconds lag, so that all terms in equation (10) involving delays of more than 20 seconds will have little effect on the region of the cross-correlation having positive values of T, and will thus not affect the solution for $h_z(t)$. Hence, the open-loop temperature auto-correlation was convolved only with the controller transfer and then subtracted from the closed-loop cross-correlation. Following this, the valve-position auto-correlation was used to solve for $h_z(t)$; the result was approximately a pure delay. Successive steps in the computation are shown in Fig. 12. There is some residual in the noise region.

Frequency domain calculations

The sampling theory for the power-density spectrum of a single series has been rather fully worked out. An excellent account is given by Blackman and Tukey.[8] Here the stochastic formula is not the best estimator, and the limitations of a finite length of data require that an extra factor be introduced. Blackman and Tukey call this a window; their terminology will be used here.

It is a peculiarity of the statistical sampling properties of the power-spectrum that the variance is of the same order as the power density at each frequency. Thus, a power-spectrum graph will show wild fluctuations. This can be remedied by averaging the results at neighbouring frequencies when the variance is reduced, but frequency resolution is lost in the same proportion. It is always necessary to do this averaging to get an intelligible result; it can be effected simply by modifying the window factor.

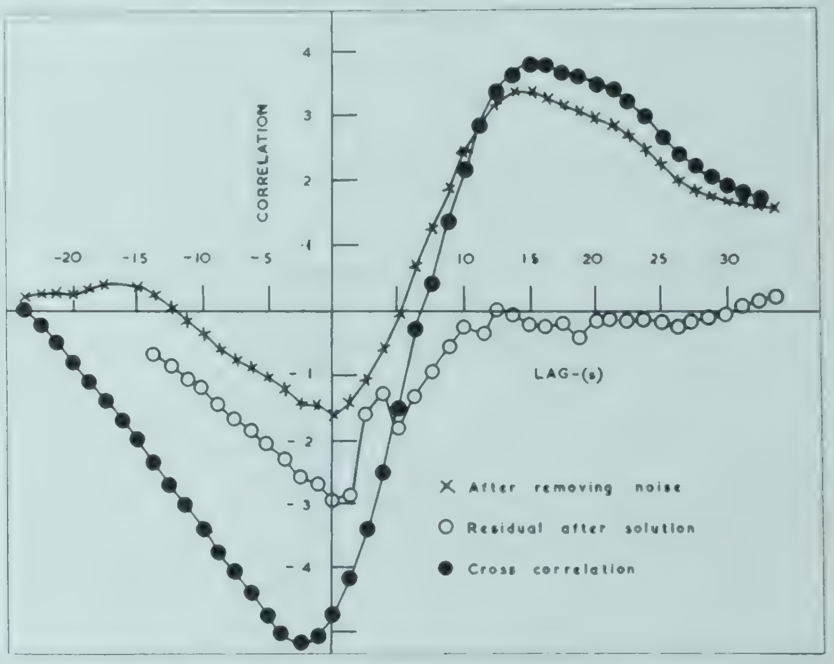


Fig. 12.—*Solution of valve-position to temperature dynamics showing subtraction of effects of noise*

The formula to estimate the power-density spectrum from experimental records is

$$\Phi_{xx}(\omega) = \int_{-\infty}^{\infty} D(t)\, \varphi_{xx}(t)\, e^{-j\omega t}\, dt \quad . \quad . \quad (11)$$

where $D(t)$ is the lag window, and $\varphi_{xx}(t)$ is the auto-correlation function calculated from the experimental record.

The averaging caused by the window factor can be introduced in several ways: in equation (11) it is a function multiplying the correlation function. This is entirely equivalent to averaging the spectrum over frequency. The precise form of the window function has to be chosen to suit the requirements of each particular experiment. The choice is, in effect, that of a weighting factor to be applied to power density estimates at neighbouring frequencies during the averaging process. Practical windows attenuate the correlation functions at large lags and thereby emphasise the centre portions, which are more accurate.

In this problem the broad trend of the spectrum was required rather than details of resonances and a very simple function was satisfactory. This is the triangular window first proposed by Bartlett. Figs. 13A, and 13B, show the window in the time (lag) domain, and also, the equivalent weighting factor in the frequency domain. Notice that the frequency weighting is always positive.

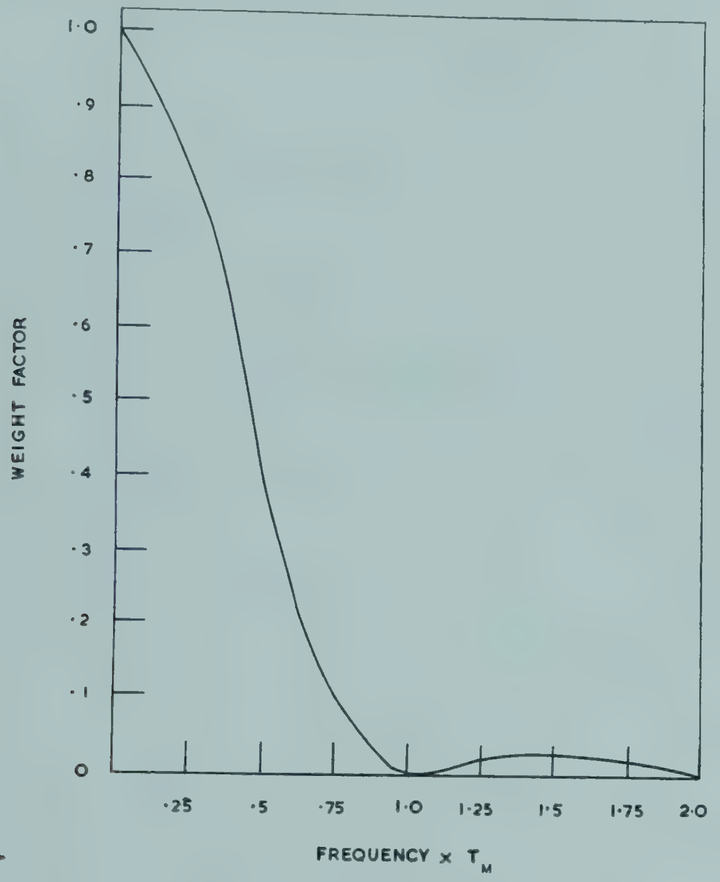


Fig. 13A.—*Spectral window*

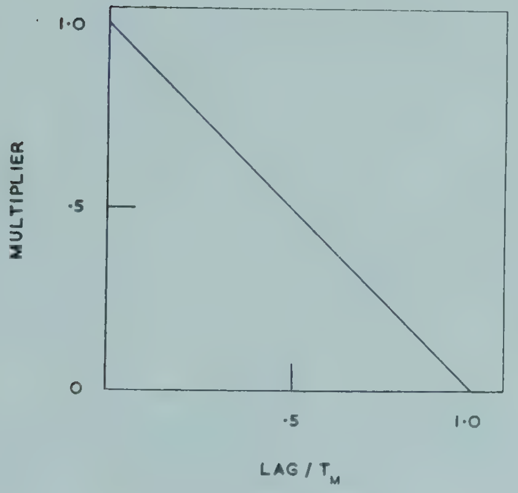


Fig. 13B.—*Lag window*

Equations (14) and (15) give expressions relating the width of the window to the frequency resolution and also the variance of the resulting estimate. Here the best compromise appears to be a frequency resolution of 0·05 c/s, and an error limit such that 80% of the estimates would lie within ± 25% of the true value.

The open and closed loop auto-correlation functions were multiplied by this window, and then Fourier transformed numerically on the DEUCE computer. Figs. 14 and 15 show the resulting auto-spectra.

The statistical sampling theory of the cross spectra is not known, but it is clear that some similar averaging procedure must be used. In this paper an *ad hoc* approach is used. It is possible to regard the cross-correlation function as a shifted and modified version of the auto-correlation function, for if say, the network h(*t*) had been a pure delay, the cross correlation would be exactly the same shape as the auto-correlation but with a time-shift. Thus, the same window shape was used as on the auto-correlation, but the window centre was moved to the peak of the cross-correlation. Such a simple device would, of course, have been ineffective if the cross-correlations had not had a clearly identifiable peak.

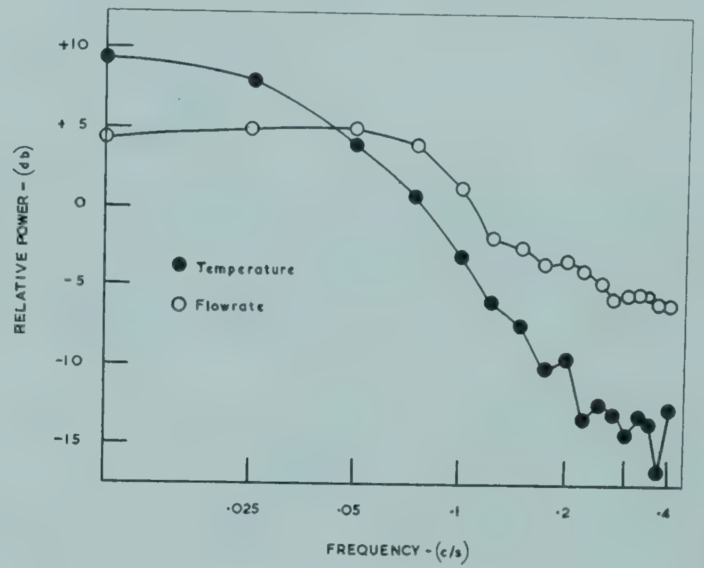


Fig. 14.—*Auto-power spectra. Open loop results*

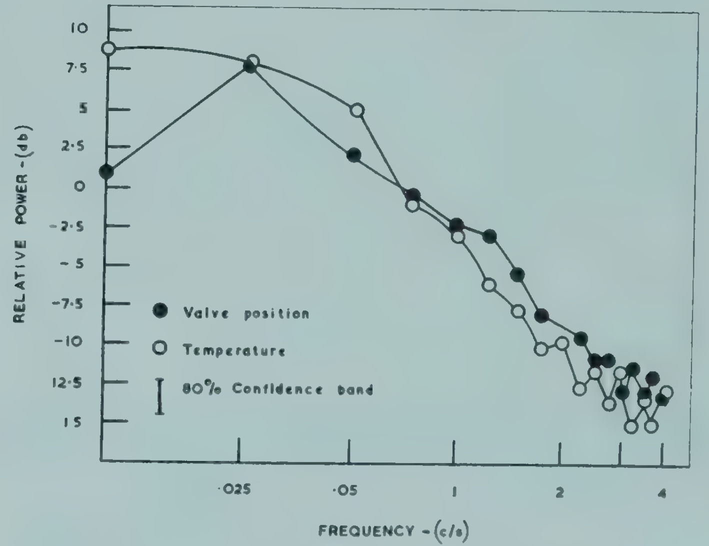


Fig. 15.—*Auto-power spectra. Closed loop results*

When putting the window on the valve-position/temperature cross-correlation, two separate positions had to be used, one for getting the controller solution and the other for the steam-valve/temperature solution. In placing the window for the valve-position/temperature dynamics solution it was found that the noise region of the cross-correlation was almost eliminated. No further steps were therefore taken to remove the effects of noise in the loop. The cross-spectra are shown in Figs. 16, 17, and 18. One special difficulty encountered was that the computer actually produces the sine and cosine coefficients only in the transform. The phase thus produced is between −90° and +90°. The experimenter then has to decide what multiple of 2π to add.

The large delays in this example made it very difficult to decide what quantity to add, and the phase results are, accordingly, only given over the range which is reasonably certain.

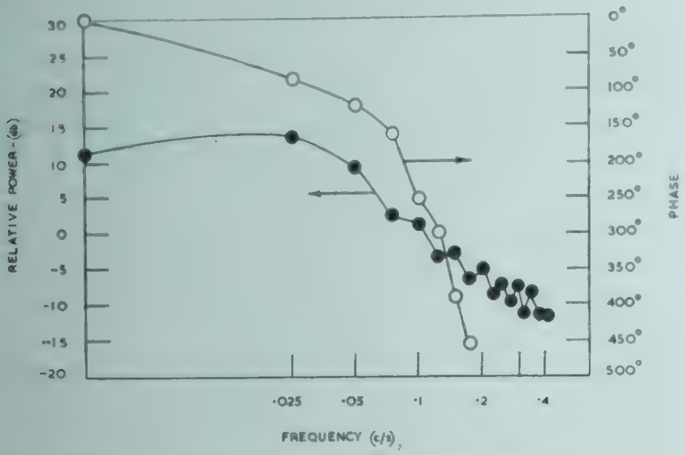


Fig. 16.—*Cross-power spectrum: flowrate-to-temperature*

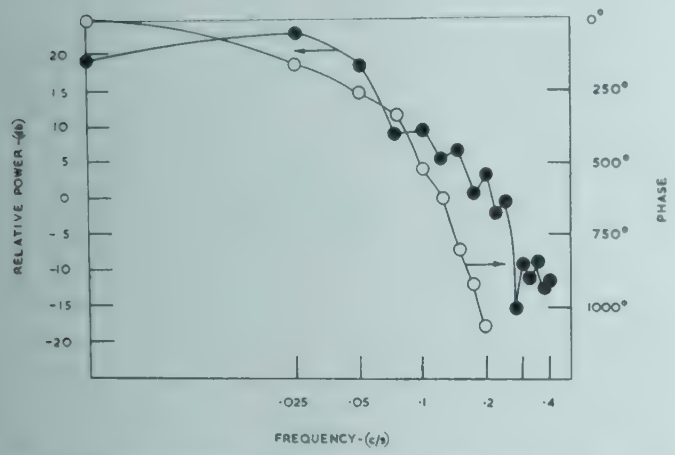


Fig. 17.—*Cross-spectrum. Steam-valve position and temperature. Valve-to-temperature dynamics region*

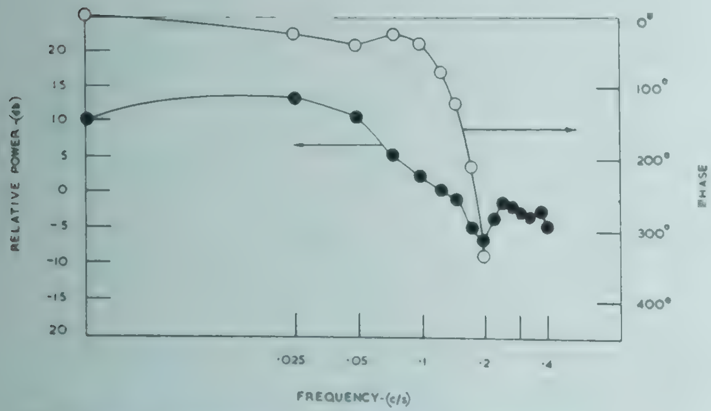


Fig. 18.—*Cross-power-density spectrum. Steam-valve position and temperature. Controller region*

The frequency responses follow immediately by subtraction between the auto- and cross-spectra expressed in logarithmic form, these are shown in Figs. 19, 20, and 21.

Error Levels

Errors due to non-stationarity

It will be realised that all the formulae used above depend for their validity on the time series being stationary at least to a second order. As yet no simple test is known which will check this easily. In this problem several auto-correlations were repeated using half the data (actually 256 terms). These compared very well with the function computed from the whole series, and it is assumed that these prove the series to be stationary. One example is shown in Fig. 9. The agreement is well within the sampling error.

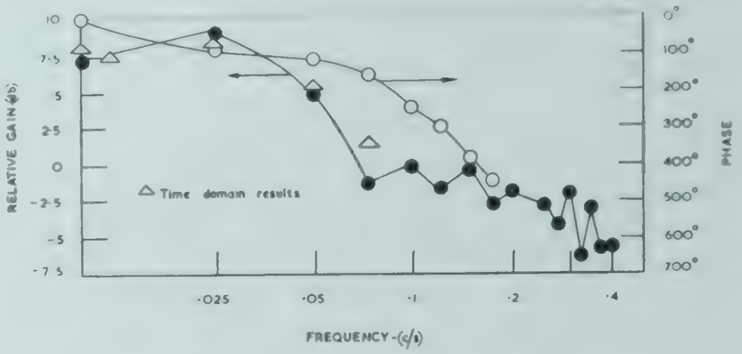


Fig. 19.—*Flowrate-to-temperature frequency response*

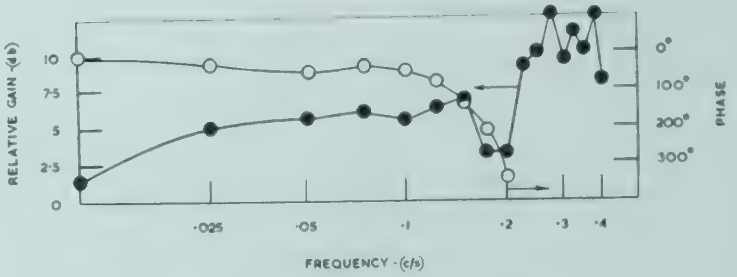


Fig. 20.—*Controller frequency response*

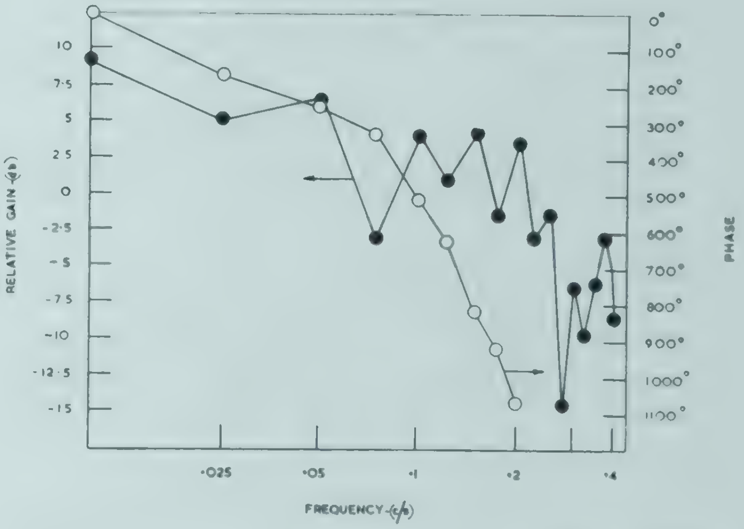


Fig. 21.—*Steam-valve position-to-temperature frequency response*

Errors due to misreading the records

The records were read by eye at quite close intervals. If one or two points had been missed a time-shift would have occurred which would have invalidated all subsequent calculation. Fortunately some comparable results were available computed by an analogue machine directly from the records. The analogue machine accepted input through a time-constant which eliminated the mean-level term. Thus the level of the computed function would change but its general shape should remain the same. A comparison of the functions showed this effect. An example is shown in Fig. 10.

Error due to data-sampling interval

If the records were not sampled often enough the higher frequency components would be falsely mixed in with lower frequency components, or, as Tukey puts it, "aliased".

This can be checked by noting that the maximum frequency in the auto spectra allowed by the sampling rate is 0·4 c/s. In practice the spectra fell off at 0·2 c/s, showing an adequate sampling rate.

Error in the noiseless case

Little noise was assumed to enter the controller between input and output, and thus the solution will show an error due only to the limits of the instrumentation and the reading of the records.

The errors in the instrumentation are not known. The records were read to magnitude intervals of 5%, and the average magnitude of the resultant error should be given by the Sheppard's correction of $d^2/12$, where d is the magnitude interval. This correction is negligible here, and a more realistic estimate is given by noting that the residue on solving equation (2) is 2–5%. Thus an error of 5% seems reasonable for the magnitude of the impulse weighting function.

Error level of the auto-correlation function

Bartlett[3] has shown that, under fairly general conditions, the variance of the auto-correlation is given by:

$$\mathrm{Var}\,[\varphi_{xx}(s)] = 1/(n-s)\sum_{V=-\infty}^{+\infty} \varphi^2{}_{xx}(V) \quad . \quad (12)$$

where n = total number of terms in the series, and s = lag.

Here the variance was found to be approximately 0·02, so that it can be said that with 90% probability the true value is within 0·23 of the estimated value.

Error level of the cross-correlation function

Bartlett has given a result for the variance of the cross-correlation of two Gaussian sequences:[3]

$$\begin{aligned}\mathrm{Var}\,[\varphi_{xy}(s)] = 1/(n-s)\sum_{-\infty}^{\infty} \{&\varphi_{xx}\varphi_{xy} + \varphi_{xy}\varphi_{yx} \\ &+ \varphi^2{}_{xy}(\varphi^2{}_{xy} + \tfrac{1}{2}\varphi^2{}_{yy} + \tfrac{1}{2}\varphi^2{}_{xx}) \\ &- 2\,\varphi_{xy}(\varphi_{xx}\varphi_{xy} + \varphi_{yx}\varphi_{yy})\} \quad . \quad (13)\end{aligned}$$

The results here give a variance of approximately 0·09 so that with 90% probability the true value lies with 0·12 of the estimated value.

Error levels in the power spectrum

Tukey gives the following simplified formulae for the variance of the power spectrum

$$\frac{\mathrm{Var}\,[\Phi_{xx}(\omega)]}{[\text{Ensemble Average } \Phi_{xx}(\omega)]^2} = \frac{T_m}{T'_n} \quad . \quad (14)$$

where

T_m = the width of the lag window,
T_n = total length of record, and
$T'_n = T_n - p/3 \,.\, T_m$
p = number of separate sections of record.

Also the χ^2 distribution of the errors has a number of degrees of freedom given by

$$\frac{2\,[\text{Ensemble Average } \Phi_{xx}(\omega)]^2}{\mathrm{Var}\,[\Phi_{xx}(\omega)]} \quad . \quad . \quad (15)$$

In the present case the width of the lag window was chosen as 20 seconds giving a frequency interval of ·05 c/s

Variance = 0·034
Number of degrees of freedom = 58

Using the χ^2 table it is found that 80% of measured values will lie within ± 1·1 db of the true value.

Comments on the Results

All results given here are given without the absolute magnitude of the gain; in fact the gain was determined in an auxiliary set of step change experiments.

Open loop flowrate/temperature transfer

The step response shows an overshoot, and the frequency response a corresponding resonance peak. This is unexpected on ordinary heat exchanger theory, and must indicate some resonance, either mechanically in the flow system or in the heat transfer mechanism itself.

Controller transfer

The time-response can be interpreted as a proportional-plus-integral, with a time constant of 56 s (using the gain value found in the auxiliary experiment, of ·72). The kink near 10 s is possibly connected with a derivative action, but this cannot be evaluated.

The frequency response shows a rising high-frequency gain suggesting a derivative action but it is so irregular that the time-constant cannot be found. No sign is found of the integrating action. This is not surprising, as the break frequency would be of the order ·003 c/s and this is lost in a frequency band of ·025 c/s. This is an indication of possible pitfalls in analysis methods.

These results are closer than 10% to the known values of the controller setting.

Steam-valve position/temperature transfer

Time domain analysis gave the result

$$3{\cdot}12\,(0{\cdot}3\,\mathrm{e}^{15p} + 0{\cdot}1\mathrm{e}^{22{\cdot}5p})$$

The frequency-response is flat over the main part of the working band, substantiating the hypothesis of a pure delay. The phase curve is close to that for a pure delay of 13 s.

Comparison with other work

This same heat exchanger has been analysed, by two other different methods. Hainsworth, Tivy, and Paynter[5] have measured the steam-valve position/temperature transfer function using sine and step-wave methods. Their Fig. 2 may be compared with the present Fig. 21. It is not possible to claim agreement but, their Fig. 4 may be compared to the result quoted above for the time-domain response. Their result is approximately a single time constant of 15 s, which is in line with the above result of a delay of 15 s.

Lees and Hougen[6] measured the water flowrate/temperature transfer function using the pulse testing method. Their Fig. 6 may be compared with the present Fig. 19. Both frequency responses follow the same general trend, but the present one is much more irregular.

A detailed discussion of the discrepancies is not worthwhile because of the large difference in magnitude of test signal used.

Comments on Analysis Methods

Time domain advantages

The advantages of basing analysis on time domain are:

(i) The criterion of physical realisability in the experimentally determined weighting function is easily maintained by solving only in the region of positive values of T.

(ii) The stationary nature of the data can be checked by comparing the auto-correlations of different stretches of the data.

(iii) Clear and unambiguous results are obtained when there are large delays.

(iv) Some attempts at removing the effects of noise in a feed-back loop can be made.

Time domain disadvantages

The disadvantages of the time domain procedure are:

(i) Extraction of the weighting function is lengthy.

(ii) A simple method of solution only works when the correlations are single sharp peaks. Thus it would not work well in a system with strong resonances.

(iii) No indication of the adequacy of the data sampling rate is given.

Spectral analysis advantages

Spectral analysis has the following advantages:

(i) It gives clear indication of the adequacy of the data sampling rate.

(ii) It shows up resonances clearly.

(iii) Extraction of the frequency response easy.

Spectral analysis disadvantages

Spectral analysis has the following disadvantages:

(i) Ambiguous results are obtained when there are large delays.

(ii) The initial decision on the frequency resolution, if unwise, may obscure an important feature.

(iii) There is no practical way of checking the physical realisability of a derived transfer function.

Acknowledgments

The authors are indebted to Mr. R. H. Tizard for freely giving his time to supervise the machine computation, and for providing a tested programme for calculating the correlation functions.

Thanks are due to the English Electric Co. Ltd. for generously giving computing facilities on their DEUCE high speed electronic computer.

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The authors thank the editors of the *Bell System Technical Journal*, and Dover Publications Inc. for permission to reproduce Fig. 13.

Symbols Used

$B(p)$ = temperature disturbances/steam - valve position transfer function on closed loop (relative units).
$D(t)$ = lag window function (numerical factor).
$e(t)$ = error time series (relative units).
$G(p)$ = controller transfer function.
$\left.\begin{matrix}H(p)\\H(w)\end{matrix}\right\}$ = steamer-valve position/outlet temperature transfer function.
$h(u)$ = impulse weighting function.
$n(t)$ = disturbance time series (relative units).
T, t = time (s).
T_m = width of lag window (s).
T_n = total length of time series analysed (s).
Var = variance.
$x(t)$ = input time series.
$y(t)$ = output time series.
μ_x = mean value of the series $x(t)$.
σ_x = variance of the series $x(t)$.
$\Phi_{xx}(\omega)$ = auto-spectrum of series $x(t)$.
$\Phi_{xy}(\omega)$ = cross-spectrum of series $x(t)$ and $y(t)$.
$\varphi_{xx}(\tau)$ = auto-correlation function of series $x(t)$.
$\varphi_{xy}(\tau)$ = cross-correlation function of series $x(t)$ and $y(t)$.

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SUPERFRACTIONATOR CONTROLLABILITY DATA OBTAINED BY THE USE OF A DIGITAL COMPUTER

By J. M. KEATING, B.Sc.,* and D. S. TOWNEND, A.C.G.I., B.Sc., A.M.I.Mech.E., A.F.Inst.P.*

SUMMARY

Steady state relations between the independent variables and the product quality have been calculated by the use of a computor over a limited range for a superfractionator column having a throughput of 9000 bbl/stream day, producing *iso*-pentane from stabilised gasoline.

The work is the first step of a comprehensive study of the controllability of the column.

The aspects of controllability of particular interest are:

- **(1) Selection of independent variables to be controlled.**
- **(2) Siting of quality analyser sample points.**
- **(3) The possible advantages of an optimising control system.**

The paper records graphically the results of a number of computer runs. The results are presented in two forms:

- **(1) Variation in the column composition profile with variation of plant parameters.**
- **(2) Graphs indicating the effect of plant parameter variations on product quality.**

A preliminary discussion of the value of this information is included and possible lines along which the study may be extended are indicated.

The work so far completed has taken no account of the dynamic characteristics of the column.

Introduction

A knowledge of the effects of changes in process variables, *e.g.* reflux ratio, feed-temperature, yield *etc.*, and of disturbances, *e.g.* feed-composition, ambient and cooling water temperature, *etc*, on the controlled variable (usually product purity) in both steady and transient states is required by the control engineer in designing a control system for a given fractionation column.

At present the design of such systems depends almost entirely on qualitative knowledge derived from past experience and discussion. During recent years much attention has been focussed on the transient response of fractionation columns[1–4] and little apparent effort has been directed towards the work initiated by Uitti[5] in obtaining quantitative information on the inter-relationship of the variables in the steady state condition.

It is considered that quantitative knowledge of steady and transient states is of equal importance to the proper design of control systems and to the operation of the plant. The information on the steady state may be extremely useful as an operating guide. Furthermore, whilst quantitative steady-state data are useful in deciding which variable should be controlled when using the present day environmental control systems, it is considered that this information is essential for the successful future application of control by computer.

Uitti's approach was to obtain the relations between the variables, using McCabe-Thiele diagrams for a binary feedstock. It is considered that this line of study should be enlarged and extended to multi-component feedstocks without the limitations imposed by the McCabe-Thiele method.

The object of the present series of studies on fractionation is to ascertain, for steady state conditions, quantitative parameter relations for the analysis of specific control applications and to investigate plant flexibility.

Description of Plant Studied

In order that quantitative and realistic results could be obtained the study was based on a real plant, namely a 9000 bbl/day *iso*-pentane superfractionator. This plant was designed to produce *iso*-pentane $({}_iC_5)$ with a maximum contamination of 6% by weight with normal pentane $({}_nC_5)$ and heavier components, from a feed stock of stabilized gasoline, debutanized to 0·2% by weight normal butane and containing *iso*-pentane varying from 10·4 to 15·7% by weight according to the source of crude oil. The original specification also called for a minimum recovery of 80% by weight of the *iso*-pentane from the feed stock.

A schematic diagram of the superfractionator is shown in Fig. 1. The parameters which affect the purity of the product are:

- (1) Feed rate, condition, composition, and entry point.
- (2) Number and efficiency of column plates.
- (3) Column pressure.
- (4) Reboiler heat input rate.
- (5) Overhead distillate rate of flow.
- (6) Reflux ratio.

The last three of these are interdependent.

Bearing in mind the object of the exercise and the nature of the expected disturbances to the column, it was decided that the parameters most suitable for study were those given in Table I.

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* British Petroleum Co. Ltd., Beaufort House, Gravel Lane, London, E.1.

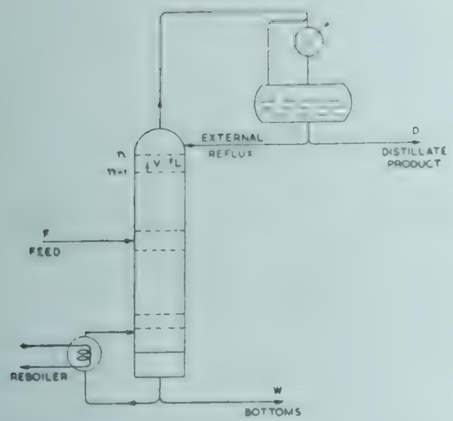


Fig. 1.—*Superfractionator—basic flow diagram*

TABLE I.—*Parameters Studied*

(V represents a variable, C represents a constant.)

Feed rate	C	C	C	C
Feed entry point	C	C	C	C
Feed composition	C	C	V	C
Feed point	C	C	C	V*
Yield	V	C	C	C
Reflux ratio	C	V	C	C
Number of theoretical stages	C	C	C	C
Column pressure	C	C	C	C
Distillate contamination	Dependent variables			
Iso-pentane recovery				
Reboiler load				

* See Appendix II.

The problem is to present the reactions of the dependent variables to one varying independent variable with all other independent variables held constant (the procedure to be repeated for all independent variables in turn) in a form convenient for the study of plant controllability. Graphical presentation would be a suitable form if sufficient parameters could be included and if extracting information did not involve excessive or difficult interpolation. The selection of parameters was influenced by what were considered to be the major disturbances to which the plant may be subjected. These include change in feed composition, change in ambient temperature, and change in cooling water temperature. The two latter may be regarded as changes in internal reflux.

To obtain this information a large number of solutions to equations is required. Each equation would involve the conventional plate-to-plate calculation for a distillation column. Longhand methods of solutions are very tedious and would be prohibitive in this case owing to the large numbers of results involved. The decision to use a digital computer followed as a natural consequence.

Several computer programmes for distillation problems are under consideration by the authors' company, but the only programme which had been used within the company and found satisfactory was that used by Bonner for the solution of plate-to-plate calculations for use with I.B.M. 650 Digital Computer.[6] The form of the Bonner programme is more suitable than the conventional plate-to-plate and short-cut methods in that it takes a specified number of theoretical stages and calculates product purity. This is an essential requirement to the writers' approach in contrast to most other methods which give the number of theoretical stages for a given split. The programme is not sufficiently flexible for the work envisaged in this present study, but the information sought was modified to accommodate the Bonner programme limitations.

Description of Computer Data

From its input data, and from a series of mass balance and heat balance equations, the machine makes a plate-to-plate calculation starting from the top of the column and working to the feed plate. It makes a similar calculation working from the bottom to the feed plate, thus completing one trial. The resulting mismatch between the two values obtained for each component of the composition is compared with the maximum allowed in the input data, and an adjustment is made to the top composition to begin the next trial if convergence has not been achieved. A corresponding adjustment is calculated for the bottom from a mass balance equation. The temperature and vapour rate are computed at each stage for each trial and are stored for use in the succeeding trial. The reflux condition is taken into consideration during the calculations and tests carried out during each trial. For the first trial the machine uses a distillate rate calculated from the distillate-to-feed ratio and the feed rate specified in the input data. The machine is programmed to assume the initial split between components to be perfect in the proportion of the distillate and bottom rates. All feed components thereby excluded from the assumed distillate and bottoms compositions (heavier components in distillate and lighter components in bottoms) are then given an initial value of 10^{-40} mols per hour. The use of this extremely small rate illustrates the advantages of digital computer techniques —it avoids the necessity of adding in these components at the appropriate stages as is necessary in the manual plate-to-plate calculation. The computer is also programmed to assume a constant initial vapour rate and an initial linear temperature gradient. Convergence is subsequently achieved by adjusting the individual component flow rates. On convergence, the machine makes final heat and material balances and the temperature profile is calculated from each stage composition. The output, as detailed in Table II, is punched out and tabulated.

TABLE II.—*Computer Output Data*

Trial Summaries	These give the distillate and bottoms rate and maximum mismatch at feed stage for each computer calculation during convergence.
Results of Last Trial	Includes the feed stage composition in mol. fraction as calculated from the top down and bottom up, the resulting error in each component, the approximate error in each component in the bottoms, and the final distillate and bottoms rates.
Final Profiles	These give the temperature, vapour rate, and concentration of one specified component, at each stage of the column.
Final Material Balance	In the form of a flow rate for each component in the bottoms, distillate and feed, and percentage composition for distillate and bottoms.
Final Heat Balance	Indicating the heat input from feed and reboiler, and the heat output by distillate, bottoms, and condenser.

Programme limitations

It will be seen from the input data detailed in Table III that a distillate rate tolerance has to be specified. The programme's first choice of method of adjusting distillate compositions—at the beginning of each trial after the first—is by altering the value of D by changing the contribution of each component to D. If the value of D thus obtained falls within the tolerance the machine proceeds with the trial: if not, each component flow is changed so that the sum of the changes equals zero and this results in a slower convergence. Thus to obtain a reasonably rapid convergence a large value of ΔD is desirable. The significance of this is that the machine may accept all, part, or none of the tolerance thereby causing difficulty in planning a run to include a given yield in its results. Since V/F is a constant for each run, the same difficulty was experienced in obtaining the desired reflux ratio. This lack of flexibility proved a handicap in correlating earlier results, but it was soon observed that the probability of the machine's accepting all the tolerance was high, and advantage was taken of this fact in planning subsequent runs.

TABLE III.—*Programme Input Data*

Feed rate . . . . F lb/h

Distillate yield (ratio of distillate rate to feed rate) D/F (mol basis)

Distillate rate tolerance (maximum allowable difference between specified value for distillate rate and the final computed rate expressed as a ratio) . $\Delta D/D$ (mol basis)

Vapour feed ratio (ratio of vapour rate at a specified stage to the feed rate) . . . . . . V/F (mol basis)

External reflux condition . . . . T (° F) or bubble point

Feed condition and composition

Feed point (stage at which the feed enters column)

Vapour loading point (stage to which V refers in V/F)

Number of theoretical stages (includes reboiler, excludes condenser)

Number of components

Allowable mismatch at feed plate (maximum acceptable discrepancy between any component composition values on feed stage as calculated by computer from top to feed and bottom to feed stages. *i.e.* convergence termination)

Equilibrium constant data in the form of an equation,

$$T = AK^3 + BK^2 + CK + D$$

where T is the equilibrium temperature

Relative volatility data in the form of an equation

$$a = ET^3 + FT^2 + GT + H$$

where T is the equilibrium temperature

Enthalpy Data

Liquid specific heat data in the form of an equation

$$C_p = KT + L$$

Vapour latent heat data in the form of an equation

$$H_v = MT^2 + NT + P$$

where T is temperature

Other programme instructions.

Values of the reflux ratio for each run were calculated from the formula:

$$\frac{V}{F} = \frac{D}{F}(R + 1) \qquad \text{(mol. basis)}$$

where $R = L/D$ internal reflux ratio (see Fig. 1).

Recovery values were calculated from the material balance of each run.

Presentation of data

Tabulated results would be too cumbersome to present and difficult to discuss because of the large volume of data accumulated from the computer runs. The results are therefore presented in graphical form. Several graphs may be obtained from the same information by using different combinations of the same parameters. Final selection is a matter of personal choice influenced by the nature of the intended analysis. For example, Fig. 2 shows $_nC_5$ in distillate plotted against reboiler heat load with D/F and reflux ratio as parameters for a feedstock of 75° C T.B.P. (see Index of Data) and Fig. 3 shows $_nC_5$ in distillate plotted against recovery with the same parameters and feedstock.

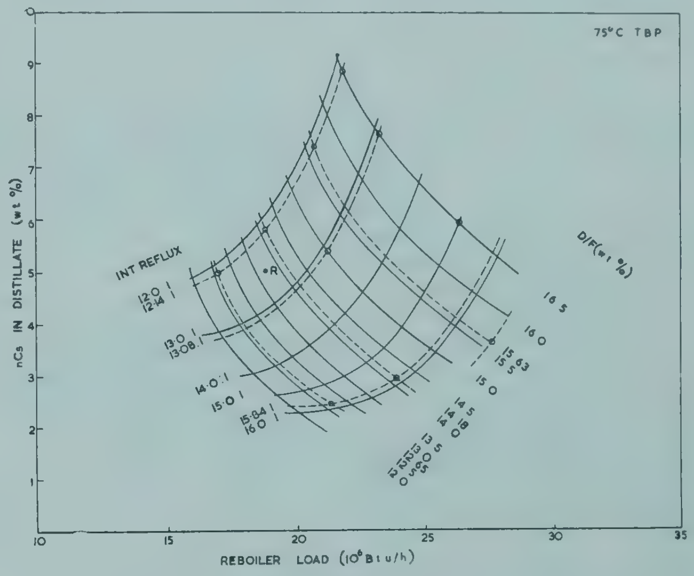


Fig. 2.—*Heat load*—n-*pentane overhead, 75° C T.B.P. feed*

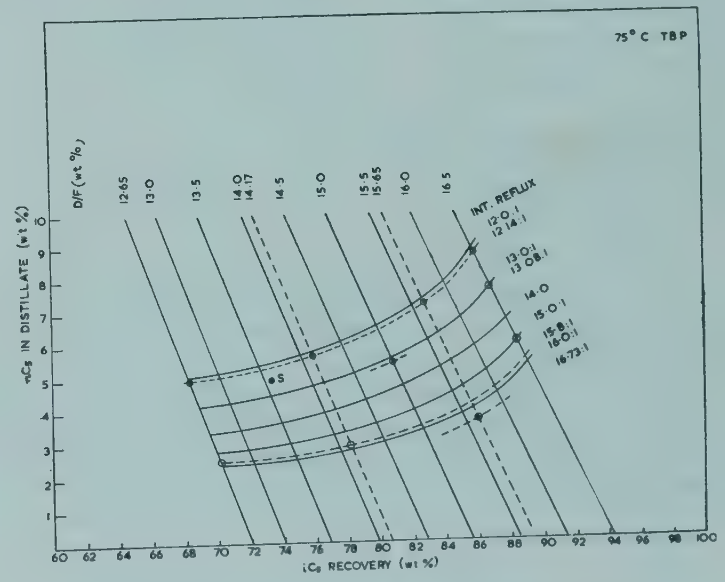


Fig. 3.—Iso-*pentane recovery*—n-*pentane overhead, 75° C T.B.P. feed*

The graphs were not derived directly from the results of the computer runs but were constructed by a procedure outlined in Appendix I. It will be noted that the full lines on the graphs were constructed for round figures of the parameters and the dotted lines were constructed to accommodate results obtained from the computer runs.

All the graphs constructed up to the time of submission are included as a matter of interest although not all are referred to in the text. It is anticipated that many more could be constructed from the available data as dictated by the course of any particular aspect of the study. It will be observed that the majority of graphs are on a weight basis. The programme carries out the main calculation on a mole basis but has the option of mole, volume, or weight basis for the feed, making the terminal calculations in units consistent with the feed basis. Weight basis was used for the feed in view of the specification of the plant. Where graphs employ mole basis for parameters, it is because the information comes from a stage in the main calculation.

Table IV.—*Index of Data*

Feed stock	Light straight Run Gasolines from Kuwait Crude having True Boiling Points (T.B.P.) as below and containing a maximum of 0·2% by weight normal butane.

Feed composition (*wt.* %)

	75° C T.B.P.	85° C T.B.P.	95° C T.B.P.
Normal butane ($_nC_4$)	0·2	0·2	0·2
Iso-pentane ($_iC_5$)	17·3	15·7	12·5
Normal pentane ($_nC_5$)	24·8	22·4	17·8
Hexane (45–85° C T.B.P.)	40·8	41·0	40·5
Heptane (85–114° C T.B.P.)	15·2	17·2	22·3
Octane (114–137° C T.B.P.)	1·7	3·5	5·0
Nonane (> 137° C T.B.P.)	—	—	1·7

Feed rate	6500 lb/h
Feed condition	Temp. 150° F (liquid)
Feed point	At column stage 14
Number of theoretical stages	36 (Including reboiler, excluding condenser)
Vapour loading point	At column stage 35
External reflux temperature	100° F
Distillate rate tolerance (ΔD)	Varies from ·002 to ·05 for different runs
Allowable mismatch at feedplate	0·002
Specific gravities, relative volatilities, and enthalpies	Constant for all runs

Use of Graphs

It is intended that the graphs be used as tools in the study of plant flexibility, and in evaluating and analysing proposed control systems.

Most of the graphs are in a readily usable form. For example, if plant settings are required to give 6% $_nC_5$ in the distillate with 80% recovery when taking feed from a stabilised Kuwait gasoline at 85° C T.B.P. then point Q on Fig. 4 indicates directly that the reflux ratio and yield settings are 12·5 : 1 and 13·6% approximately.

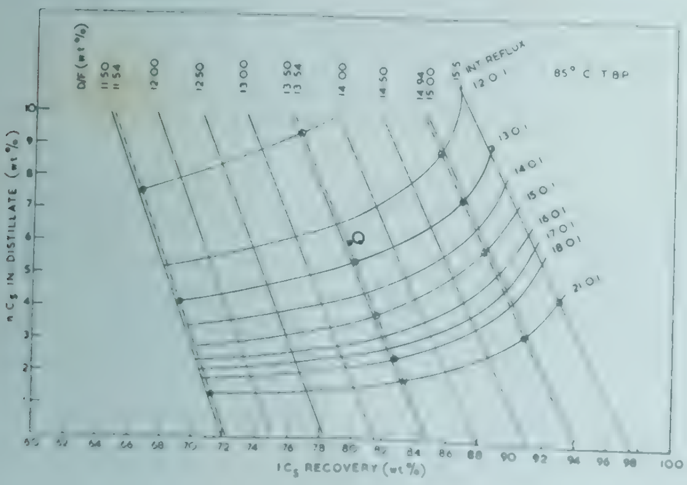


Fig. 4.—*Iso-pentane recovery—n-pentane overhead, 85° C T.B.P. feed*

The graphs can be used to give more information than appears at first sight. This becomes apparent in the following observations on control systems.

Data for Control Systems

The following is intended to demonstrate how information can be accumulated from the graphs for the ultimate purpose of analysing various control systems. Only conventional systems will be considered although the method of data extraction is similar to that for other systems.

Volume Control

In essence, volume control involves controlling the maximum number of flow rates at fixed values consistent with the number of degrees of freedom of the plant. In consequence, it is generally considered to be suitable only with feed of constant composition. Fig. 5 illustrates the particular system used in this example.

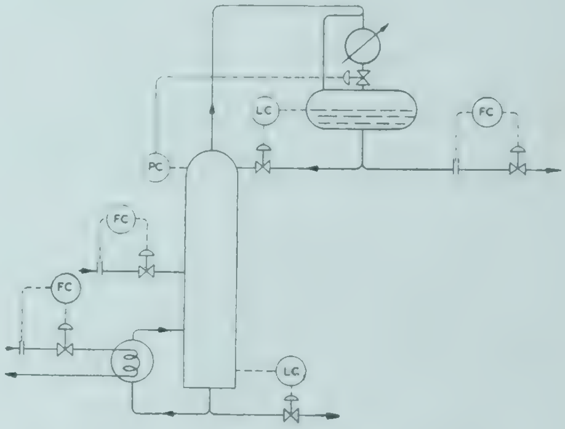


Fig. 5.—*Superfractionator volume control system*

If it is intended to ascertain the effect of a change in feed composition on the overhead purity and recovery, then Figs. 2, 3, 4, 6, 7, and 8 will be required. Assuming the plant to be producing specification distillate of 6% $_nC_5$ and 80% recovery of $_iC_5$ with an 85° C T.B.P. feed, then Fig. 4 indicates that reflux ratio and yield will be 12·5 : 1 and 13·6% respectively (point Q). For these values (Fig. 7) shows that the reboiler heat load will be 19 × 10⁶ B.t.u./h (point P). If the feed composition now changes to that of a 75° C T.B.P. and

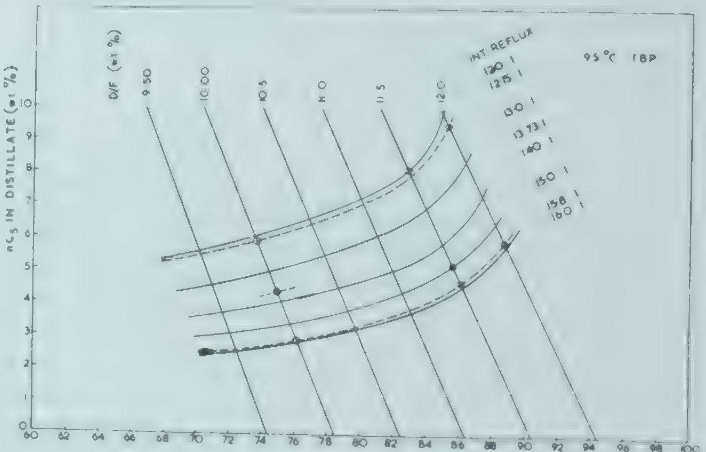


Fig. 6.—*Iso-pentane recovery—n-pentane overhead, 95° C T.B.P. feed*

the plant settings are not changed, then Fig. 2 indicates that for the same reboiler heat load and values of D/F, the overhead contamination decreases to 5·0% (point R). Fig. 3 shows that the recovery has decreased to 73·4% for these values of $_nC_5$ and D/F (point S).

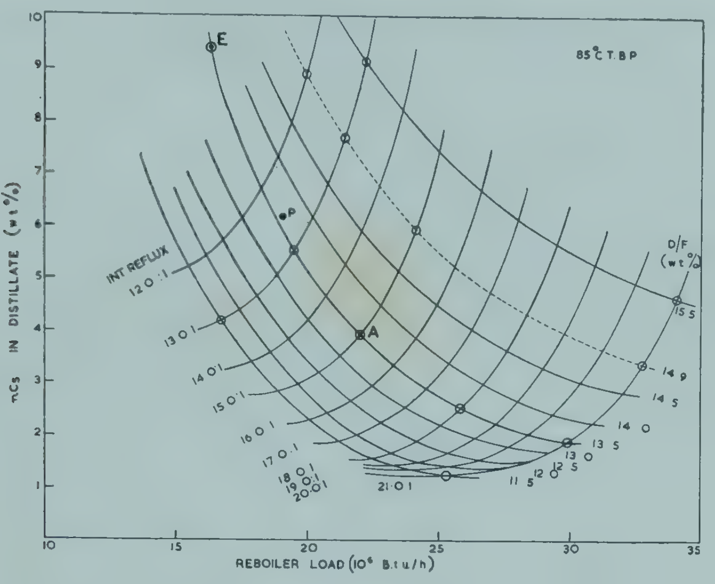


Fig. 7.—*Heat load—n-pentane overhead, 85° C T.B.P. feed*

If the feed composition changes to that of 95° C T.B.P., inspection of Fig. 8 will show that impurity of the distillate has increased to a point not represented on the graph. Specification tolerance, and the expected uncontrolled feed composition variations, will determine whether this type of control is suitable. In this particular example volume control is unsuitable if a feed variation of 85° T.B.P. to 95° T.B.P. is anticipated.

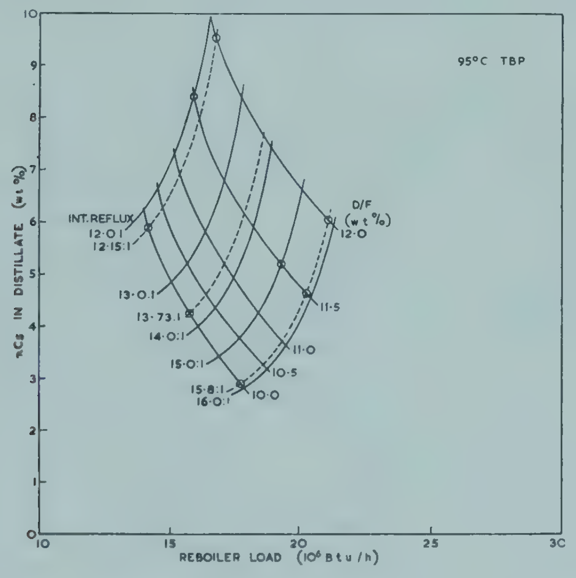


Fig. 8.—*Heat load—n-pentane overhead, 95° C T.B.P. feed*

Temperature control

Of the many forms of temperature control for distillation columns, that indicated in Fig. 9 will be considered. Similar reasoning can be used for other forms of temperature control.

With this system, the overhead purity is indirectly controlled by controlling the temperature of one column stage through regulation of the reboiler heat input.

One important characteristic to be determined for control analysis purposes is that of plant system sensitivity, the

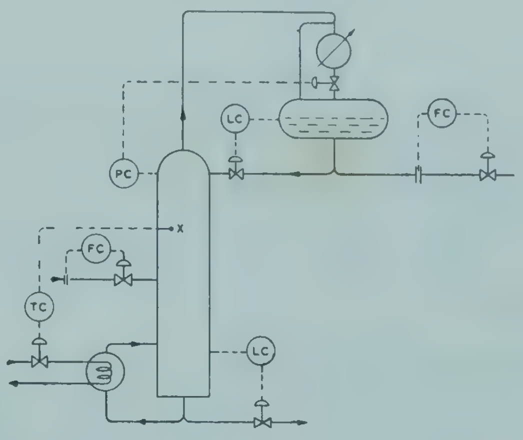


Fig. 9.—*Superfractionator temperature control system*

variation of overhead purity per unit temperature change at the controlled point

i.e.

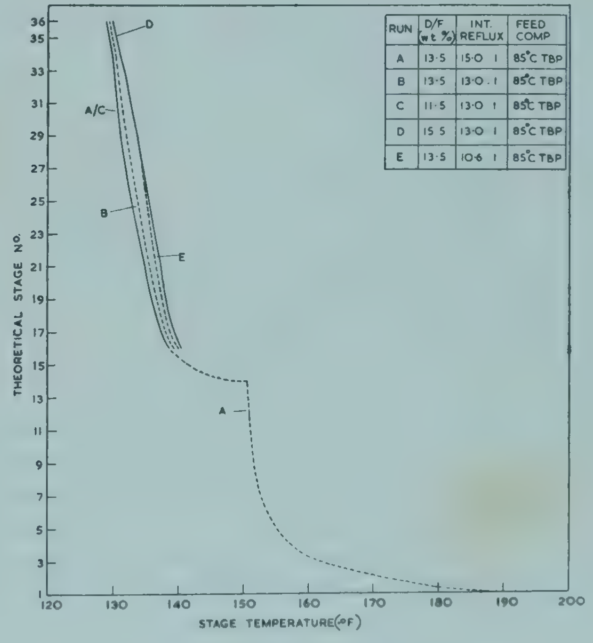
$$\frac{(\text{Change in } \% \ _nC_5 \text{ in overhead})}{(\text{Unit change in temperature at point x}).}$$

A second important characteristic is the plant control sensitivity.

i.e.

$$\frac{(\text{Change in reboiler heat input})}{(\text{Unit change in temperature at point x}).}$$

As we are dealing with an indirect control system, these two sensitivities are different.



RUN	D/F (wt %)	INT. REFLUX	FEED COMP
A	13·5	15·0 1	85°C TBP
B	13·5	13·0 . 1	85°C TBP
C	11·5	13·0 1	85°C TBP
D	15 5	13·0 1	85°C TBP
E	13·5	10·6 1	85°C TBP

Fig. 10.—*Column temperature profile—85° C T.B.P. feed*

Since the system is non-linear the sensitivities will vary according to the operating point. Those near the specification point will therefore be considered. Figs. 10 and 7 will be required. As the system is operating under constant ratio of D/F (13·6%) with the reboiler heat load varying about the specification point (19 × 10⁶ B.t.u./h *i.e.* point P on Fig. 7, senstivities will be obtained by straddling this point. Runs

A and E on Fig. 7 give the following information:

Change in reboiler heat load 22 − 16·1
= 5·9 × 10^6 B.t.u./h.
Change in $_nC_5$ in distillate
= 9·4 − 3·9 = 5·5%.

Assuming that point X in the column is at stage 26, run nos. E and A of Fig. 10 give the following information:

Change in stage temperature
= 134·5° − 132·4° = 2·1° F.

From the values obtained above the following sensitivities can be calculated:

Plant Control Sensitivity

$$= \frac{5 \cdot 9 \times 10^6}{2 \cdot 1} = 2 \cdot 81 \times 10^6 \text{ B.t.u./h F.}$$

Plant System Sensitivity
= 5·5/2·1 = 2·62%/° F.

The specification allows a contamination of 6%, and the latter sensitivity shows that contamination will vary by 45% of its maximum for each deviation of the controlled point by 1° F.

Other characteristics may be determined. The effect of changes in feed composition may be obtained in a manner similar to that discussed in the case of volume control. In this case a temperature profile similar to Fig. 10 would need to be derived for the 95° and 75° T.B.P. feeds (or other values according to the extent of the disturbance being investigated).

Quality control

"Quality control" here refers to automatic plant control and it requires a composition detector and a measuring device transmitting to a conventional receiver-controller.

Several monitoring points and regulating points may be considered as in the case of temperature control, but it is proposed to comment only on the most difficult (from the aspect of graph manipulation), *i.e.* a measured variable in the tower, there being a regulator on the reboiler heat input. Data for other methods can be similarly extracted.

As in the case of temperature control, the system and plant control sensitivities will be determined.

Fig. 11 indicates that stage 26 is a sensitive point in the column.

Sensitivities will again be obtained by straddling the specification point. Point Q, Fig. 4 indicates the specification point with reflux ratio and yield of 12.5 : 1 and 13.6% respectively. Fig. 12 shows that the $_nC_5$ concentration at stage 26 is 32% with the column operated to give specification distillate. It is necessary then to straddle the 32% point on the 13.6% D/F line on Fig. 13 to obtain plant control sensitivity. Considering runs A and E the following information is given:

Change in reboiler heat load
= 22 − 16.1 = 5.9 × 10^6 B.t.u./h.

Change in $_nC_5$ on stage 26
= 41 − 25·3 = 15·7%.

From Fig. 12 for the same change in $_nC_5$ on stage 26,

Change in $_nC_5$ in distillate
= 9.4 − 3 9 = 5·5%.

From these values the following sensitivities can be calculated:

Plant Control Sensitivity (Reboiler to stage 26)

$$= \frac{5 \cdot 9 \times 10^6}{15 \cdot 7}$$

= 0·38 × 10^6 B.t.u./h 1% on stage 26.

Plant System Sensitivity (Distillate to stage 26)

$$= \frac{5 \cdot 5}{15 \cdot 7}$$

= 0·35% in distillate/1% on stage 26.

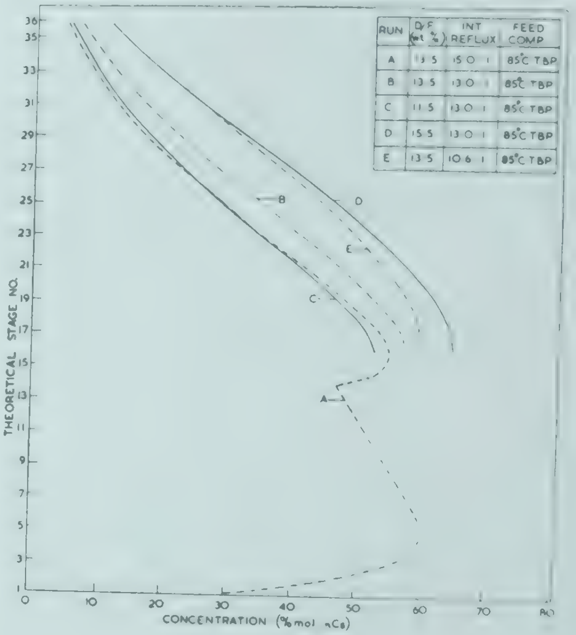


Fig. 11.—*Column n-pentane concentration profile—85° C T.B.P. feed*

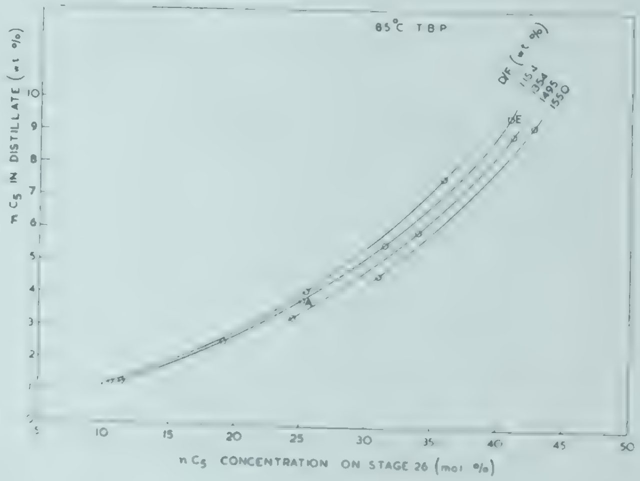


Fig. 12.—*n-pentane in liquid on stage, 26—n-pentane overhead, 85° C T.B.P. feed*

Similar sensitivities may be obtained for monitoring points at the feed and distillate for use in evaluating the various control methods. From the value of Plant System Sensitivity obtained above it might appear advantageous to monitor stage 26 rather than the distillate. This may be true, but it must be borne in mind that evaluation on a single factor is unwise and other factors such as plant lag, prevalent disturbances, and instrument sensitivity/range characteristic would have to be considered. This aspect will be included in future study but is outside the scope of this paper.

As in the case of temperature control, the effect of feed composition changes may be investigated. Profiles similar to Fig. 11 for the expected uncontrolled values of T.B.P. may be constructed to assess the most sensitive stage and graphs similar to Fig. 13 would be needed for these.

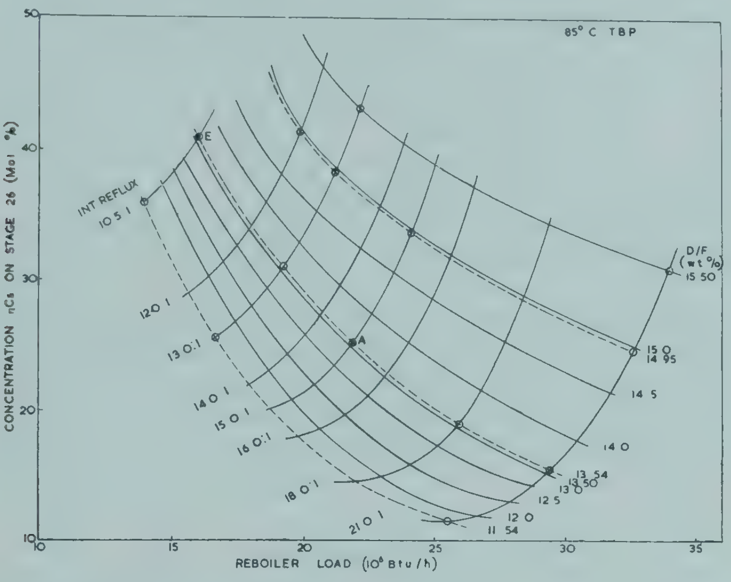


Fig. 13.—*Heat load—n-pentane in liquid on stage 26, 85° C T.B.P. feed*

Plant flexibility

It is considered that the data contained in the graphs may be manipulated to extract a considerable amount of inherent information. In considering plant flexibility for design or operating purposes it is important that the terms of reference within which the plant will be operated should be clearly defined. These may include operating to an optimum condition.

If it is possible to define an optimum, the graphs may be used to determine the optimum operating area. This will, by its nature, involve the variation of more than one parameter and the effect of variations within the operating area can be quantitatively determined.

Opinions on the definition of a particular optimum may vary considerably and it is a matter of specifying proper conditions to suit each individual case. For example, the following conditions may apply in some cases:

(1) Maximum distillate production to specification purity—to meet an emergency demand.

(2) The most economic production to specification purity—for normal production.

(3) Maximum distillate production to specification purity with minimum feed—to deal with a shortage of feedstock.

Plant settings for the above conditions can be obtained from the graphs. In determining the operating settings other factors, such as limiting vapour and liquid loading, costing factors, *etc.*, would need to be taken into account.

Future Study

In the immediate future, study will be concentrated on further analysis of the control systems already mentioned. The graphs will be used to obtain information on steady-state conditions for various combinations of measured and controlled variables, and the effect of several disturbances, such as feed temperature and feed composition, will be investigated.

The effect of changes in feed temperature and feed point on column operation for optimising purposes will also be studied.

Acknowledgments

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Appendix I

Method of constructing graphs

At the start of the work it was intended to obtain sufficient results from the computer to be able to plot all graphs directly. It soon became apparent that this method would require an extremely large number of computer runs and consequently a less costly method of construction was sought. The following method was eventually adopted.

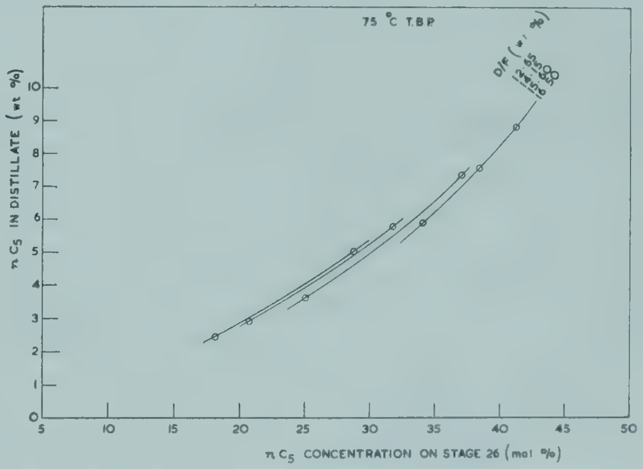

Fig. 14.—*n-pentane in liquid on stage 26—n-pentane overhead, 75° C T.B.P. feed*

It was expected, and confirmed by results, that the relationship between V/F (mol basis) and reboiler heat load was linear for a given feedstock and given feed condition. Values for this relationship were obtained from computer results and Fig. 18 was constructed for the three feedstock compositions. Since V/F is related to D/F and the reflux ratio by the equation

$$\frac{V}{F} = \frac{D}{F}(R + 1),$$

the reboiler heat load can be obtained for any pair of values

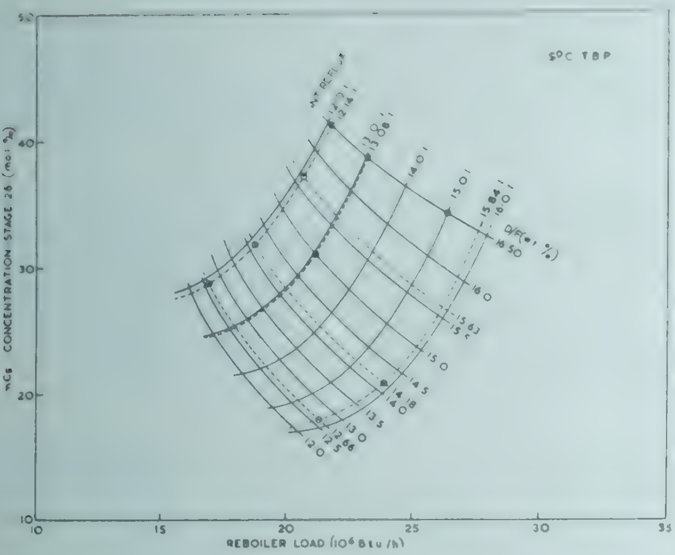

Fig. 15.—*Heat load—n-pentane in liquid on stage 26, 75° C T.B.P. feed*

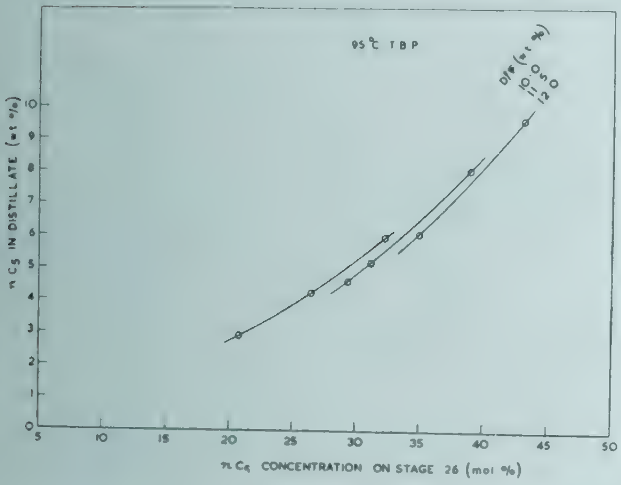

Fig. 16.—*n-pentane in liquid on stage 26—n-pentane overhead, 95° C T.B.P. feed*

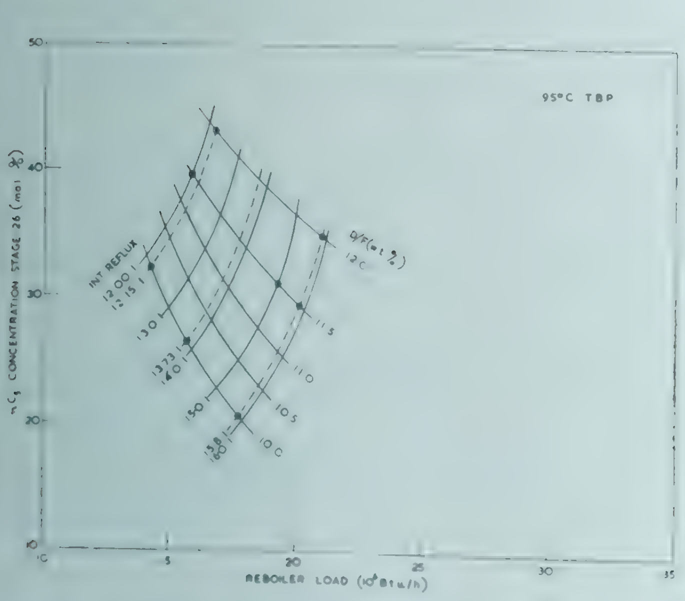

Fig. 17.—*Heat load—n-pentane in liquid on stage 26, 95° C T.B.P. feed*

of D/F and reflux ratio. For example in constructing a graph similar to Fig. 2, the x co-ordinate of the intersection between any pair of D/F and reflux ratio lines can be found by calculating V/F from the above equation and obtaining the reboiler heat load from Fig. 18.

No such simple relationship was found for the intersection of a D/F line, a reflux ratio line, and the corresponding Y co-ordinate ($_nC_5$ in distillate), without resorting to an artifice. A relatively large number of computer runs was made with a constant value of D/F (13.54% by wt.) and with different reflux ratios. It was found that plotting the reciprocal of the values of $_nC_5$ in the distillate on an arbitrary straight line gave an artificial reflux ratio scale, only slightly non-linear, and one which could be interpolated with a good degree of confidence (Fig. 19). It was then discovered that over the ranges covered by the graphs, the same reflux ratio scale produced linear results for other values of D/F. The Y co-

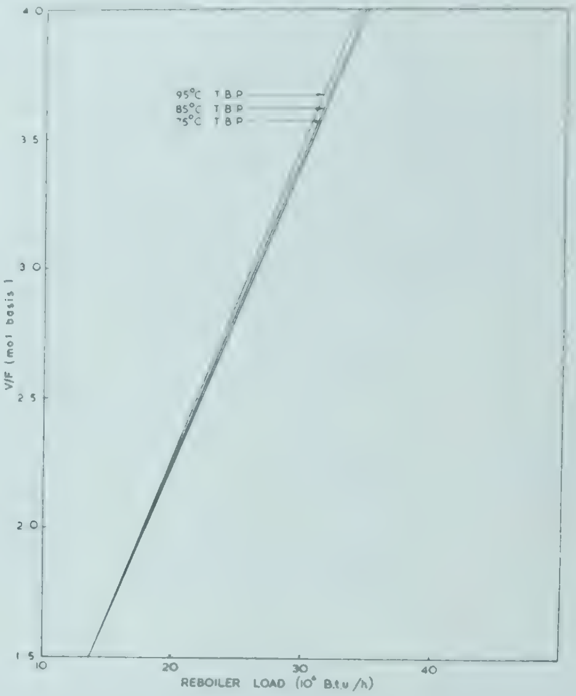

Fig. 18.—*Heat load—vapour feed ratio stage 35*

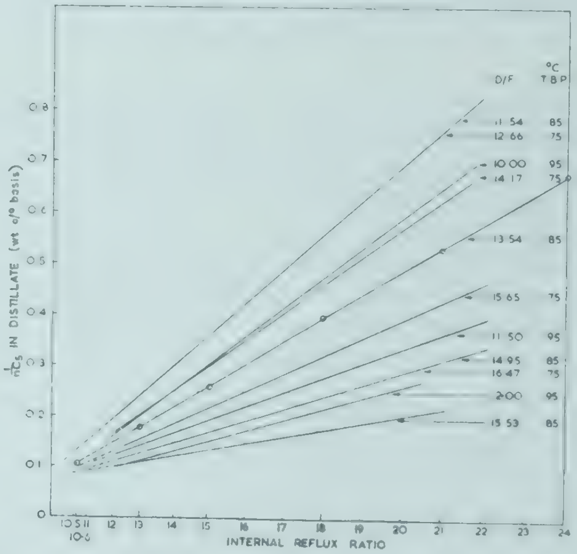

Fig. 19.—*Linear plot-internal reflux ratio—reciprocal of n-pentane in overhead*

ordinate referred to above can thus be obtained directly from Fig. 19 for any *D/F* line which has been constructed and any reflux ratio. The construction of any *D/F* line requires at least two computer results with common values of *D/F*.

It was found that the above method was extremely sensitive and that a high degree of accuracy was required to plot graphs such as Fig. 2. Surprisingly little smoothing of the plotted points was found to be necessary. There are however indications that the method in this particular instance is not reliable for reflux values below approximately 11 : 1.

Graphs with composition on stage 26 and reboiler heat as co-ordinates, *e.g.*, Fig. 13, were constructed on similar lines using Fig. 12, which gives the relationship between ${}_nC_5$ in distillate and composition at stage 26, as an intermediate step.

Development of graphs involving recovery, *e.g.* Fig. 4, was straightforward, bearing in mind that there is a linear relationship between ${}_nC_5$ in distillate and recovery of ${}_iC_5$, for a given value of *D/F* and feed composition, since the ${}_nC_4$ content is fixed and, for all practical purposes, appears only in the overhead product.

Appendix II

Change in feed point

EFFECT ON OVERHEAD PURITY

The effect of a change in the stage at which the feed enters the column on the overhead purity was investigated.

All previous computer results related to a feed point at stage 14. It was decided that one of these results near to specification should be straddled by further results using stages 9 and 17, as a first attempt, to indicate the direction and extent of change in overhead purity. These led to two other runs with stages 19 and 16 as feed points in order to obtain a smooth curve over the range of stages 9 to 19. The input data for all these runs were identical to that for the selected stage 14 feed point run (in particular reflux ratio and *D/F* were 13·0 : 1 and 13·54% respectively) so that any change in overhead purity could be attributed to change in feed point only. The results obtained were plotted in Fig. 20 which indicates that a flat optimum occurs at stage 14 or perhaps just below. The fact that the feed stage selected for previous runs coincided with the optimum is not surprising since the study was based on the specification of a real plant and the selection of feed point was naturally influenced by the design of the actual plant.

Some idea of the effect of a selection of feed point away from the optimum may be obtained by comparing the reflux ratio required to give the same overhead purity with that for a feed point at optimum. For example, from Fig. 20 it may be seen that the ${}_nC_5$ content of the product for a feed point at stages 17 and 14 is 5·9% and 5·6% respectively. From Fig. 4, which refers to feed entering at stage 14, it can be seen that for a *D/F* of 13·54% an ${}_nC_5$ content of 5·9% may be obtained at a reflux ratio of 12·6 : 1, a saving of approximately 3% in reflux or reboiler load.

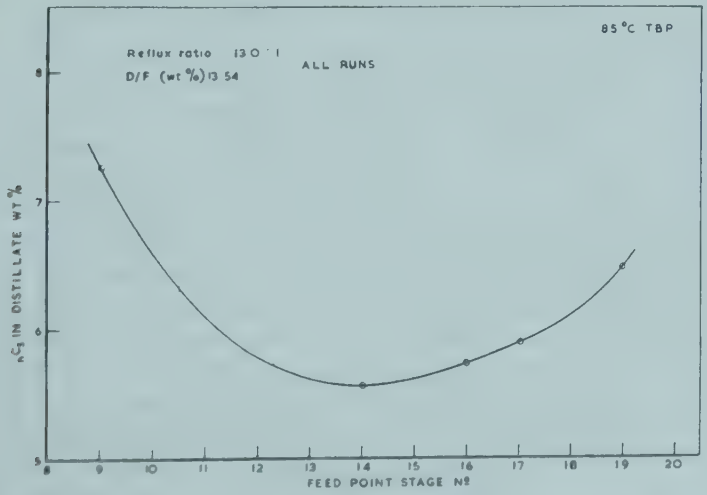

Fig. 20.—*Variation in distillate composition with feed point*

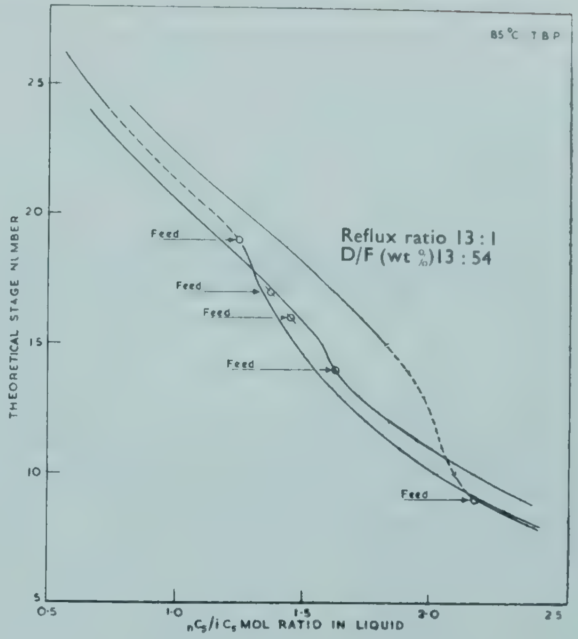

Fig. 21.—*Variation in ${}_nC_5/{}_iC_5$ ratio profile with feed location*

EFFECT ON COLUMN PROFILE

The column profiles of the mol ratios of *n*-pentane to *iso*-pentane in the tray liquid was plotted for three of the five feed stages as shown in Fig. 21. Only the ratios at the feed points of the other two were inserted for the purpose of clarity.

For the case of the feed point at stage 14, an extra computer run was made with the same input information, but calling for an *iso*-pentane concentration profile to be punched out in place of the *n*-pentane profile. This enabled an accurate ${}_nC_5/{}_iC_5$ curve to be constructed on Fig. 21. In the case of the other feed point stages, for economy reasons the curves were obtained by estimating the ratio from a knowledge of the ${}_nC_5$ content only. The hexane and heavier content in the upper half of the column is negligible from a few stages above the feed point.

The butane content is also negligible except for a few stages below the top (outside the range of Fig. 21).

The concentration of the heavy component is approximately constant below the feed point except for a few stages from the bottom of the column. The temperature profile, shows the regions over which these conditions hold. These regions are indicated by the full curves in Fig. 21.

It may be inferred from Fig. 21 that the distortion of the profile about the feed point is related to the optimum feed position. For example, Fig. 20 shows that stage 14 is at the optimum, and Fig. 21 indicates that the profile for this feed point is less distorted than that for a feed point either above or below the optimum.

An interesting point to note is that for a given error displacement of the feed stage point from the optimum, the deterioration in the overhead purity is less if the displacement is above the optimum rather than below, as indicated in Fig. 20.

DISCUSSION OF PAPERS PRESENTED AT THE FIRST SESSION

Dr. F. R. **Himsworth** said that he had been struck with the wide variety of the papers. That, he supposed, was inevitable because of the great generality of the subject under discussion. Some authors had dealt primarily with process development and control, mentioning incidentally the help of computers. Others had very rightly described new computation and control techniques, illustrating them with rather simple process control problems. One result was that for the not-too-highly-mathematical chemical engineer some of the problems, which he knew were difficult, appeared rather easy, and some which he recognised as very simple and almost trivial appeared to be rather difficult. He suggested that when control and computing experts were writing on new techniques they should include an explanation in everyday language of what they were setting out to do, what assumptions they were making on the way, and what conclusions they had reached.

A performance index was introduced on page 11 of the paper by Kalman, Lapidus, and Shapiro. It was entirely rational and as good an index as one could choose, but as presented it seemed rather arbitrary and the problems of asking management to decide on values of $\mathscr{P}$, α, and possibly a semi-definite matrix, appeared formidable. The explanation of what the performance index meant, as it would be explained to the management—as it must be explained to management some day—would have added to the value of the paper.

The paper by Coutie gave the reverse side of the picture. It was a good example of a complex problem—because the plant problems which the author was discussing were complex—explained in simple language. Coutie, however, might have fallen into the error of making it appear too simple. The work which he described was based on a great deal of theoretical work by Dr. Box and himself and on many years of patient work in co-operation with plant manufacturers. No one should be misled into thinking that what had been described very simply was in fact a simple problem. The evolutionary operation mentioned was not simply a black box which churned out results but rather a new philosophy of operating plants based on the good old principle that in this life one got nothing for nothing. All plants in operation showed changes—random changes and non-random changes—and it was often thought that by analysing those accidental changes meaningful conclusions could be drawn. In simple and rather fortunate cases that might be so, and from the analyses of such changes one might get meaningful results. But the old principle of nothing for nothing often applied and elaborate computational methods did not change the principle. In fact, there was a danger that by doing bigger sums they might just multiply a number of wrong conclusions. That was not a condemnation of such methods of analysis but merely a cautionary word that the apparent results were not always as good as they might seem

Kalman, Lapidus, and Shapiro had touched on the same question in their Fig. 1 where they showed a black box C which fed in perturbations. Presumably it was part of the evolutionary operation in which, from time to time, a number of changes in the variables were made and their effect discovered. That was likely to be a more rewarding method than taking the changes which happened to occur and finding their apparent effects.

One point which had struck him was that there was singularly little mention of instrumental errors and inaccuracies. It might not seem important to achieve instrument accuracy to within 1% but instruments were the basis of all the techniques described. He was not suggesting that the problem had been overlooked but it was not emphasised sufficiently and he hoped that its importance was not being forgotten. For process development and control problems, especially the more advanced control problems, a different order of accuracy was wanted from that which was acceptable in conventional control.

The four papers were, each in its own way, very interesting examples of the different lines of development being followed. All led eventually to the same goal of improved process efficiency. He emphasised, however, the need for control engineers, computer engineers, chemical engineers, and managers, to talk to some extent the same language.

Kalman, Lapidus, and Shapiro hoped that the new concepts which he had discussed would become the bread and butter of the chemical engineer. This was extremely desirable although it might be rather far away. It reminded him of an anecdote told by Sergeant Sullivan about an Irish barrister who was representing a farmer in some land dispute. In the middle of the case the judge said to the barrister, "I take it your client is familiar with the doctrine *ignoratio non excusat*?" "Your Honour", replied the barrister, "In the woilds of Connemara the talk's of nothin' else".

Mr. B. D. **Dagnall** said that the papers by Coutie and by Kalman, Lapidus, and Shapiro, were of high quality and great interest but they differed considerably in treatment. One was practical and the other theoretical. He found it difficult to reconcile some of the theoretical claims of the second paper with the results obtained in practice and quoted in the first.

In particular, Kalman and his colleagues, in describing the function of box C in their hypothetical computer, stated that it was "used to generate and inject special calibration signals into the process. These signals are of such a level and form that they do not affect the normal operation of the process significantly, yet they perturb the process in such a way that useful information can be obtained". From Coutie's examples of evolutionary operation, however, it was clear that this process might well take months or years. The time taken depended on the plant and not on whether a human being or a computer was used to analyse the responses. It was merely a function of the characteristics of the plant. Were they to understand, therefore—it was not quite clear in the paper—that the authors proposed that a large digital computer should be linked continuously to a chemical plant for this purpose, or was it to be used intermittently? If it were to be used intermittently a whole host of practical difficulties arose. On the other hack, if it were to be used continuously they must think about the economics of the process.

It had been suggested that the computer needed for this purpose would be of the IBM 700 series, and the IBM 701 had been quoted. For the price of a 701 one could employ over 100 engineers, build a new laboratory, buy a tanker, or even buy half a million pairs of surplus boots from the Ministry of Supply! They must consider whether the money might be better spent.

That led him to the question of economics, which he felt had not received the attention which it deserved. It was not sufficient to show that a proposal was technically feasible: it must be shown to be economically attractive.

Both papers brought out the value of running a plant as long as possible on one operation and on uniform feedstock. If conditions were changing rapidly one did not have time to optimise the plant's operation. Those who were used to operating with running feed tanks would appreciate that point, because it was difficult for them to use the method of evolutionary process control.

Coutie's paper was valuable because it showed what could be achieved by very simple methods. Although the work was based on complex theory, the application of evolutionary process control was simple. Finally, whilst on the subject of simplicity, he made a plea for the more general use of quality control charts in the chemical and petroleum industries. They were delightfully simple. They had been used for years in the engineering industry. In some applications in the chemical industry they could be of considerable value in helping the operator to reduce the variation in the quality of the product.

Dr. H. H. Rosenbrock commenting on the papers by Florentin, Hainsworth, Reswick, and Westcott, and by Keating and Townend said that the practical difficulties of the alternative approach of injecting sinewaves or square waves into a plant were well known. Probably one of the biggest difficulties was the vigorous response from the plant operator before the disturbance was put in!

There was a great incentive to find out as much as one could about plants by using the random fluctuations which were always there in practice. So far, despite its attractiveness, the method had not been much applied. The technique had not been used to obtain information from plants, and if the authors could clear away some of the difficulties they would be doing a valuable service.

One difficulty which was often quoted was the fact that unless the frequency band of the disturbance covered the frequency band in which one was interested, one could not find out anything about plant behaviour at those frequencies. Absence of particular frequencies would have just the same effect if one worked in the time domain, but would, he thought, be much more difficult to detect.

The technique which Florentin *et al* had developed for dealing with noise was ingenious. Noise had been one of the stumbling blocks in applying the theory in practice. He was not sure whether he had understood their method correctly. They seemed to assume that there was no noise except the bare minimum necessary to make their transfer function realisable and compatible with the recorded disturbances. That meant that errors in measurement were treated as noise—as they should be—but it also seemed to mean that errors which arose from using a finite length of record appeared partly in the noise. If there were actual noise present in the process it partly appeared in their analysis as noise but, as far as he could see, it also partly appeared by modification of the transfer function which the authors obtained. Perhaps he had misunderstood the point, and he would welcome clarification.

Turning to the paper by Keating and Townend, Dr. Rosenbrock said that the outstanding requirement at present was to get simpler techniques for obtaining information about the behaviour of plants when one wanted to control them. It was one thing to measure the response of a plant once it had been built. That was extremely useful and could confirm or modify theory, but it was no use in trying to design a plant for control or to improve the design of a plant, because it applied only to the particular plant on which the measurements had been taken.

The paper illustrated the great difficulties, and the differences between the process control field on the one hand and mechanical servos on the other hand. Mechanical servos were largely amenable to mathematical analysis, at least as a first approximation, and they could also be simulated fairly directly. But even a distillation column, which was a fairly simple piece of equipment—the processes going on were entirely physical—needed the full resources of digital computers applied for long periods of computation.

He had made some similar calculations, although not in such great detail, and had reached the conclusion that there was a great need for simpler methods of dealing with the problems because without them there was little hope of obtaining economically the results which they needed in order to design the system. It was no use spending more money on saving a controller than the controller itself was worth.

The information which the authors presented was extremely interesting and he wondered whether, as a result of cogitation over their results, the authors had come to any general conclusions about the control of distillation columns. One point which seemed to have arisen from his own work was that if the temperature in the column were to be controlled it was usually better to control the top product off-take rather than the reflux back into the column, or similarly better to control the bottom product take-off than the boil-up. He wondered whether the authors agreed, and whether they had reached other general and simple conclusions.

Mr. J. W. Broadhurst said tha the and his colleagues had used a similar approach to the problem, similar to that used by Keating and Townend but with manual computation. Expressions similar to the authors' "Plant Control Sensitivity" and "Plant System Sensitivity" were used to assess:

(*a*) The extent to which it was necessary to install additional control systems to minimise disturbances entering distillation columns.

(*b*) The form of control which was likely to be most satisfactory.

(*c*) In borderline cases, whether the proposed control equipment was likely to have adequate proportional band width.

Distillation columns were best controlled by determining and controlling the position of the composition pattern, *e.g.* by measuring the temperature at a point in the column and using it to regulate the heat supply to the reboiler. In many cases a single controller was adequate, but consideration of the accuracy with which the heat supply to the reboiler had to be maintained to give the required product purity might indicate that small changes in the pressure of the steam supply were sufficient to produce unacceptable changes in product composition. In such cases a steam-pressure controller might be installed so that the temperature controller might more adequately correct for other disturbances which could not be eliminated at source. Alternatively a cascaded system employing a steam flow controller might be used if the pressure of the available steam was low and the pressure loss resulting from two control valves in series could not be tolerated.

If a column was controlled by fixing the position of the composition pattern then various forms of measuring element could be used such as temperature, temperature difference, vapour pressure, and analysis instruments. The choice between them was largely governed by the changes which occurred with plant load changes in the static pressure of the column, at the point of measurement. Consideration of the change in product composition with static pressure—assuming the column to be controlled in turn by each of the variables considered—would show which was the most suitable detecting element, bearing in mind that it was advisable to choose the simplest system which had the required sensitivity. The approach was similar to determining the authors' "Plant System Sensitivity" for each of the measuring elements and determining the change in them with static pressure in the column. In that case, however, consideration of the static conditions in the column alone was inadequate because the ultimate quality of control would depend on the dynamic characteristics of the plant. It was there that, at present, the theoretical approach broke down because of lack of knowledge of the dynamics of distillation columns.

The examination of borderline cases was really an extension of the choice of measuring element and again applied particularly to columns producing high purity products. The above considerations might have led to the choice of, say, temperature as the measured variable but the accuracy of temperature measurement required might be such that an instrument with a narrow span and high sensitivity was necessary. The proportional band adjustment was limited on conventional controllers and if the span of the instrument had to be suppressed too far it might be impossible to obtain a sufficiently wide proportional band to achieve stable control. Even though the dynamic characteristics, and therefore the attenuation, of a distillation column were not known it was still possible to calculate for the worst conditions by assuming that the attenuation was unity. That provided an indication of the maximum proportional band required for stable control and therefore avoided the possibility of instability due to lack of sufficient mechanical adjustment in the chosen controller. The assumption of unit attenuation was really a factor of ignorance because the degree attenuated was not known and clearly the assumption must lead to over-design, and possibly to the rejection at the drawing office stage of a measuring system which would have operated successfully.

The need for further information on the dynamics of distillation columns was pressing. Did the authors envisage any programme to obtain information on that aspect; in particular did they propose to set up any mathematical models to obtain information on the dynamic performance or had they any other approach in mind?

Mr. H. B. **Wiener** said that he was much impressed by the analysis of the way in which digital computers would be applied to the running of chemical plants given by Kalman, Lapidus, and Shapiro, and he wanted to say something about the gap between that kind of thinking and its practical application. Mr. Broadhurst had referred to the point very forcibly, and he wished to return to it because he thought that someone should put in a word for the digital method, which was so powerful in principle that in the end it would certainly win.

There were two obstacles to the application of computer control. One was fundamental and the other was economic. The fundamental objection was that the behaviour of a large-scale chemical reaction could be extremely complicated when side reactions and heat and mass transfer effects were taken into account, and that changes could occur so rapidly that quite large adjustments to control valves might have to be made after one programme cycle. A complete optimising programme would need to be very elaborate, and he doubted whether the cycle could be completed quickly enough on a machine of economic size.

The economic obstacle was that the computer would save neither labour nor instruments, since the kind of plant in which it could be applied would already be highly instrumented and thinly manned. One would not risk putting all one's eggs into one basket and suddenly having to control half-a-dozen streams manually if the computer failed. Conventional controllers would therefore be kept working as a second string, to be switched in when a failure occurred. The computer would therefore have to earn its keep entirely by improving the operation of the plant.

The retention of conventional controllers might suggest that the computer should operate on their setpoints rather than on the plant itself. He thought that that would be a mistake because control would be less effective with an additional mechanical element with its own dynamic properties included in the loop. If the computer were also to adjust proportional bands, integral action, etc., the amount of electro-mechanical equipment became alarming. In addition, the output of the measuring instruments could not then be linearised, corrected, and compounded before being applied to the controllers, unless additional mechanical devices were used.

In the first instance, therefore, the computer should take over the functions of the assembly of conventional controllers plus an experienced human operator, maintaining steady operation with a number of set conditions and catering for upsets due to mechanical failures and temperamental behaviour of the chemical reaction. He thought that only then would computers be widely applied in chemical plants—when they justified themselves economically by doing that job better before any optimising procedure were applied. Although, as Coutie pointed out, the set conditions found by experience were often very close to the optimum, there was still a lot to be gained in many processes by getting better control action. If the process were so well behaved that one could not improve it by getting better control action against the normal upsets which occurred, then there was no case at all for a computer on the plant because one could make a statistical study such as had been described to determine the best operating conditions for the plant.

What seemed to be needed for progress was a package which required no further development by the user and little maintenance, not necessarily of very high capacity but supplied with standardised units for applying the output direct to the plant controls. The first crude control programme should be provided by the makers—the programme for taking over the functions of the conventional controllers—because the makers would have acquired experience in writing such a programme. It should not be ambitious but should be based on the known dynamic behaviour of the measuring instruments and controls of the plant plus some of the predictable interactions which were known to occur when these systems were controlled by separate loops which could not be tied together because of the nature of the conventional controllers. The users would then operate on that crude programme in the first instance, just as management now operated on the human operator, by feeding in different kinds of set conditions. Later they would use the spare capacity of the machine for testing optimising programmes which would probably have to be of the statistical, evolutionary-operation type, working from one group of set conditions to another. Later still if they understood the process they could use the more sophisticated, dynamic optimisation kind of programme.

He thought that they must have that kind of package which paid for itself without years of research before there were too many confirmed sceptics who had installed a large general-purpose machine on a plant, which had generated a lot of work and proved very instructive, but which had never been intended as an economic proposition at the time, and which now seemed to prove conclusively that it never could be an economic proposition.

Mr. R. A. Woodrow said that he had been particularly interested in the correlation analysis of the heat exchanger.

His first comment concerned the type of solution which the authors sought. The system with which they were dealing was taken to be a linear active system as demonstrated by the form of equation (1) of the paper while the solution sought was a best linear passive approximation. That raised the question of what was the "best" linear passive description of the data.

Goodman and Reswick (Ref. 2 of the paper) had assumed that in the case of data taken from open loop systems such as in Fig. 1, the best linear passive description of the data was that one in which n(t) and x(t) were uncorrelated, while for data taken from closed loop systems such as Fig. 2 the best description of the data was assumed to be that in which s(t) and n(t) were uncorrelated.

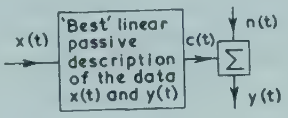

Fig. 1.—*Open loop system*

In the present paper the authors suggested an alternative definition of the best linear passive approximation to the data defined as that quantity which minimised the mean square value of n(t) of Fig. 1. That was equivalent to the formulation of the problem: given the input variable x(t) and the output variable y(t) of a physical system, find a physically realisable linear passive operator which, when operating upon the input variable x(t) produces an output variable c(t) having the property that the operator minimises the mean square value of $[y(t) - c(t)]^2$.

That formulation took no account of the origin of the data, *i.e.* whether from an open or closed loop system—and could in certain circumstances lead to quite a different form of weighting function from that sought by Goodman and Reswick.

To demonstrate that, consider the situation of Fig. 1 when representing part of the closed loop system of Fig. 2. In that case the Goodman-Reswick method seeks to find a linear passive description of the data having the property that x(t) and n(t) are correlated in some particular fashion. But, accepting the minimum mean square error criterion suggested in the present paper, the weighting function obtained was quite different. To appreciate that, consider Fig. 1 with n(t) and x(t) correlated.

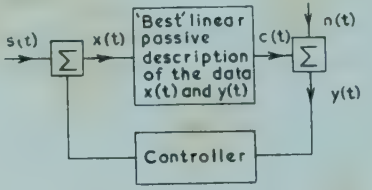

Fig. 2.—*Closed loop system*

Seek a function $h_1(t)$ such that

$$n(t) = n_1(t) + \int_0^\infty h_1(u)\, x(t-u)\, du \quad . \quad (1)$$

where $n_1(t)$ is uncorrelated with $x(t)$.

Multiplying throughout by $x(t-T)$ and averaging

$$\phi_{xn}(T) = \int_0^\infty h_1(u)\, \phi_{xx}(T-u)\, du + \phi_{xn_1}(T) \quad . \quad (2)$$

But for the overall system

$$y(t) = n(t) + \int_0^\infty h(u)\, x(t-u)\, du \quad . \quad . \quad (3)$$

which after multiplying by $x(t-T)$ and averaging gives

$$\phi_{xy}(T) = \phi_{xn}(T) + \int_0^\infty h(u)\, \phi_{xx}(T-u)\, du \quad . \quad (4)$$

Combining equations (2) and (4)

$$\phi_{xy}(T) = \int_0^\infty \{h(u) + h_1(u)\}\, \phi_{xx}(T-u)\, du \quad . \quad (5)$$

since $\phi_{xn_1}(T) = 0$.

Equation (5) was the solution obtained by a minimum mean square error criterion for closed loop systems, and yields a weighting function $h(u) + h_1(u)$ whereas the Goodman-Reswick formulation requires the determination of $h(u)$.

Mr. Woodrow said that these calculations led him to wonder why the authors stated that ". . . these formulae cannot be applied directly in a feedback system . . ." and why they then fell back on equation (5) of the present paper which assumed quite a different criterion for the best h(t) from that defined earlier in their paper. Could the authors make that point clear?

Mr. Woodrow continued that he had not attended the Dusseldorf Conference, neither had he access to Reference 7 which might account for his difficulty in interpreting the solution outlined in Fig. 7.

The method described always led to a solution of the form:

$$\phi_{xy}(T) = \sum_{r} a_r \, \phi_{xx}(T - T_r)$$

which was equivalent to the assumption that the function h(t) of equation (2) of the paper was of the form

$$\mathrm{h}(u) = \sum_{r} a_r \, \delta(u - T_r)$$

That must not be confused with the approximation

$$\mathrm{h}(u) = \sum_{r=1}^{n} a_r \, \delta(u - rT)$$

used in the method of solution of Reference 2, where nT was the memory time of the system and n was large (order 20 or more) to make T sufficiently small "so that the essential character of the response is retained" (Ref. 2 of paper).

In the present case the solution involved three terms only which suggested that the physical system could be reduced to a number of delays together with appropriate attenuators as in Fig. 3. Was it not an oversimplfication accompanied by excessive inaccuracy to suppose that the weighting function could be approximated by as few as three delays?

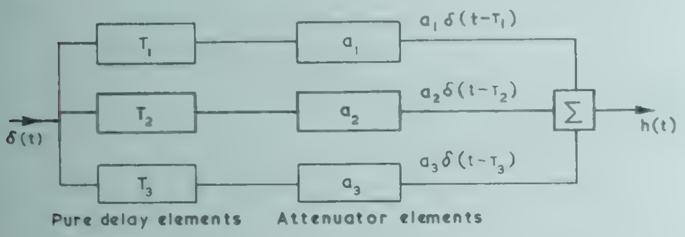

Fig. 3.—*Suggested system of a number of delays and appropriate attenuators*

Finally, he was puzzled about the question of lag windows and equations (11) and (7) of the text. These could be compared, but differed fundamentally in that equation (7) was exact if an ideal experiment of infinite time duration was assumed, whereas equation (11) involved the auto-correlation of a finite sample of x(t) (duration of 10 min in the case considered). Giving that the symbol $\phi^{s}_{xx}(t)$ to avoid confusion with the exact autocorrelation function of equation (7) and making the observation that the lag window D(t) was chosen to make the power spectral density functions of the sample [equation (11)] and of the complete record [equation (7)] closely the same it followed that

$$\phi_{xx}(t) = \mathrm{D}(t) \, \phi^{s}_{xx}(t) \quad . \quad . \quad . \quad (6)$$

The paper suggested that, with only $\phi^{s}_{xx}(t)$ as available experimental data it was possible to choose D(t) to satisfy equation (6). Could the authors say

(*a*) how, in general, D(t) may be selected,

(*b*) why it was not used in the time domain solution to improve the correlation records before attempting a solution to the integral equation (2)?

Mr. L. von Hamos, commenting on the paper by Florentin *et al.*, said that some work had been done in his laboratories on the fundamental basic features of auto-correlation functions to find out about some of the problems which apparently existed. On the one side they had the theoretical mathematics, which gave complicated formulae for such information. On the other side there was the engineer who would like to have apparatus which gave a measure in a finite time of the statistical properties of the phenomena which were met, for example, in the chemical industry.

They found that the calculations were too complicated to be carried out by the methods of the pure mathematician. Moreover, if they found the mathematical formula, they had no time to obtain a numerical evaluation of it, even by the big machines such as the IBM 701. It took too much time. His approach was therefore different. He simulated the mathematics. Noise generators and filters were used to produce the noise of known characteristics—not the unknown noise which the authors had mentioned. He produced a well known noise and made experiments with it by analogue and digital computers. Thus he could calculate the estimates and the necessary window functions in a very short time. He suggested that they must obtain a more accurate idea about such things before they could investigate the very complicated dynamical processes by statistical means. The heat exchanger alone was very complicated. Some of its dynamical properties, such as resonance in the frequency response are not yet well understood. Results of A. Hempel (Oslo) and J. R. Jensen (Copenhagen) presented at a symposium in Bergen in December, 1958 show also discrepancies between theory and experiments.

One point to be noticed in the statistical technique, however, was that if the natural deviation of a control process were used (the natural noise), they were not sure to have the uniform spectral density of the input where there was a closed loop. Some of the spectrum was emphasised, where the frequencies were badly controlled, and that was not so where the frequencies were well controlled. There was no possibility of finding out the transfer functions in the frequency regions where there was perfect control.

It should be mentioned that the estimation of cross-correlation spectra has been treated by N. R. Goodman.[1]

Mr. C. A. J. M. van der Heijden emphasised a comment made by Dr. Rosenbrock about the distillation column illustrated in Fig. 9 of the paper by Keating and Townend. The disadvantage of the scheme could be demonstrated by means of Fig. 19 of the same paper by assuming that a constant temperature in the top of the column meant a constant ${}_{n}C_5$ content of the top product at constant pressure. That, of course, was not quite true, but the assumption might be made in the example to show qualitatively what happened if the feed composition changed, for instance, from T.B.P.85 to T.B.P.95. For an operation in which D/F equalled 11·54% and $1/{}_{n}C_5$ equals 0·2, Fig. 19 showed that the control scheme would try to change the internal reflux rato from 12 to 20. In general it might be said that the control scheme tended to give very large changes of load in the column for quite normal disturbances in feed composition.

Mr. W. R. Burton wrote.

The paper by Coutie has given a lucid explanation of process optimisation by statistical procedures. It may be from consideration of space that Coutie has confined his review to consideration of process operating at steady levels of variables such as temperature, concentration, and pressure. This is characteristic of reactions where cost of materials, and consequently yields, dominate the cost of operation.

Many processes, however, involving expensive equipment and requiring product of given specification depend for their cost on throughput. This, in turn, may depend not on steady levels of operating characteristics but on the way in which the latter vary.

An example may pinpoint the problem: in the reactions

$$A + B \longrightarrow AB \quad . \quad . \quad . \quad (6)$$

$$A \longrightarrow A^* \quad . \quad . \quad . \quad (7)$$

$$A^* + B \longrightarrow AB \quad . \quad . \quad . \quad (8)$$

the required product AB is formed by reactions (6) + (8) both of whose rates increase with temperature. Reaction (8), however, is much slower than reaction (6) because A* is an inactive species of A formed by reaction (2) which has also a positive temperature coefficient.

The time taken to reach a specified conversion to AB is thus markedly dependent on the temperature-time profile followed. In fact, in an actual plant case specifications could not be reached if the initial temperatures were too high.

Could Coutie outline how the optimum time-temperature path be most rapidly pointed out by statisical—and preferably evolutionary—procedures?

Mr. R. A. Woodrow wrote:

Assuming with Florentin *et al.* in the paper "Correlation Analysis of a Heat Exchanger" that Fig. 4B of their paper "is a reasonable compromise between mathematical simplicity and engineering reality", I should like to ask why they have not exploited the mathematical simplicity of their assumption?

To make this point, let us formulate the equations relating the parameters defined in Fig. 4B. These are:

$$\mathrm{n}(t) = \int_0^\infty \mathrm{h}_w(u)\, \mathrm{W}(t - u)\, \mathrm{d}u \quad . \quad . \quad (9)$$

$$\mathrm{T}(t) = \mathrm{n}(t) + \int_0^\infty \mathrm{h}_x(u)\, \mathrm{x}(t - u)\, \mathrm{d}u \quad . \quad (10)$$

and

$$\mathrm{x}(t) = \int_0^\infty \mathrm{g}(u)\, \mathrm{T}(t - u)\, \mathrm{d}u \quad . \quad . \quad . \quad (11)$$

These three equations contain four unknowns, and thus cannot be solved as they stand. But, in this problem, both open loop and closed loop data were collected, and these provide adequate information for a solution.

Denoting data collected from an open loop experiment [g(*t*) = 0)] by a suffix o, and that collected from closed loop data [g(*t*) a function to be determined] by the suffix c, the following equations result.

(*a*) Open loop test [g(*t*) = 0]

From equation (11):

$$\mathrm{x}_0(t) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

From equations (10) and (12):

$$\mathrm{T}_0(t) = \mathrm{n}_0(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

From equations (13) and (9):

$$\mathrm{T}_0(t) = \int_0^\infty \mathrm{h}_w(u)\, \mathrm{W}_0(t - u)\, \mathrm{d}u \quad . \quad . \quad (14)$$

Equation (14) may be solved directly for $\mathrm{h}_w(u)$, this being the only unknown in the equation.

Turning to the closed loop test, and bearing in mind that, as a result of the previous test, $\mathrm{h}_w(t)$ is now known, it follows from equation (9) that

$$\mathrm{n}_c(t) = \int_0^\infty \mathrm{h}_w(u)\, \mathrm{W}_c(t - u)\, \mathrm{d}u \quad . \quad . \quad (15)$$

Equation (15) can be solved for $\mathrm{n}_c(t)$ from the known value of $\mathrm{h}_w(t)$ and the data $\mathrm{W}_c(t)$ collected during this test.

From equation (10)

$$\{\mathrm{T}_c(t) - \mathrm{n}_c(t)\} = \int_0^\infty \mathrm{h}_x(u)\, \mathrm{x}_c(t - u)\, \mathrm{d}u \quad . \quad (16)$$

Equation (16) may now be solved for $\mathrm{h}_x(u)$ the only unknown in this equation.

Finally, from equation (11)

$$\mathrm{x}_c(t) = \int_0^\infty \mathrm{g}(u)\, \mathrm{T}_c(t - u)\, \mathrm{d}u \quad . \quad . \quad (17)$$

which may be solved for g(*u*), the only unknown.

Equations (14), (16), and (17) are all of the same form, namely:

$$\mathrm{f}_0(t) = \int_0^\infty \mathrm{W}(u)\, \mathrm{f}_1(t - u)\, \mathrm{d}u \quad . \quad . \quad (18)$$

If it is assumed that:

$$\mathrm{W}(u) = \sum_{r=1}^{n} a_r\, \delta\left(t - r\frac{T}{n}\right) \quad . \quad . \quad (19)$$

where T is the memory time of the system, $\delta(t)$ is the unit impulse function, and a_r are constants to be determined (see Ref. 2 of the paper for a detailed discussion of this assumption), then equation (18) reduces to:

$$\mathrm{f}_0(t) = \sum_{r=1}^{n} a_r\, \mathrm{f}_1\left(t - r\frac{T}{n}\right) \quad . \quad . \quad (20)$$

Using equation (20) for n different values of t it is possible to formulate n different equations to solve for the parameters a_r. These form a set of equations which may be represented by the single matrix equation

$$\mathbf{F}_1\mathbf{a} = \mathbf{f}_0 \quad . \quad . \quad . \quad . \quad (21)$$

where $\mathbf{F}_1$ is an $n \times n$ matrix, $\mathbf{a}$ an $n \times 1$ column vector and $\mathbf{f}_0$ is an $n \times 1$ column vector. The solution of equation (21) is $\mathbf{a} = \mathbf{F}_1^{-1}\, \mathbf{f}_0$.

Alternatively, one could do even better than this by using equation (20) for m different values of t where $m > n$, producing an overdetermined set, and finding a best solution to this set in the minimum mean square error sense.

In this case $\mathbf{F}_1$ would be an $m \times n$ matrix where $m > n$, $\mathbf{a}$ an $n \times 1$ column vector, and $\mathbf{f}_0$ an $m \times 1$ column vector.

The best solution of this set in the minimum mean square error sense is:

$$\mathbf{a} = \{\widetilde{\mathbf{F}}_1\mathbf{F}\}^{-1}\, \widetilde{\mathbf{F}}_1\, \mathbf{f}_0 \quad . \quad . \quad . \quad (22)$$

where $\widetilde{\mathbf{F}}_1$ is the transpose of $\mathbf{F}_1$ and $\{\widetilde{\mathbf{F}}_1\mathbf{F}\}^{-1}$ is the inverse of the product of the $\mathbf{F}_1$ matrix and its transpose.

Looking at Figs. 8 and 11 of the paper it appears that it was considered adequate to express the impulse responses as the sum of 9 (in Fig. 8) and 6 (in Fig. 11) impulses. If n were of this order in the above matrix equations, the whole problem reduces to the solution of sets of simultaneous equations of order 10 or less—not nearly such a tedious task as the form of analysis here adopted.

Mr. Coutie replied that he agreed with Dr. Himsworth about the difficulties associated with using naturally occurring random plant variations when trying to derive information about the response surface. It was true that such variations contained much information about the plant but they seldom gave useful information about how to control a plant, and it was largely because of that that evolutionary operation had been introduced.

Mr. Dagnall had made a plea for the more general use of quality control charts in the chemical industry. One difficulty associated with their use on chemical processes, apart from the physical processes of the engineering industry, was that although the chart could indicate periods of non-normal running of the plant, it was often difficult to find out what corrective action should be taken unless the response surface was known. In other words, they were more useful for providing information about what had happened to a process, rather than for controlling it. Recently, Professor G. A. Barnard of Imperial College suggested the use of a different type of control chart, in which the cumulative sum of response, rather than a grouped mean, was plotted. That method was appreciably more sensitive to fairly small changes in the mean level of a response than the conventional charts, but the problem of interpreting the information from the chart for purpose of control remained.

With reference to Mr. Burton's example, Mr. Coutie thought it likely that for a reaction having such a simple model, the methods described in Ref. 8 of his paper could be used to derive an approximation to the optimum temperature-time profile from a study of the kinetics of the system. For the more complicated systems, however, for which it was known that the temperature-time profile was important, an empirical approach by evolutionary operation was still possible. Essentially all that need be done was to determine the effect on a response of changing the profile. If operating costs were important, the response would probably be the cost per unit of product. It would then be necessary to describe the class of possible profiles in terms of a small number of parameters; for example, the initial and final levels of temperature, and the rate of increase of temperature between them. More complicated profiles could be considered by allowing a quadratic representation of the rate of increase of temperature and variations in those parameters could then be made in a normal evolutionary operation scheme. Rates of temperature or pressure build-up had in fact, already been included in several evolutionary operation schemes.

Messrs. **Kalman**, **Lapidus**, and **Shapiro** commented in writing on the background of their paper before replying to comments. They said that they wanted to give an outline of their views on the probable future development of a new technology of automatic control in the chemical industry, in terms of ideas, problems, and possible methods of solution. They were looking at least five years ahead. They had no ideas to contribute in regard to matter of practicability, financing, or management psychology.

They could not give a completely satisfactory answer to the latent doubts embodied in Mr. Dagnall's phrase, that it was not sufficient to show that a proposal was technically feasible, it must be shown to be economically attractive. They did not know, however, of any instance historically where such philosophy had resulted in significant advances. While day-to-day improvements were important, really great technological developments did not come about as a result of economic prudence. The "accidental" invention of nylon by Carothers was a familiar example. Money was not a good measure of the quality of pure research for it was precisely the unpredictable elements of pure research which, in the long run, provided the economic justification.

It was possibly unfortunate and certainly inevitable that the development of the new technology which they had tried to anticipate in a small way in their paper demanded a sharp increase in the general scientific competence of the engineers concerned or more precisely in the tolerance of long-range scientific attitude as against the short-range practical attitude. No doubt one of the prerequisites of progress was the broader mastery of modern statistical techniques as represented by Coutie's paper.

While the majority of the engineering profession might be inclined to discount its "angry young men" of today, one could not ignore the possibility (and present practice) of powerful states directing their resources to accelerate unprofitable pure research in the sciences and technologies. The impact of those forces was already all too apparent in the American electronics industry where financial rewards of new knowledge often overshadowed old experience.

That led to an observation of Mr. Wiener that "the users would operate on the crude programmes in the first instance. Later, they would use the spare capacity of the machine for testing optimizing programme". That was the "wrong" way of doing research. The elaborate adaptive or self-optimising controller hinted at toward the end of their paper was primarily a research tool. Once that tool was available and the necessary investigations had been completed, great savings in cost and complexity might be anticipated in "projecting" the methods and principles found by research onto a practical problem. The converse procedure, as suggested by Mr. Wiener, was unlikely to come to grips with the basic problems in a reasonable length of time.

The authors echoed Dr. Rosenbrock's warning that "it is no use spending more money on saving a controller than the controller itself is worth". The only way of getting better control was to have a more detailed mathematical description of the process itself. If it required, say, 200 operational amplifiers and dozens of nonlinear units, it was silly to look for the best settings on 1, 2, 3, or even a dozen conventional controllers. In fact, their experience indicated that the equations describing the controller should be roughly 1–50 times as complex as the description of the process in order to be able to incorporate features such as evolutionary operation. At present rate of progress, by 1970 or sooner they should have digital computers with speed and storage capacity about 10^2–10^3 times better than present ones at no increase in cost. Far from being awestruck by the economic enormity of the big computers as Mr. Dagnall seemed to be, they felt that the "per bit" cost of computation could be expected (as it certainly had in the past) to continue to decline drastically. In controlling big, simple pieces of equipment (chemical plants, submarines, rockets, etc.) by small, complex pieces of equipment (computers) the long-run economic trends always favoured the small things since material costs would tend to be negligible and the production process was intrinsically capable of a very high degree of mechanisation.

From the authors' point of view, questions of safety were not paramount at that time since much more basic questions were as yet unanswered. But in the last few years one had ceased to take the colloquial point of view quoted by Mr. Wiener; and therefore they were sceptical of Mr. Wiener's forecast that conventional controllers would be kept working as a second string, to be switched in when a failure occurred. Much work had been done during the past two years (at least in the United States of America) on the scientific as well as engineering aspects of fail-safe equipment and it seemed that solutions other than those quoted above were likely to emerge; in fact, they were already incorporated in recent process control equipment. The same reply could be given to Mr. Dagnall's questions about continuous *versus* intermittent operations. Those things depended strictly on the exigencies of the moment and they did not think they had an important bearing on the research that they had reported.

The specifications of x^d, $\mathscr{P}$, a as mentioned were essentially management decisions, depending on market conditions, particular requirements of particular customers, etc. Those matters were included in the paper to emphasise the important (but often overlooked) point: the mathematics of control problems could be solved only after some arbitrary assumptions had been made about how the system should (as distinct from could) behave. To wit, x^d represented (in a crudely abbreviated form) how much and what quality material and plant should produce. The matrix **Q** represented management's view as to how requirements of quantity, quality, cost, etc. should be compromised when they were (as usual) in conflict. The constant a was a purely technical trick which was discussed in the more detailed expositions of their methods referred to in the bibliography.

A number of questions were raised concerning evolutionary operation and the possibility of obtaining information about process dynamics in a reasonable length of time without unduly upsetting the process. To date, methods of proposed ways for doing that were based on results from pure research as of about 1945.[1] Considerable further progress had been made since then in statistical communication and detection theory, but little or no application had been made to process dynamics. To mention a specific point, the autocorrelation type of detection procedure suggested by Goodman and Reswick was not adequate when small signal-to-noise ratios were encountered (*i.e.*, small process interference); in such cases, cross-correlation type of detection might be much more efficient, but could not be used without first having a fairly good model of the process itself. Needless to say, the possibility of identifying process dynamics was closely related to the quality of available measuring instruments. To be able to control, the instruments must transmit information about the process, but very little was known just how much information was needed to accomplish a given objective. Those were some of the unsolved or untouched problems which must be looked into more deeply before technological hopes could be translated into economic facts.

Perhaps the most urgent problem, however, was finding better means of representing dynamic systems. The linear representation used in the paper became poorer and poorer as the volume in the state space over which statistical averages of input-output data were taken increases. There were at present no acceptable methods of representing a nonlinear system; merely piecing together many linear systems obtained for different regions was too inefficient. That problem, too, had connections with information theory as shown by recent Russian work.

The authors concluded by saying that they did not disagree in substance with the various complaints which had been raised from the standpoint of the "practising" engineer. Rather, they tried to provide a small glimpse at the broad spectrum of research currently under way in many places and try to guess some significant aspects of the coming new technology.

Dr. Westcott replied that he agreed with Dr. Rosenbrock that it was very bad if frequencies were missing, but the same defects were found in the correlation function. The essential point was that the frequencies they obtained were those which the control received in normal circumstances and they were therefore the only frequencies in which they were interested.

The question of short records and whether errors were treated as noise posed the question as to what reliance could be placed on the records? That again was a compromise between accuracy and the time taken. What happened, in fact, was that they used a very inefficient process, continuing it so long as to make sure of the results.

Mr. Woodrow had pinpointed many of the difficulties but he had slightly misunderstood one point: in their realisation of the transfer function they had dealt with the individual impulses and three impulses had been sufficient. In the particular case, and only that case, the three pulses had been accurate to ± 2%. They could get very good results with a delay line synthesiser having as few as ten sections.

Referring to the lag window, Dr. Westcott said that as they were using short records and calculating the correlation function, as they continued with longer and longer delays the series went wild and was entirely unreliable. It thus was not satisfactory to transform the whole series which must be broken off somewhere. The question of the lag window was one of where and in what manner they broke it off and rejected inefficient data.

Messrs. Florentin, Hainsworth, Reswick, and Westcott later wrote:

Mr. Woodrow has raised the important question of the choice of the best solution for the transfer function. This is a general question which arises in any statistical work, and is a key problem in the development of the present method. In other applications several criteria are in use, *e.g.* maximum likelihood estimators, confidence intervals, least squares estimators, etc. Up to now it has proved very difficult to apply these criteria to regression problems involving several time series. The least squares estimator used in the paper is one of the few which has been worked out.

In the closed loop situation two equations are required to describe the system, one for the forward path, and one for the return path. To apply least squares error minimisation both equations must be taken into account, and this has not yet been done. It is possible, however, to deduce a relation between the population values of the correlation functions, this relation was used here with the hope that the records taken were long enough to make the errors small.

In choosing the window factor in spectral analysis, one makes a compromise between scatter in the magnitude of the results, and frequency resolution. In practice the choice will be influenced by the experience and judgment of the experimenter, and it is to be hoped that future use of the method will provide the experience which will be the best guide.

The window factor was introduced after a thorough analysis of the estimation properties of the power density spectrum. No analysis of the time domain approach of equivalent thoroughness has yet been carried out, and it is not known if an extra factor could be introduced which would improve the results.

Mr. Woodrow has raised the further point that this problem may be approached as a data-fitting problem, as opposed to a statistical analysis. This is an important philosophical distinction. Taking the statistical viewpoint, one assumes that the process is governed by a fixed set of parameters and probability distributions. This means that all variables in the system will be statistically fluctuating time series. Any measured record will be one of many possible ones, and generally every repetition of the experiment will produce a different result. One then wants to use the observations to calculate the best approximation to the underlying fixed parameters. There is here a basis for comparing different methods of using the observations to calculate the results, and a foundation for a discussion of the errors in the results.

In the data fitting approach one assumes fixed records with superposed experimental error. One then gets the best fit between the records, possibly minimising the effect of the experimental error, but disregarding the possibility of getting different results in a repetition of the experiment due to the records themselves being different. The error here can only relate to errors in measurement, and in the usual formulation of the method, auto-correlation in the error is not considered.

Often in engineering this distinction is of little practical importance. For instance, in the present example, in the open loop case one could include all the noise, regardless of its source, as experimental error. On applying the data fitting technique the same least squares equations would be obtained, but there is no longer any meaning to be attached to an ideal error-free (population) correlation function. There is no question of one method being better than the other, only the question as to whether one accepts a statistical model for a particular system, or a deterministic model with superposed error. This decision will vary from system to system.

In the authors' opinion, recognition of the statistical nature of the variables encountered in the typical plant parameter estimation problem will lead to a fuller understanding, and more fruitful use, of the experimental data.

Considering the first data fitting method proposed, no attempt is made to minimise the error, and in some circumstances it is excessively susceptible to instrumental error[2]. On examining the second method for least squares error minimisation, it will be noticed that the coefficients in the "normal" equations are the covariance functions, *i.e.* non-normalised correlation functions. If a normalising factor is used it is interesting to note that the data fitting approach involves the same arithmetic as the statistical least squares estimator. It is also noteworthy that the more recent techniques of least squares data fitting (also discussed by Lanczos)[2] use a Fourier analysis of the data followed by a smoothing procedure which is very similar to the window method.

The authors are pleased that Mr. Woodrow has raised these fundamental issues, because the present method will be of most use on large and complex plants; and where there are many aspects of plant behaviour to be considered, it is important that the underlying assumptions of the measuring technique should be put clearly before the experimenter.

Mr. **Keating** agreed with the need for simpler techniques and said that a more simple way of assessing controllability of plants was required especially in the design stage. He emphasised, however, that he was not suggesting that the method included in the paper should apply only to a particular plant. What he and his colleagues had hoped to achieve eventually was a technique which would be applicable to a wide variety of distillation plants. On the question of simpler techniques, he would point out that what was complicated today usually became simple tomorrow. He agreed that it was much easier to simulate mechanical control rather than process control because the process parameters were usually more difficult to define. However, he and his co-author intended to attempt simulation of a distillation process following on the present work.

The authors had not drawn as many general conclusions as they would have liked although they agree that controlling the top product rather than the reflux seemed to be the correct method. The reason might be that the reflux could be considered as having high inertia and consequently a large damping effect. It was dangerous to draw general conclusions too carelessly, however, because what might be good for one type of disturbance might be bad for another. He also pointed out that in the case of the present work, temperature was not sensitive, which was to be expected when dealing with close fractionation.

Dealing with the non-linearity of the system indicated in Fig. 4 on page 32, he pointed out that D/F was rather sensitive at the higher values. From below a recovery of 70% up to about 84% it was possible to increase D/F and thereby increase recovery without an excessive increase in reflux. That was one conclusion and he had no doubt that many others could be drawn.

Mr. Broadhurst had asked whether the authors had any envisaged programme on the dynamics of a column. They agreed that any programme must be modified by current results but they hoped to obtain theoretical transient responses. That would be followed by some practical transient response measurement on the plant, and they were designing apparatus for that purpose. Any steady state practical data would naturally have to be obtained statistically and control charts were already in use in the particular refinery.

Replying to Mr. van der Heijden, Mr. Keating pointed out that unintentional feedstock changes of 75–85–95° C. were not normal. The paper quoted them to give some clearly seen results. In fact, the plant was actually being controlled by volume control and that type of disturbance was not experienced. He also pointed out that constant temperature at point x in the column was not equivalent to the constant overhead purity at different feeds but in the particular case the difference was small. The maintenance of overhead purity by controlling the temperature in the column by regulating the heat load of the reboiler, that is with constant D/F would result in a column loading which varied with feed composition, and might result in overloading the column.

Reference

[1] Goodman, N. R. *Scientific Paper No.* 10 *of the Engineering Statistics Laboratory, University of New York*. March, 1957.

[2] Lanczos, C. "*Applied Analysis*," 1957 (London: Sir Isaac Pitman & Co., Ltd.).

SESSION 2
PROCESS CONTROL SYSTEMS

Chairman: **Mr. J. S. Brough, A.M.C.T., M.I.Chem.E.**

Chairman of Group A, British Conference on Automation and Computation

THE CONTROL OF AXIAL FLOW COMPRESSORS: THE ESTABLISHMENT OF A CONTROL LOOP TO MEET UNUSUAL PERFORMANCE SPECIFICATIONS

By J. E. SAMSON, B.Sc., A.Inst.P.*

SUMMARY

An analogue computer was used to study the unusual problems that were encountered in establishing control loops around axial-flow compressors; the specific object of the controls was to prevent the compressors from being damaged under certain operating conditions. Practical operating considerations, however, led to modifications of the optimum control adjustments and it was found necessary not only to use the analogue computer to evaluate the loop performance, but also to provide a complete picture of expected performance under these conditions as they were not amenable to treatment in any other way.

Introduction

The plant and the problem

Six 22 000 horse-power, electrically-driven, constant-speed, axial-flow, air compressors are arranged so that they may be used singly, or in combinations of series, parallel, or series-parallel. They are primarily utilised for the establishment of sub-atmospheric pressures in the loads connected to the suction of the machines.

The compressor characteristics are such that the volumetric delivery is substantially independent of the suction pressure and of the pressure ratio across the machines. They are subject to the normal limitation of their type, in that there is a discontinuity in the characteristics when a critical value of pressure-ratio is exceeded. Beyond this pressure-ratio the performance collapses and an unstable condition of surging pressure-ratio develops. This will not only upset the suction, but will seriously damage the machine unless the pressure-ratio is quickly reduced to eliminate the surge conditions. A satisfactory method of surge control is to use a by-pass valve around the compressor between the delivery and suction line.

The object of the control system is to provide an anti-surge protection. Whilst this could be achieved by tripping the by-pass valve fully open at a pre-determined value of the pressure-ratio, this would seriously interfere with the available performance of the machine. A continuous form of control is therefore desirable so that only a minimum mass flow need be sacrificed to obtain control under any operating condition. Further, to obtain maximum performance from the machines over as wide an operating range as possible, it is desirable that no opening of the by-pass valve should occur until the pressure-ratio rises to close to the surge point.

It must be stressed that in the event of a surge, the control system cannot be expected to bring the system out of surge: the normal control action if a surge occurs would, in fact, tend to aggravate the condition.

Under normal conditions the control will be on automatic, but the load will cause the pressure-ratio to be lower than the limits of the control band. Progressive increases in pressure-ratio due to changes in the load are required to have no effect until this band is entered. Progressive opening of the by-pass valve is then required to cause partial recirculation.

Under some conditions of approach to surge the pressure-ratio may change from 2 to 4·5 in 3 or 4 seconds; higher rates can occur, as when a compressor outlet valve suddenly closes causing a pressure-ratio rise from 2 to 4·5 to occur in about ½ second; such extreme cases, however, are considered as emergencies and can be dealt with by supplementary action.

Each compressor is required to be started-up on recirculation under automatic control. A series of interlocked automatic isolating valves is provided to change the arrangement of the machines or to start up another compressor whilst under automatic control. Since these operations could cause the control system to demand that the by-pass valve be closed due to a low pressure-ratio (whilst, due to the action of the interlocks, the by-pass valve is opened) a time-delay is provided between the controller output signal and the by-pass valve. Thus when these sequences have been carried out there remains a time delay between the restoration of the signal to the control valve and the operation of the valve. This is necessary because the rate of rise of the pressure ratio would be so rapid that a surge would be almost inevitable if the control action was restored without any time delay.

Operating Conditions and Control Requirements

Pressure ratio

The pressure ratio is required to be continuously measured, and it was found necessary to design a new instrument to meet the following performance specifications:[1]

Ratio Scale:	2·0–4·5.
Operating Range:	0·25–60 lb/in² abs.
Required Accuracy:	±1% Pressure Ratio.

These are very wide operating conditions; however a ratio computer was built, the performance of which was up to this specification. Its accuracy was ±1% Pressure Ratio down to a suction pressure of 2 lb/in^2 abs; its repeatability was 0·5% over the whole operating range. This ratio computer was arranged to provide a pneumatic output signal of 3–15 lb/in^2 g varying linearly with pressure ratio between the limits of 2–4·5. The instrument was able to measure a ramp of full scale in 3·5–4·5 seconds.

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Control loop

Fig. 1 is a schematic diagram of the control loop. The ratio transmitter sends a signal to the pressure-ratio controller which feeds its signal into an air/oil pressure convertor. This permits a pneumatic receiver controller to operate an hydraulically operated by-pass valve.

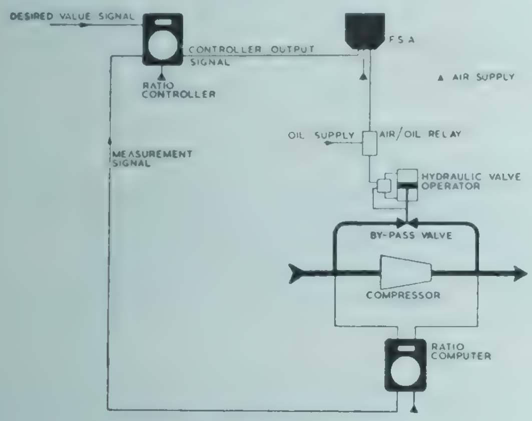

Fig. 1.—*Closed loop schematic*

As it was not anticipated that the general frequency of operation of the whole loop would be higher than ½ cycle per second, it was decided to use pneumatic control systems together with an air/oil convertor for the hydraulically operated valve. Preliminary considerations suggested that a proportional-with-derivative controller would be satisfactory. It was not anticipated that the noise which might be present would be sufficient to affect the use of derivative action.

The proportional-band width of the controller was required to be a minimum because recirculation was required to start as near to the surge level as possible. The possibility of using integral action in the controller was rejected because of saturation problems when running under normal operating conditions. A further feature prohibiting the use of integral action was the fact that parallel or series operation could cause "fighting" between systems if that control function were used. Equally an integral action cut-out mechanism would be unsatisfactory because it would lower the position of the band width and would cause recirculation to occur at too low a pressure-ratio.

A safety hydraulic trip mechanism that opened the by-pass valve independently of any control action was provided in case a surge occurred. When this oil circuit was restored provision was made that the time delay mentioned earlier should come into operation so that the by-pass valve did not tend to close too quickly.

A further source of difficulty was that the by-pass valve did not have a fixed relationship of lift against pressure ratio. This characteristic necessitated a shift of stem position which must occur to cause a constant pressure ratio at differing degrees of recirculation. The by-pass valve was a 24 in. double-beat plug type, with angle body, and was capable of moving full stroke in under 1 second.

An additional complication from a control aspect was that the duct-work configurations for different compressor arrangements were capable of altering in a volumetric ratio of approximately 300 : 1 between normal load conditions and full recirculation. The effect of a given volume of duct-work would also differ considerably at different suction pressures.

Initial Operational Results

When the control loop was first set up under full recirculation conditions it was not found possible to achieve proportional band widths less than 160%, and no significant derivative action was acceptable. This was partially due to the presence of considerably more noise than had been originally anticipated. This wide band, even allowing for the noise, was unexpected and attempts to narrow it by damping the measurement impulse did not improve the settings until excessive damping had been introduced. There was severe objection to damping the impulse lines because the maximum speed of response of the measurement was required for the anticipated fast transients. These results suggested that the plant itself had a very high gain and further investigation of its characteristics thus appeared essential to indicate how a narrow band could be achieved. Further, if the system had a large gain, the presence of noise would not be an excessive contribution to the large band width, but the same noise could be troublesome with a narrow band unless steps were taken to filter it, at least as far as a signal to the valve was concerned.

Initial Plant Investigation

Under full recirculation conditions, high speed photographic records were made of the controller output signal, the pressures around the suction and delivery of the compressor, the valve-stem position, the oil output pressure from the air/oil relay, and the ratio computer output signal. It was at first suspected that the noise pattern was due to non-uniform distribution of pressure around the duct-work, but it was soon established that there was very little variation in pressure distribution across any section of it. Investigations of the records under recirculation conditions and with manual step disturbances indicated that there were two second-order, under-damped components in the loop (the valve and the ratio computer) and a certain dead time due to the associated duct work, hydraulic mechanisms, and impulse lines. These explained the wide band, since it was apparent that such a system would have a large phase lag whilst its gain would be high, thus not only requiring a wide band width but also offering very little attenuation to noise. It appeared from the step disturbances that the compressor itself had very small effect on the overall loop response.

This characteristic response of the plant is a good illustration of the principle that a system with an attenuation slope of more than 20 db/decade where the gain is unity will be unstable unless the controller gain is low. The recognised technique for altering the plant characteristic to enable a high controller gain to be used is to introduce some dominant lag in the control loop (*e.g.* damping) so that the attenuation rate is 20 db/decade where the gain is unity.[2, 3] It was decided to investigate the possibilities of various types of transfer function on an analogue computer, not so much because of the simplicity of this method but because it was necessary to be as certain as possible of the settings before trying any device on the plant. It was essential to safeguard the compressor and to avoid costly operation of the machines during setting-up operations.

The transfer functions derived from the step responses were set up on a simulator,[4] and the behaviour to set-point disturbances was investigated. The simulation was extremely simple, the whole problem being treated as a linear set of functions. As would be expected, it was found that putting various capacities at the pneumatic input of the air/oil convertor permitted a reduction in band width to about 75%. However the introduction of a pure lag introduces a 90° additional phase shift into the loop at higher operating frequencies and thus slows down transient response of the loop. It was thought that the result obtained on the simulator could be used in practice to establish the validity of the simulation.

A more satisfactory technique that avoids this increase in phase-lag is to use an equalising network in the controller output. This produces a slope of 20 db/decade, and can be arranged to have unity gain at zero frequency, and a fixed attenuation at high frequencies. The phase lag of this equaliser will be zero at low and high frequencies, and will reach a maximum between these limits, the basic time constant of the equaliser determining the relative values of "high" and "low" on a real frequency scale.

An investigation of the possibility on the analogue computer showed that an equaliser transfer function of

$$(1 + 0{\cdot}2s)/(1 + 4{\cdot}0s)$$

enabled the band width to be brought down to 20% and gave a step response without overshoot for a set-point change. It was then necessary to design a device that would produce a function of the form $(1 + aTs)/(1 + Ts)$ using pneumatic components. In this function a would be continuously adjustable (preferably between zero and unity), and the value of T would be unaffected by changes in a.

This has setting-up and operating advantages in comparison with a standard electrical equalising network which usually has a transfer function of

$$(1 + R_1Cs)/[1 + (R_1 + R_2)\,Cs]$$

and which consequently requires changes in both R_1 and R_2 to perform as above. The advantage of the desired transfer function is that once the basic time constant has been selected the value of a can be adjusted to "tune" the process when the band width has been set to a required value.

Such a device was designed,[5] and Fig. 2 is a schematic diagram of it. It produces the required transfer function, with the incidental advantage that a is adjustable from 1/40 to 400 by one continuous movement of the lever. If a is set at unity, the instrument [a "Frequency Selective Attenuator" (F.S.A.)] acts as 1 : 1 repeater.

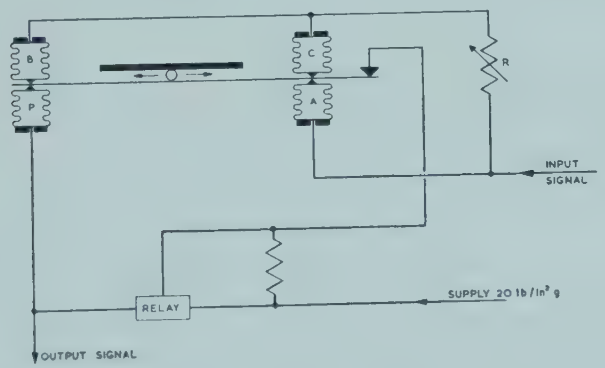

Fig. 2.—*F.S.A. schematic*

Advantages of F.S.A.

An advantage of this type of function is that if a is small (1/10), the time constant may be adjusted so that the transient response of the loop falls within the frequency range determined by the break point of the time constant T and that of $T/10$. This will provide a measure of integral action by virtue of the high gain at low frequencies without the large phase lags due to integral action with such large gains. Further, this benefit may be obtained beyond the "mechanical reach" of the proportional band in the controller; thus, saturation of the F.S.A. may occur, but the initiation of control action from the controller will still be at the same place on the scale of the instrument because the band width has not altered its position with regard to the desired value. A further advantage of this equalising network is that a large signal/noise ratio is not required as the rather large basic time constant will provide a filtering effect. This time constant, however, can slow the transient response of the loop, and the F.S.A. was designed with a "plug-in" derivative term, to give an overall transfer function expressed by:

$$\frac{(1 + aTs)(1 + TdS)}{(1 + Ts)(1 + Td/20s)}$$

This should improve the loop response. It was also thought that the derivative action would only be satisfactory (if at all) after the filtering of noise. In any case the analogue computer study indicated that derivative action would only be of marginal value.

Experimental Confirmation of Predicted Settings

A variable capacity in the air line to the air/oil relay and an F.S.A. were installed on site. The loop was put on to automatic control on full recirculation. The effect of the capacity was first tried and indicated that an improvement to 75–80% band-width was possible, but increasing the lag further showed no immediate improvement. Although this was then abandoned, it confirmed the simulation.

The F.S.A. was put into the circuit, and it was found that the band-width could be narrowed to 16% with a time constant of 3·8 s. This was further confirmation of the accuracy of the simulator model, and also of the improved performance possible by use of the F.S.A. It was found that 20% band width gave the best results.

It was then suggested that the F.S.A. be fitted to another compressor where there was more noise. This was done, and it was found that a time constant of 8 s gave a band width of 30%, and a setting of 12 s gave 20%. The value of a on each compressor was about 1/10. This indicated that there was considerable difference between the two compressors tested. Over a period of time it was confirmed that derivative gave no advantage either in the controller or the F.S.A.

Operating Complications due to Introduction of F.S.A.

Certain complications now presented themselves in the other conditions of operation apart from full recirculation.

The hydraulic trip that opened wide the by-pass valve in emergency presented no problem on remaking the circuit, since the time constant in the F.S.A. acted as an additional delay in the event of a build-up of pressure to the valve, and thus supplemented the existing delay. However, when the compressor was uncoupled from its load, the input of the F.S.A. was vented and the output from the controller shut off so that the by-pass valve was wide open when the compressor outlet valve was closed. When this circuit was restored, the controller output was a maximum and this tended to build up the F.S.A. output. However, there was only a specified time interval between this venting and remaking, which could not easily be altered; if the F.S.A. time constant was greater than this time interval, its input side did not have time to fall to an appreciably lower value than it was originally; consequently when the circuit was remade the output of the F.S.A. immediately restored itself to the residual value left in the input bellows. This meant that the by-pass valve instantly took up a position more closed than was desirable thus minimising the effect of the time delay in the output from the controller to the F.S.A. It was thus essential to compromise on the F.S.A. time constant so that in the time interval between venting and remaking the F.S.A. input circuit, the residual pressure would fall to such a low value that the repositioning of the by-pass valve on restoration of control would be small. This set a maximum value of 9 seconds for the time constant. A further consequence of this constant was that under the normal operating conditions on load, the F.S.A. would be saturated at the maximum controller signal, and when recirculation was necessary the F.S.A. would have to desaturate before the by-pass valve could be moved. This effect would reduce with the F.S.A. time constants. A help in overcoming this saturation was to restroke the operating range of the output signal so that with a supply pressure of 20 lb/in² g the control range was 7–19 lb/in² g.

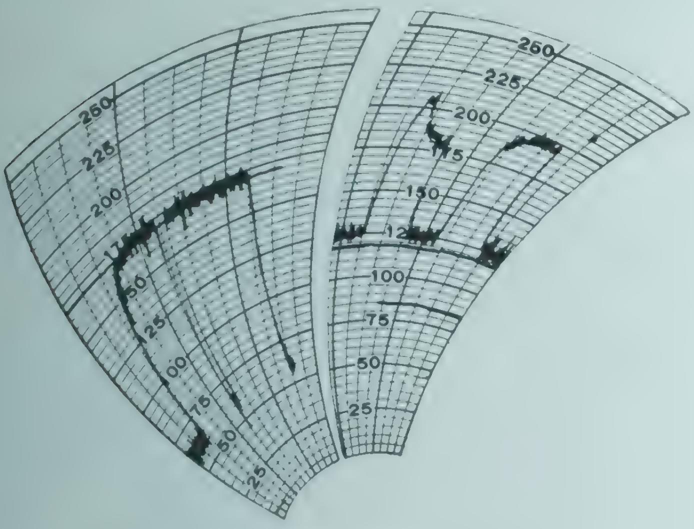

Fig. 3.—*Control system disturbance responses*

Fig. 3 illustrates on the left-hand chart the responses to a start up under automatic control and a return after the control circuit has been remade when the by-pass valve was hydraulically tripped wide open. The right-hand chart on Fig. 3 shows the responses to a test devised to give the system as large a disturbance as possible whilst on recirculation. This consisted of turning the control system to manual control and raising the pressure ratio manually until it was outside the band width; on returning to automatic control, there is a maximum discrepancy between the valve position and the controller command signal.

Further Plant Investigation

All this testing was of necessity carried out on recirculation on one machine. It became necessary to carry out operational tests with straight-through running. The initial success of the simulation encouraged an investigation on the analogue computer to indicate the anticipated performance. An open loop frequency response analysis was carried out on the machine that had appeared to be most noisy in a similar manner to the original plant tests. This enabled the transfer functions of the air/oil relay, the compressor and associated duct work, and the ratio computer to be determined under recirculation conditions at a pressure ratio of 3·2 and with an input about this value of 10% of scale band width overall. These transfer functions enabled the difference between the F.S.A. settings on the compressors to be explained, due to the differing frequency response.

The transfer functions derived from these sine wave disturbances were as follows:

Controller output air line and air/oil relay

$$3{\cdot}5\ e^{-0{\cdot}18s}\ \text{lb/in}^2\ \text{oil/lb/in}^2\ \text{air}$$

By-pass valve . . . $$\frac{0{\cdot}298\ e^{-0{\cdot}09s}}{(1 + 0{\cdot}17s)}\ \text{cm/lb/in}^2\ \text{oil}$$

Associated duct-work, compressor, impulse lines

$$\frac{0{\cdot}4\ (1 + 0{\cdot}6s)\ e^{0{\cdot}2s}}{(1 + 1{\cdot}1s)}\quad \text{P R/}^1\ \text{cm}$$

Ratio computer . . $$\frac{4{\cdot}8}{(1 + 0{\cdot}45s + 0{\cdot}10s^2)}\ \text{lb/in}^2\text{/PR}$$

These transfer functions are only valid in the frequency range 0·5 to 8·0 rad/s. The compressor was simulated only by the transfer function given above.

Improved Analogue Simulation

With these transfer functions it was possible to set up the more complete simulation shown in Fig. 4. This is diagrammatic only, and does not seek to identify all the units used in the computer.

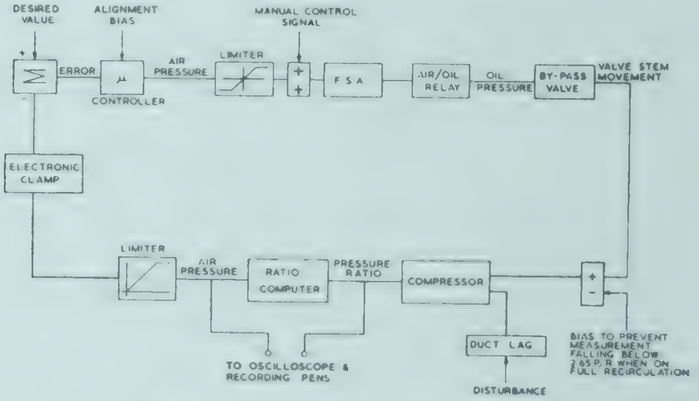

Fig. 4.—*Simplified analogue simulation*

Limitations of the simulation

The following limitations in the computation compared with the actual system must be noted.

(1) The transfer functions apply to recirculation only.

(2) The simulation uses cascaded transfer functions because these are the only ones available. Thus it does not take into account the impedance changes in the compressor, valve, and computer at different pressure ratios and pressure levels at various valve openings. These being all in parallel, it is to be expected that there will not be constant terms in their transfer functions. Since, however, the simulation is concerned with the successful operation under recirculation conditions it was thought reasonable to ignore these impedance changes.

(3) The valve characteristic is assumed linear and to correspond with that of full recirculation at its steepest slope of movement with signal change.

(4) The compressor characteristic above surge is unknown. Thus control action above surge point in the analogue will be incorrect.

(5) The valve shows velocity limitations and its transfer function is a time-dependent non-linearity. In the absence of data this was ignored.

(6) No information about the noise on the plant was available in a form suitable for putting into the simulation. Practical experience, however, indicated that with band widths of not less than 20%, the noise with the F.S.A. in circuit did not significantly affect computed F.S.A. time constants.

(7) The disturbance characteristics at the compressor were unknown since the effect of the duct-work was unknown.

It was therefore decided to use step and ramp inputs of the various rates theoretically calculated, and to feed these into the compressor in the simulation *via* a first order time constant of adjustable value to approximate a duct lag as a lumped time constant.

It should be stressed here that the major purpose of the analogue investigation was to evaluate the loop response to disturbances occurring from outside the proportional bandwidth, since normal frequency response plots could not deal with such conditions in view of the saturation that can occur in the F.S.A. and of the limitation in pressure ratio that can actually be indicated on the ratio computer. The analogue simulation thus included the two limiters shown in Fig. 4, the additional limiter representing the signal limits of the controller output. Since non-linearities of this type occur in the simulation, individual gains in each part of the loop must be accounted for and distance velocity lags cannot be lumped. The non-linearities considered have no bearing on the disturbances that occur within the band width, and their only effect is in those disturbances which the control system is intended to handle.

The analogue simulation confirmed the settings which had been found necessary on the "noisy" plant in practice, with the limitation of F.S.A. time constant taken into account. The final settings were band width = 30%, F.S.A. time constant = 9s, and $a = 1/9$.

Results

Figs. 5 and 6 illustrate the good agreement between the computed and practical responses, the measurement of the ratio computer output being taken from a damped signal in parallel with the normal signal. Fig. 5 illustrates a typical response to a set-point disturbance, the actual value of disturbance being designated as a step of unit magnitude and being confined to within the proportional band width. Fig. 6 shows the response to a load disturbance that makes recirculation necessary to prevent a surge.

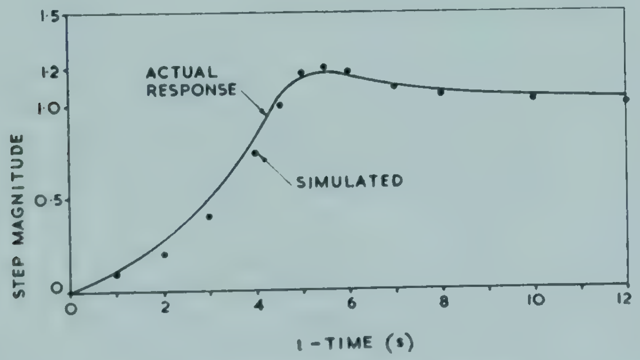

Fig. 5.—*Comparison of step responses*

Fig. 7 illustrates the open-loop responses of the system. Curve a represents the measured response of output air line, air/oil relay, by-pass valve, compressor and duct-work and impulse lines to ratio computer. The triangular marks represent the derived transfer functions for these components, and indicate the closeness of fit obtained.

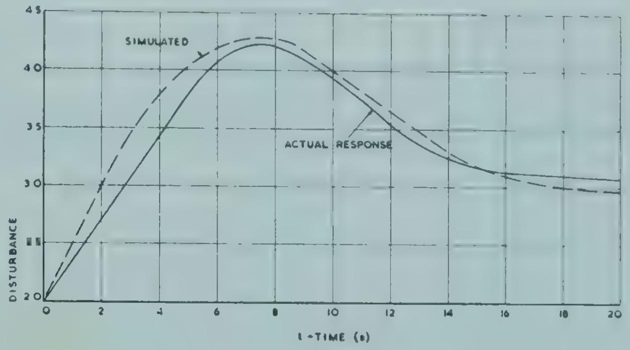

Fig. 6.—*Comparison of disturbance responses*

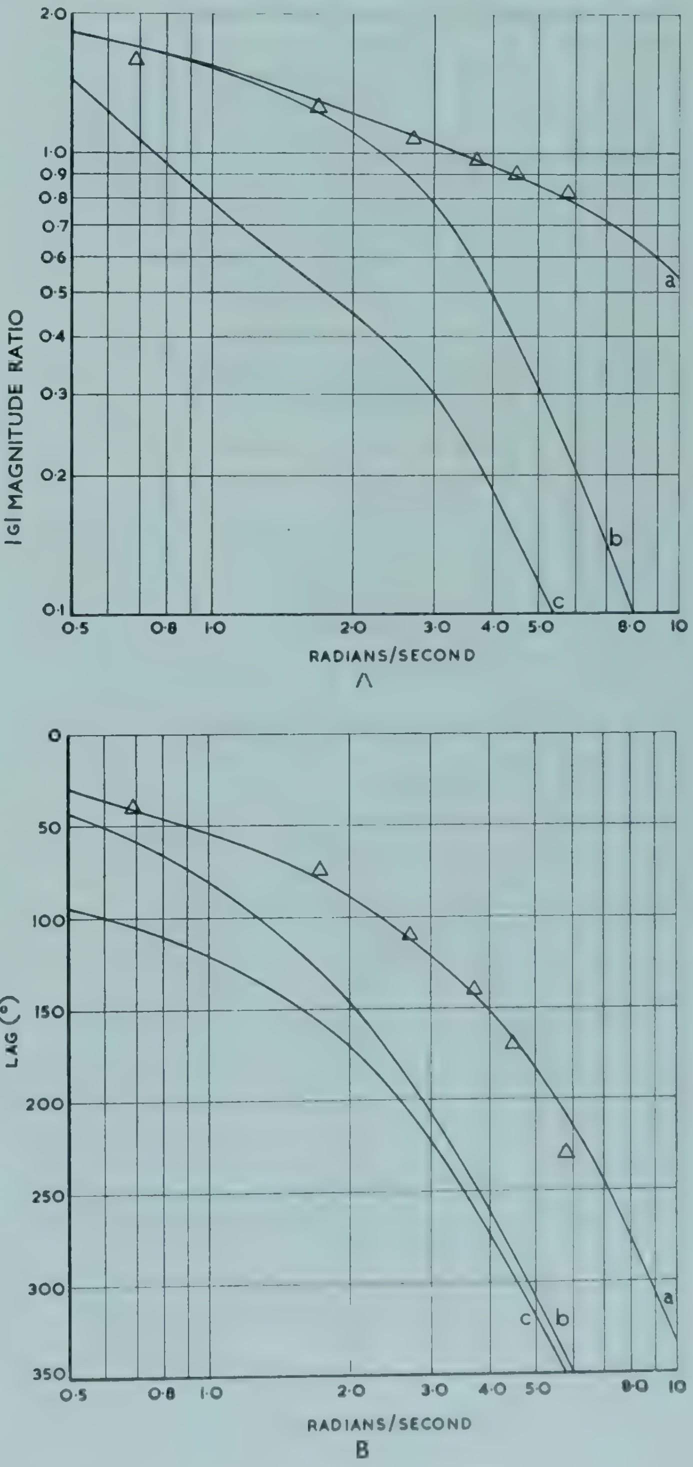

Fig. 7.—*Open loop responses*

Curve B shows curve A modified by the addition of the ratio computer.

Curve C shows curve B modified by the addition of the controller and F.S.A., set at $\mu = 3{\cdot}3$ and

$$\text{F.S.A.} = (1 + 1s)/(1 + 9s).$$

It will be seen that at 180° phase lag, curve B has a gain of 0·95, indicating that a gain of less than unity will be required for stability using a proportional controller; whereas curve C at the same phase lag has a gain of 0·4 even with a controller gain of 3·3, whilst the frequency at which this phase lag occurs is not greatly different between the two curves.

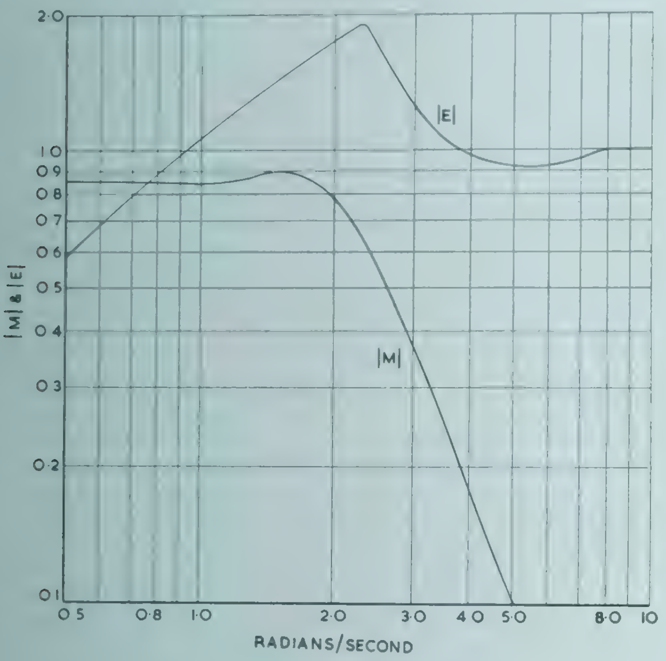

Fig. 8.—*Closed loop responses*

Fig. 8 illustrates the magnitudes of the closed loop control ratio and error ratio (| M | and | E | respectively) for the system of Fig. 7c. It will be noted that these apply to disturbances within the band width, and that | M | does not exceed unity, indicating that the system will not overshoot for set-point disturbances. The peak error ratio is less than 2: this represents a satisfactory condition.

It will also be noted that the rate of attenuation of | M | is more than 40 db/decade: this indicates that satisfactory rejection of noise at high frequencies will be obtained.

Behaviour under surge conditions

In assessing the performance of the loop as an anti-surge control, the minimum overshoot beyond the desired value is necessary when a load disturbance occurs that could cause a surge.

To obtain a guide to the behaviour of the system under these conditions, a three dimensional surface of the expected overshoot E as a percentage of the applied disturbance potential value was plotted against both rate of change of applied pressure ratio and various duct lags, at desired values of 3·0 and 3·5. This is shown in Fig. 9. A difficulty encountered in the simulation of the disturbance was that no potential value for comparison with practice could be calculated. This was due to the limitation of the characteristic of the compressor above surge. However, by extrapolating the valve characteristic used in the simulation, a potential value of 7·0 pressure ratio was used. It is probable that this figure was high, but this was not important since the overall pattern of the control loop behaviour was more essential in this case.

The predicted behaviour was that a maximum rate of disturbance within the expected practical range of operation occurred in the part of the plot that showed a distinct flattening off of the expected overshoot even without duct lag. The presence of duct lag helped this, and it was thus concluded that the controls would prevent the surge from occurring. This was substantiated in practice on this particular machine, operating alone.

Multiple Compressor Operation

Obvious difficulties arose in connection with variable impedances when seeking to extend this simulation to more than one compressor on load. However, as far as parallel operation was concerned, it was thought that a

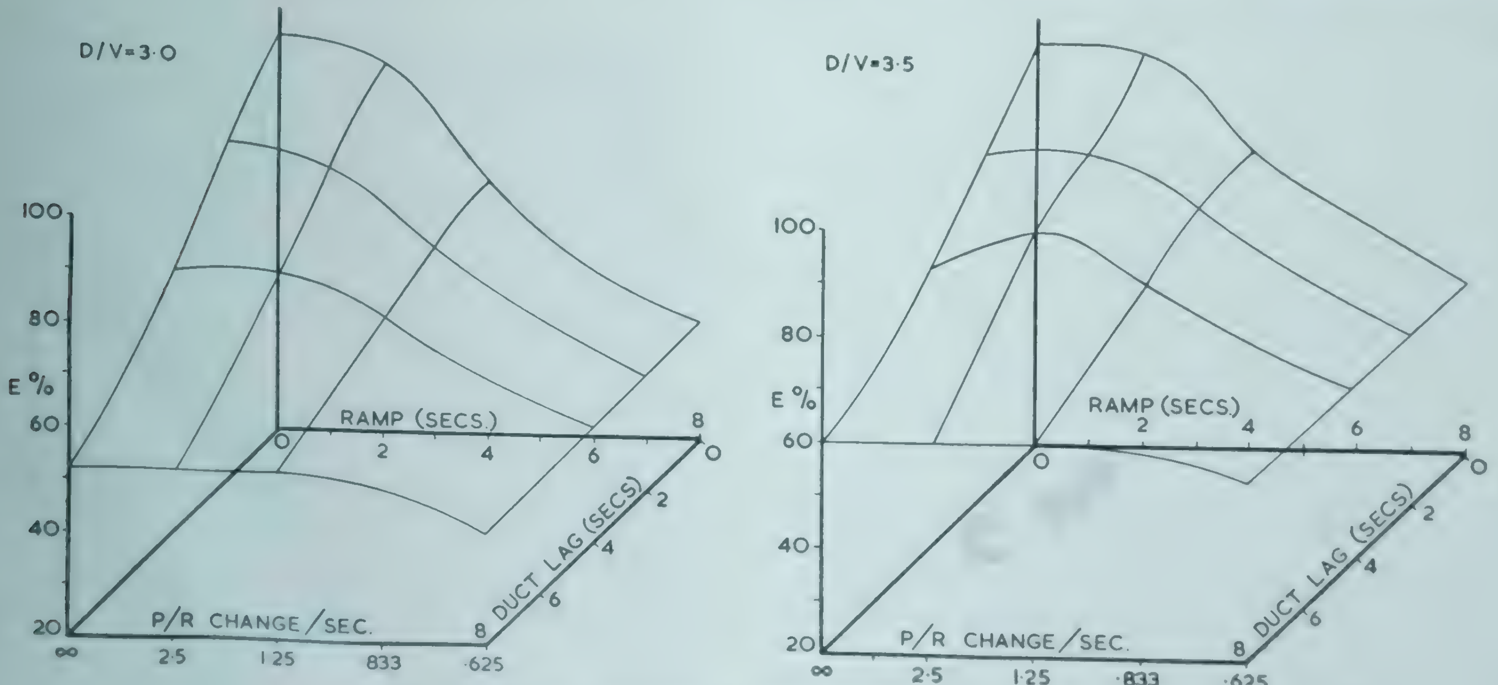

Fig. 9.—*Predicted surfaces of performance of single compressor*

knowledge of the actual rate of disturbance as it occurred at the compressors would permit applying this rate as a disturbance to one computer in the simulation. This has been accomplished, and the actual response of three compressors in parallel has been measured and this disturbance applied to the original simulation. The agreement between the simulated and actual response is indicated in Fig. 6 and appears to justify the simplifying assumptions made in the simulation.

Conclusions

The use of a simple analogue simulation based entirely on step response analysis has been shown to yield quantitative information about controller settings that can be extremely accurate. It can show a saving of costly operating time for a modest outlay in computation time and expense.

The use of a more elaborate simulation, where all instantaneous non-linearities are taken into account, has been shown to yield accurate qualitative and quantitative data about control system performance under conditions of disturbance that cannot be uniquely described. The flexibility of an analogue simulation in permitting this data to be accumulated rapidly under widely differing types of dsturbance is of great value in providing a comprehensive coverage of the possible variety of operating disturbances. The required characteristics of an instrument that has to be designed to meet a specific application are immediately obvious, and this greatly facilitates the implementation of a mechanical design to fulfill these requirements.

The advantage of this considerable saving in operating time by the use of a simulation is in this case exceeded only by the complete safety afforded to an expensive component of a plant. However, it should be stressed that the simulation must first be verified before elaborate results are attempted.

Acknowledgments

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RATIO FLOW CONTROLLER FOR IN-LINE BLENDING

By H. H. IDZERDA, A.M.I.E.E.*

SUMMARY

A completely novel instrument for comparing signals transmitted by measuring devices is described. This comparator, the so-called differential integrator, is of the electronic type and is able to handle as input signals, frequency, position, and d.c. voltage or current. The usefulness of the circuit is shown by the description of a flow ratio controller for in-line blending, an instrument that incorporates the differential integrator.

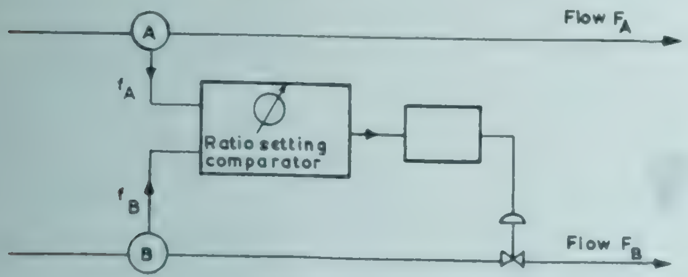
The introduction of electronic instruments in the oil and chemical industries is very often prejudiced by the existence of perfected pneumatic techniques. In this case, however, the situation is different. No pneumatic solution is available for accurate fluid-flow ratio control using positive displacement meters as the measuring elements, although a completely mechanical comparator exists. A comparison of the mechanical and electrical systems is made and it is shown that the electrical type is superior on all important points.

Introduction

An increase in the number of "in-line blending" installations in oil refineries is to be expected. Batchwise mixing in tanks will then be superseded by continuous mixing to specification of components either from storage tanks or directly as they flow from the distillation units. The advantage of this is a saving in storage tank capacity.

For accurate blending operations accurate flow measuring devices are needed. The commonly used orifice in a pipe line in combination with a differential pressure transmitter has an accuracy of 2 to 3%. Rotameters may be used for an accuracy of 1%: in these the output is almost linear with volume flow. This is of importance for integration or when blending over a wide range of ratios. For greater accuracy, positive displacement meters are used, which can have an accuracy of 0·2% of volume flow under favourable conditions. The output of such a meter is a rotating shaft, the speed of which is proportional to volume flow. Flow integration is simply carried out by a totalizing counter. If, however, accurate blending with this type of instrument is required, several problems are encountered.

As can be seen in Fig. 1, which shows the general control circuit of a ratio blender for two flows, both flows are measured and have to be compared, taking into account the desired ratio setting. Some kind of transmission of the speed of a rotating shaft to a central point is needed. If a solution to this transmission problem is sought by means of well-developed pneumatic techniques, a converter is needed to translate shaft speed into a pneumatic pressure. Such converters are available, but their accuracy is not better than 1% and it does not seem sound engineering to lose the accuracy of the expensive positive displacement meter in the transmitter following it. A simple electrical solution might be to convert the shaft speed into a d.c. voltage by means of a tachogenerator, but the accuracy of the tachogenerator is too low.



The signals f_A and f_B delivered by meters A and B are weighted in the comparator according to the desired ratio setting. A difference in the weighted signals acts on one of the flows by means of a controller and valve.

Fig. 1.—*Ratio control circuit for two flows*

Mechanical solution

A complete and accurate mechanical solution to the problem exists. If two liquid streams are to be blended, the shaft speed of one of the two positive displacement meters is geared down in the desired ratio. The speed of the output shaft of the gearing box is then compared with the speed of the shaft of the other positive displacement meter in a normal differential gear. The output shaft of this differential gear is the input of a controller correcting one of the flows.

The main drawback of this system is that a stable, infinitely variable mechanical gear box is needed for a wide range of ratio settings. Also, the output shafts of the positive displacement meters must be brought physically together, making it necessary to build the meters close together. This last point could be circumvented by the use of "electrical shafts", but this would increase the complexity of the system considerably.

Electronic solution

After a careful analysis of the problems involved an elegant and accurate electronic solution to this blending problem was found. The heart of this device is the electronic differential integrator. One ratio blender incorporating this differential integrator has been on field test for over a year and has given excellent performance. Its accuracy was assessed by comparing the desired ratio as set on the instrument with the calculated ratio read from the two integrating counters of the flow meters. The accuracy of the blending was better than 0·1%. This field experience has shown that the instrument can be easily installed and that all possible blends can be obtained directly by setting the ratio on the dial.

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Principle of the Electronic Differential Integrator

The working of this unit can best be explained by a simple analogy.

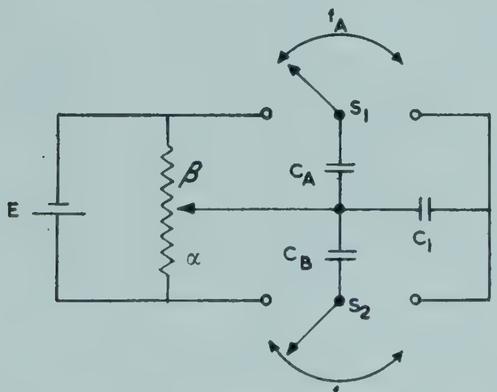
Consider a bath tub, partly filled with water. Two men, A and B, each have a bucket; A fills the bath tub by means of his bucket: B empties it. If the buckets are equal in size the average level in the bath tub can be held constant only if the men work at the same speed.

If A has a bucket twice the size of B's, B's speed must be doubled to keep the level constant.

The action of the differential integrator is an almost exact electric equivalent of the buckets and the bath tub, while the controller following it ensures that the level is kept constant.

Electronic circuit

The circuit diagram is given in its most simple form in Fig. 2. Switches S_1 and S_2 can be switched from left to right and *vice versa.* Capacitors C_A and C_B are the equivalents of the buckets, C_i the equivalent of the bath tub. In the left-hand position of the switches C_A and C_B are charged: in the right-hand position they are discharged into capacitor C_i.



Switch S_1 carries an electric charge βEC_A from left to right. If S_1 is switched at f_A c.p.s. capacitor C_i is charged with a current βEC_Af_A. Similarly a current $-\alpha EC_Bf_B$ charges C_i across S_2. If C_i is kept uncharged by means of a control circuit acting for example, f_B then $\beta C_Af_A = \alpha C_Bf_B$.

Fig. 2.—*Principle of electrical differential integrator*

If C_A and C_B are equal and the potentiometer is in mid-position ($\alpha = \beta$), they are charged to an equal value but with opposite sign and, after their discharge into C_i, C_i remains uncharged. If S_1 is switched at a frequency f_A and S_2 at frequency f_B, C_i will remain uncharged as long as $f_A = f_B$.

If the potentiometer is not in the mid-position, C_A is charged to a voltage βE and has a charge βEC_A, and C_B has the charge αEC_B. If the switches are actuated at frequencies f_A and f_B the charge of C_i in a time T will increase by Q:

$$Q = \int_0^T (\beta EC_Af_A - \alpha EC_Bf_B)\,dt \qquad (\beta = 1 - \alpha)$$

and hence, for the practical case that $C_A = C_B = C$, the voltage increase, ε, across C_i will be:

$$\varepsilon = \frac{E}{C_i}\,C\int_0^T (\beta f_A - \alpha f_B)\,dt = k\int_0^T (\beta f_A - \alpha f_B)\,dt;$$

f_A and f_B can be functions of time.

Now if by some means f_A or f_B is adjusted so as to bring ε to zero, then:

$$\beta f_A - \alpha f_B = 0 \text{ or } \frac{f_A}{f_B} = \frac{\alpha}{\beta}$$

If we suppose that f_A and f_B are proportional to the volume flows, F_A and F_B, it will be clear that the desired ratio for F_A/F_B can be set by choosing α by means of the potentiometer.

To realize synchronous switching of switches S_1 and S_2 with F_A and F_B in practice, the output shafts of the positive displacement meters carry contacting mechanisms that energize relays in the differential integrator.

It is interesting to note that the function of the control circuit is to bring ε always to zero. This is equivalent to a controller having integral action. If after a disturbance ε has been deviating from zero for some time, the total integrated blend is correct again as soon as ε has returned to zero. The bath-tub capacitor C_i acts as a memory and has, during the time of the disturbance, accurately booked how many buckets of C_A or C_B were missing from the correct amount for the chosen blending ratio.

Design Considerations

In the foregoing it was supposed that C_A and C_B were completely discharged into C_i. This supposition is only correct if there is no voltage across C_i, a condition hardly ever fulfilled in practice, because a small voltage is needed for control purposes. This affects the accuracy of the instrument. Since it was decided that the overall accuracy of the instrument should not be less than that of the primary measuring elements, the positive displacement meters, an accuracy of one part in a thousand was chosen. Hence the lowest charging voltage of C_A (or C_B) must be 1000 times the maximum voltage ($\varepsilon_{\max}$) across C_i. The lowest charging voltage for C_A is determined by the supply voltage E and the ratio setting α. The range chosen for α was 1/11–10/11, which gives a ratio range of 1–100. The minimum voltage across C_A is therefore $\sim 0{\cdot}1\ E$.

The maximum voltage, $\varepsilon_{\max}$, across C_i is governed by the following consideration. The control system will have some time lags and it is therefore essential for C_i to have some storage capacity. Let us suppose that it is sufficient if C_i can store 10 maximum charges of C_A (or C_B). Starting from zero its charge will then become:

$$Q_{\max} = \varepsilon_{\max}\,C_i \simeq 10\,EC_A \text{ or } \varepsilon_{\max} \simeq 10\,EC_A/C_i$$

If the minimum voltage across C_A should be 1000 $\varepsilon_{\max}$ then

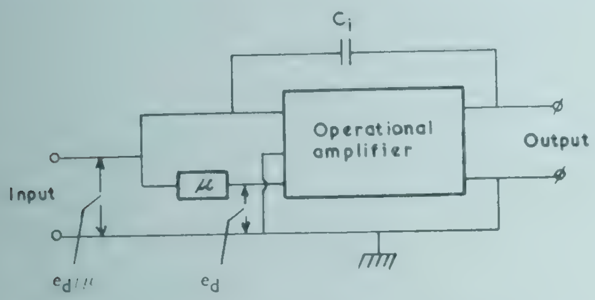
$$\frac{1}{10}\,E = 1000\,\frac{10\,EC_A}{C_i} \text{ or } \frac{C_i}{C_A} = 10^5$$

To restrict the capacitance of C_i to a reasonable value one needs to use a small capacitor for C_A. However, too small a value must not be chosen, since C_A must store the charge during the switching time and the relay contacts will have a certain conductivity to other terminals. For these reasons C_A and C_B in the practical set-up have a capacity of

0·03 μF. This would result in a value for c_i of 3000 μF, a value much too great for a capacitor of reasonable size and good insulating properties. Therefore the simple circuit of Fig. 2 cannot be used. Instead a circuit consisting of a small capacitor and an "operational" amplifier which behaves as a large capacitor was used. In fact the capacitor is, by this technique, multiplied by the amplification factor of the amplifier. A serious drawback of this capacitor transformation is, however, that the associated amplifier is a d.c. amplifier with its inherent "drift", which again would lead to intolerable errors. To deal with this amplifier drift, a drift-correcting "chopper-stabilized" amplifier had to be included as well.

Capacitor transformation

A schematic diagram of the capacitor transoformation and associated drift correcting amplifier is given in Fig. 3.



Capacitor c_i is multiplied by the gain of the operational amplifier (Miller integrator). Drift is compensated for by a stable amplifier, with gain $= \mu$, connected to another terminal of the operational amplifier.

Fig. 3.—*Practical electronic differential amplifier*

As can be seen, use is made of a symmetrical operational amplifier having two equivalent input terminals. c_i is transformed by connecting it directly across one input terminal and the output of this amplifier, whereas the other input terminal is preceded by the drift correction amplifier. Drift of the amplifier is the phenomenon that in the absence of external signals the operational amplifier needs a varying input voltage to one of its input terminals to keep the output voltage unchanged. It can be seen from Fig. 3 that if a drift voltage e_d is needed by the operational amplifier this will be supplied by the preceding stable drift correcting amplifier, leaving only the fraction e_d/μ to be supplied by the common input terminal connected to the bath-tub capacitor. The gain, μ, of the drift correcting amplifier is around 300.

Unfortunately the drift correction amplifier cannot be used alone instead of combined with the operational amplifier, since it would be too slow to respond to the fast input pulses delivered by the switching relays.

To realize the desired accuracy many more difficulties had to be solved, luckily none of them as expensive as the capacitor transformation. An intriguing problem was the influence of certain stray capacities on the bucket capacitors, c_A or c_B (referring to Fig. 2). As mentioned, switches s_1 and s_2 are driven by relays and the associated relay coils will have certain stray capacities to c_A and c_B.

A voltage step from say + to − across one of the polarized relay coils used will always be followed by a movement from right to left of switch s_1. This switch, however, will only move when this voltage step has been applied to the coil for a certain time. This means that in the right-hand position the stray capacity to c_A or c_B, which are then connected in parallel to c_i, is always discharged into c_i. In fact this is an extra current to c_i which was effectively reduced by screening and by using low-voltage, high-current coils on the polarized relays.

When used for ratio control the differential integrator should be followed by a controller, which in this instrument is an electronic controller of conventional design having proportional and integral actions. Also included in the circuit is an alarm relay, which is brought into action if the voltage across capacitor c_i exceeds a certain maximum value. This is necessary because, even when the circuit is working perfectly, it is possible to set the ratio to a value that cannot be realized by the pumps or the control valve.

Discussion

It will be clear that although the differential integrator was designed for the ratio control of two fluid streams, the system can be extended to any number of streams. For each extra stream to be blended an extra differential integrator and associated controller must be added.

From field experience it has become clear that the instrument works very satisfactorily; the only trouble arose from the associated pick-ups. These pick-ups are simple contact mechanisms located at the output shafts of the positive displacement meters. It seems to be difficult to make a good contact mechanism in the sometimes dirty atmosphere of an oil refinery. However, these difficulties have now been overcome.

Comparison with mechanical blender

The installation of the electrical system is quite simple: only a contacting mechanism need be installed on the positive displacement meters. The electrical energy selected here can be so low that the pick-ups are "intrinsically safe". These pick-ups must of course be connected electrically to the differential integrator, which is preferably located in a control room. Since this connection is by pulse transmission there is hardly any limit to the distances that can be reliably spanned. This compares very favourably with the mechanical system where it is essential that the output shafts of all the positive displacement meters concerned be brought physically together.[1]

The operation of the electronic instrument is also very easy since the desired ratio is directly obtained as soon as the accurate potentiometer is set. The ratio setting of the mechanical system on the other hand, is obtained by means of a continuous variable mechanical transmission which can only be set by trial and error methods.

In addition, the electrical system is far more flexible. In the case of ratio control with positive displacement meters, it is quite easy in principle to inject signals obtained from resistance thermometers into the differential integrator, which correct for density variations, and in this way to obtain mass flow ratio control. This will be particularly useful for in-line blending, where product temperatures can vary considerably during the operation.

The use of the circuit is by no means restricted to ratio control. In combination with a controller it can be looked upon as a computer in which the product of an input voltage or current, a frequency, and a capacitor of one half of the circuit is made equal to a similar product of the other half of the circuit. The capacitor could be changed by choosing a variable capacitor and varying its shaft position. Even if the capacitors are not varied, however, the system still has four variables, which in combination with a controller will compute the fourth variable if the other three are fed into it.

A limitation of the system might be the restricted frequency response of the relays. However, relays can be obtained that can respond to 50 c/s. If this is not fast enough the relays can be preceded by readily obtainable (transistorized) frequency dividers. Also, for lower accuracies, the relays can be replaced by electronic switches, giving a very fast computing device.

Conclusions

The differential integrator is a very useful circuit with many possibilities and its practical application to a ratio blending system results in an instrument far superior to any existing device for this purpose. It is thought that the electronic ratio controller has a promising future as an instrument for in-line blending purposes.

Acknowledgments

The author wishes to thank his colleagues, Mr. L. Ensing, Mr. A. C. Borsboom, and Mr. H. J. Nihof, for their help in developing the instrument.

Symbols Used

c_A, c_B, c_i = capacitors.
E = d.c. voltage.
e_d = drift voltage at input of amplifier.
F_A, F_B = liquid volume flows.
f_A, f_B = frequencies.
T = time.
Q = charge of capacitor c_i.
Q_{max} = maximum charge of capacitor c_i.
α, β = constants.
ε = voltage across capacitor c_i.
ε_{max} = maximum voltage across capacitor c_i.
μ = amplifier gain.

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DESIGN METHODS FOR DUAL-TEMPERATURE CHEMICAL EXCHANGE PROCESSES FOR PRODUCTION OF DEUTERIUM

By C. J. LYON, A.R.I.C., A.M.I.Chem.E.*

SUMMARY

The paper discusses the design of a dual-temperature chemical exchange process for the production of deuterium.

Liquid feed of low deuterium concentration is enriched in deuterium by flowing in a packed or bubble-cap tower at a low temperature counter-current to a hydrogen-containing gas. A portion of the enriched liquid leaving this tower is withdrawn as product; the remainder is passed through a similar tower at a higher temperature. Here the deuterium is stripped from the liquid by gas re-cycled from the top of the cold tower.

The physical data and mass-transfer equations being known, the equations have been programmed for an electronic digital computer so that the effects of different process conditions can be compared and an optimal set chosen.

Introduction

In the design of plant for the operation of cascade type processes, it is usually desired to carry out optimisation calculations in order to determine the most favourable operating conditions. The dimensions of the plant and its energy requirements will be based on these. Manual calculations, often of the successive approximation type, are laborious and time-consuming. Errors easily occur, and checking is as laborious and time-consuming as the original computations. A computing machine can be effective in reducing the time and in removing sources of human error: the example given below is a fairly straightforward application of such a machine to the optimisation calculations for a dual-temperature—(H_2S–H_2O)—chemical exchange process for the production of heavy water.

The present paper deals with the first stage only of a cascade process of the type shown in Fig. 1, in which L_1 represents the large input flow of, in this case, natural water with its low atom fraction X_1 of deuterium, while P is the small flow of product, highly enriched water with atom fraction X_p of deuterium. The separation is made possible by an equilibrium process taking place between the input water stream, L_1, and the rejected stream, $(L_1 - P)$, of depleted concentration $Y_1 = kX_1$, k being less than unity.

Taking a deuterium balance around the general or nth stage, and the last or Nth stage:

$$L_n X_n - (L_n - P)\, k\, X_n = P X_p$$

Since $L_n \gg P$, in the early stages, this equation simplifies to

$$L_n = \frac{P X_p}{X_n(1 - k)}$$

indicating that for a given separation process the rate of flow to the nth stage varies inversely as the deuterium concentration. In the ideal cascade with an infinite number of stages, the flowrate is reduced continuously towards the enriched end, the tower cross-section diminishing in proportion.[1] The obvious advantages of the ideal cascade are reduction of total volume and progressive reduction of the holdup of increasingly enriched water. On the other hand, the process could, in principle, be carried out in a single stage without increase in heat requirements. Practical requirements lead to the "stepped" cascade shown in Fig. 1, in which each stage for the H_2O–H_2S exchange process includes a hot and a cold tower each of which comprises a large number of transfer units; the reduction in tower cross-section takes place at successive stages only. Beyond a certain position in the cascade there are also practical advantages in changing from the exchange process involving hydrogen sulphide to a different exchange process, in particular because of the risk of losses from the circulating systems of hydrogen sulphide highly enriched in deuterium. Thus either water distillation or electrolysis can be used for the later stages with advantage, for while the energy requirements of these processes are impracticably high for natural water, they are

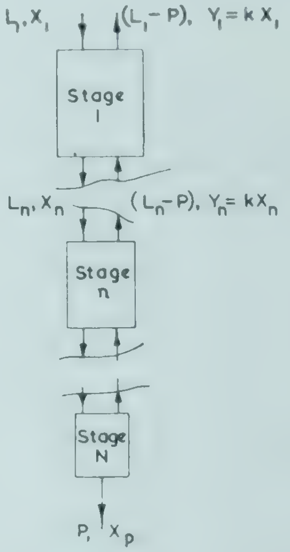

Fig. 1.—*Stepped cascade*

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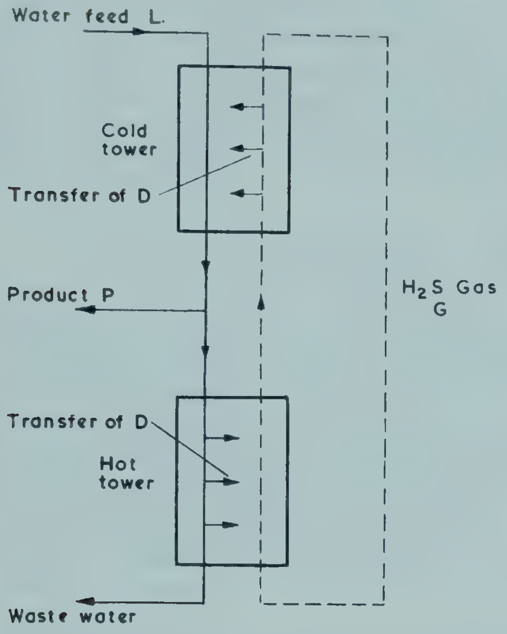
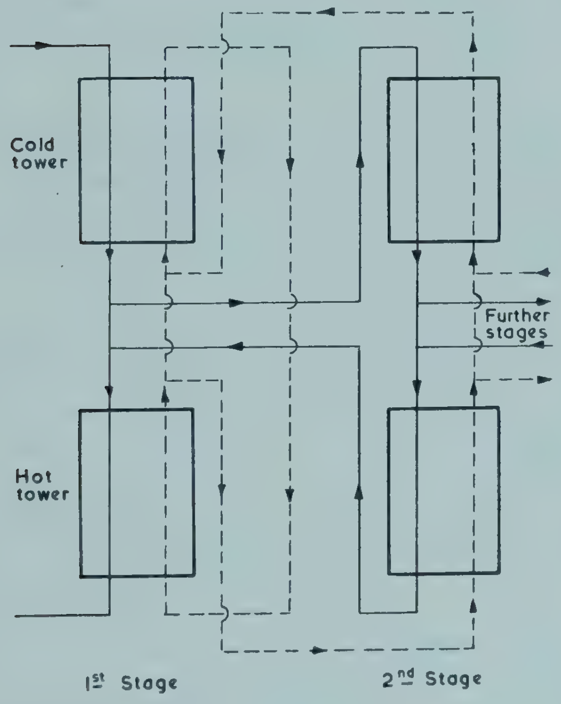
negligible when the atom fraction of deuterium is 0·1–0·2. However, this is part of the wider aspect of optimisation which will not be treated here; the main point is that it is fair to assume that the first stage of the cascade is the most costly part of the plant if the enrichment per stage is more than about 5. The enrichment obtained in this first stage will also depend on practical considerations such as the number of theoretical stages possible in a tower of reasonable height.



A *Simplified flow diagram H_2S–H_2O exchange process*



B *Coupling of stages*

Fig. 2.—*Flow sheets*

Another point to be emphasised is that in this paper references to precise optimum values have been deliberately avoided as many aspects of the present work are still of a confidential nature.

Basis of process

The chemical exchange process between water and hydrogen sulphide can be represented by the equation:

$$H_2O_{(l)} + HDS_{(g)} = HDO_{(l)} + H_2S_{(g)}.$$

Hydrogen in ordinary water is replaced by deuterium in hydrogen deuterium sulphide to give hydrogen deuterium oxide or deuterium-enriched water together with ordinary hydrogen sulphide. The equilibrium constant can be written:

$$K_{lg} = \frac{(HDO)_l\,(H_2S)_g}{(H_2O)_l\,(HDS)_g}.$$

K_{lg} has the value 2·3 at 25° C and 1·92 at 100° C: that is, enrichment is in the water phase and decreases with increase in temperature. Better separation factors are attainable with other liquid-vapour deuterium exchange reactions, notably that between ammonia and HD, but the H_2O–H_2S reaction has the great advantage that no catalyst is needed. Such separations result from differences in the zero-point energies of the isotopic molecules, and equilibrium constants are, in principle, calculable for the gas-phase reactions by statistical mechanics, provided that the relevant spectroscopic data are available; such procedures have been described by Biegeleisen and Mayer[2].

Process Description

The principle of the process is shown in Fig. 2A. Each stage of the process consists of two towers, a cold tower and a hot tower. Hydrogen sulphide gas is circulated round the two towers counter-currently to water which is fed at its natural concentration (atom fraction deuterium/hydrogen $\sim 1{\cdot}45 \times 10^{-4}$) at the top of the cold tower. By virtue of the different separation factors at the two temperatures, the net effect in the cold tower is a transfer of deuterium from the gas to the water, and the net transfer in the hot tower is in the opposite direction: here the water stream is depleted before discharge. The maximum concentration of deuterium in the water is at the bottom of the cold tower, where an enriched product can be removed or passed on to a further stage of concentration (as in Fig. 2B).

Heat conservation by appropriate exchangers is obviously required, but for simplicity these are not shown in the diagrams.

There are temperature limitations; the cold tower should operate at as low a temperature as is practicable, and this is limited to about 30° C in order to avoid deposition of solid H_2S-hexahydrate. Pressure is also limited because liquefaction of the hydrogen sulphide must be avoided. A temperature limitation is imposed in the hot tower by the need to restrict the humidity of the gas. Benedict[3] gives an excellent survey of heavy water concentration processes and has been drawn on extensively in the subsequent theoretical treatment (see also Ref. 1).

Representation of the process by a McCabe-Thiele diagram

IGNORING THE EFFECTS OF HUMIDITY AND SOLUBILITY

The operation of the system can be represented by a diagram of the McCabe-Thiele type; a simplified version of such a diagram appears in Fig. 3. The atom fraction of deuterium in the liquid phase is represented by x and in the vapour phase by y. In this first approximation, which gives an idea of the large flow-rates involved, the partial pressure of steam in the hydrogen sulphide and the solubility of hydrogen sulphide in the water will be ignored. In this case the equilibrium relations for the low range of deuterium concentrations which are of most interest can be represented by straight lines, or

$$x = Ky; \; K > 1$$

where K decreases with temperature. This is shown in Fig. 3 where the lower of the two equilibrium lines is that for the cold tower. The ratio between the molar flow rates of water and hydrogen sulphide fixes the slopes of the operating lines for the two towers. The molar feed rate of water can be written as L and of the enriched water product as P, with that of the circulating hydrogen sulphide stream as G. Thus the "cold" operating line has a slope L/G, and the "hot" operating line a slope $(L - P)/G$.

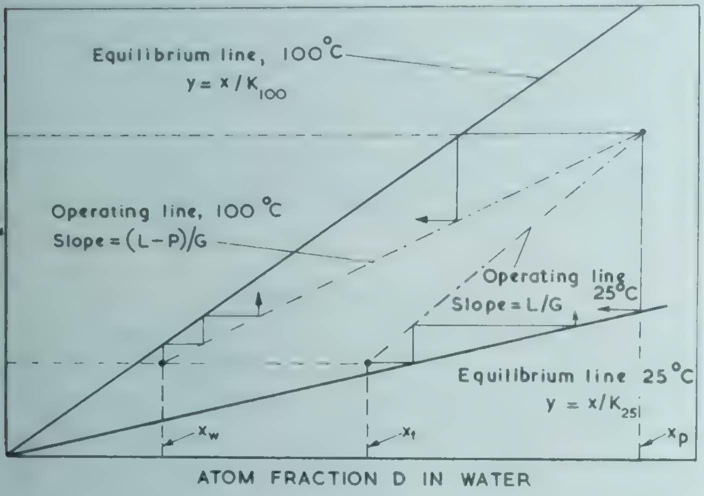


Fig. 3.—*Simplified McCabe-Thiele diagram for H_2S–H_2O exchange process.* (*After M. Benedict*)

The operating lines are chosen to lie between the equilibrium lines, and in this simplified diagram they enclose a narrow inverted V with its apex at the product composition. The lower ends of the operating lines have a common ordinate—the composition of the gas leaving the top of the cold tower and entering the bottom of the hot tower. The operating line for the cold tower intersects this ordinate at the point representing the D_2O concentration in ordinary water. Starting at this point, theoretical stages* can be stepped off in the usual way between the operating and equilibrium lines, up to the product composition represented by the point of intersection of the operating lines. This is done, in reverse, for the hot tower in which water is depleted in D_2O as it passes downwards, until it leaves the hot tower as waste water with a concentration of D_2O less than the natural value.

To simplify the analysis further it can be assumed that the operating lines intersect the equilibrium lines at the common ordinate, y_f. This implies an infinite number of plates in each tower and the greatest possible difference between feed and waste water concentrations for that particular combination of temperatures. In this case $x_w/x_f = K_{100}/K_{25}$, say, where the suffixes to K represent the temperatures in degrees Centigrade at which K is measured. The highest possible percentage recovery is $100(1 - K_{100}/K_{25})$. In round numbers, $K_{100} = 2$ and $K_{25} = 2{\cdot}5$, so that recovery is about 20% (practical figures, it should be added, are nearer to 15%).

The minimum value for the molar rate of water feed required for one mole of product D_2O in unit time is

$$1/(x_f - x_w) = 1/0{\cdot}2\,x_f = 1/(3 \times 10^{-5}),$$

or about 33 000.

Further, the slopes of the operating lines are nearly the same, since x_p is well to the right of the diagram. The minimum ratio of the molar flow rates of hydrogen sulphide and feed water is now very nearly the reciprocal of the slope of the steepest operating line, or about 2. Thus the minimum value for the molar rate of hydrogen sulphide circulation is about 66 000 for one mole of product D_2O in unit time.

ELABORATION OF THE DIAGRAM TO INCLUDE THE EFFECTS OF HUMIDITY AND SOLUBILITY

It has already been indicated that practical recovery figures—which take account of saturation of hydrogen sulphide with water vapour and of water with hydrogen sulphide gas—are considerably less than in those of the first approximation in which these effects were ignored. Practical analysis of the system begins with re-definition of equilibrium relationships in the gas and water phases. Instead of x and y will be written X and Y, corresponding to the mol fraction of deuterium-containing compounds in the liquid and gaseous phases, and instead of K will be written β, which is defined as:

$$\beta = \frac{\text{atom ratio of D to H in liquid}}{\text{atom ratio of D to H in vapour}}, \quad \text{where } \beta > 1$$

so that $X = \beta Y$ (see below).

For low concentrations of deuterium in water, the amount present as HDO is much greater than that present as D_2O.

The equilibrium relationship between D_2O, HDO, and H_2O can be expressed as:

$$\frac{[\mathrm{HDO}]^2}{[\mathrm{H_2O}][\mathrm{D_2O}]} \simeq 4 \quad \text{(where the quantities in square brackets are molar concentrations).}$$

Putting $[H_2O] = x'$, $[D_2O] = y'$ and [HDO] as $1 - x' - y'$, substitution in the above equation gives the symmetric quadratic:

$$(y')^2 - y'(2x' + 2) + (x')^2 - 2x' + 1 = 0$$

Solving for y:

$$y' = x' + 1 \pm 2(x')^{\frac{1}{2}}$$

The solution

$$y' = x' + 1 + 2(x')^{\frac{1}{2}}$$

is greater than 1 and therefore trivial. Thus

$$y' = x' + 1 - 2(x')^{\frac{1}{2}}$$

is the solution required.

Inserting values of x' or $[H_2O]$ gives y' or $[D_2O]$ while [HDO] is $1 - x' - y'$. Since the quadratic is symmetric, the concentrations of H_2O and D_2O can be reversed to give the same concentration of HDO. A few typical values are quoted in Table I.

TABLE I.—*Concentrations of H_2O, D_2O and HDO*

$[H_2O]$	$[D_2O]$	[HDO]
0·64	0·04	0·32
0·81	0·01	0·18
0·9025	0·0025	0·095
(and 0·0025	0·9025	0·095 etc., by reversal)

A similar relationship holds between D_2S, HDS, and H_2S.

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* These theoretical stages are, of course, not to be confused with the complete hot- and cold-tower system.

In defining β for low concentrations of D, the following approximation is justifiable:

$$\beta = \left[\frac{[\mathrm{HDS}]_l + [\mathrm{HDO}]_l}{2[\mathrm{H_2O}]_l + [\mathrm{HDS}]_l + [\mathrm{HDO}]_l}\right] \times \left[\frac{[\mathrm{HDS}]_g + [\mathrm{HDO}]_g}{2[\mathrm{H_2S}]_g + [\mathrm{HDS}]_g + [\mathrm{HDO}]_g}\right]^{-1}$$

in which the species D_2O and D_2S are ignored. Further, since $[H_2O]$, which is nearly unity, is much greater than [HDO], and $[H_2S]$, which is nearly unity, is much greater than [HDS], it is permissible to write:

$$\beta = \frac{[\mathrm{HDS}]_l + [\mathrm{HDO}]_l}{[\mathrm{HDS}]_g + [\mathrm{HDO}]_g} = \frac{x_{35} + x_{19}}{y_{35} + y_{19}} = \frac{X}{Y}$$

in which x and y represent atom fractions in the liquid and gas phases respectively and the suffixes the molecular weights of the compounds they represent (X and Y are as defined above.)

It is now necessary to define a "solubility", S', as moles of the most abundant sulphur compound in solution per mole of the most abundant oxygen compound in solution, or

$$S' = [\mathrm{H_2S}]_l/[\mathrm{H_2O}]_l = x_{34}/x_{18},$$

so that

$$\frac{x_{34} + x_{18}}{x_{18}} = \frac{1}{x_{18}} = 1 + S' \text{ or } x_{18} = \frac{1}{1 + S'}$$

It is also necessary to define a "humidity", H', as moles of the most abundant oxygen compound in the gas per mole of the most abundant sulphur compound in the gas or

$$H' = [\mathrm{H_2O}]_g/[\mathrm{H_2S}]_g = y_{18}/y_{34}$$

so that

$$\frac{y_{18} + y_{34}}{y_{34}} = \frac{1}{y_{34}} = 1 + H' \text{ or } y_{34} = \frac{1}{1 + H'}$$

The equilibrium reaction for the components of interest is:

$$\mathrm{H_2O + HDS \rightleftharpoons HDO + H_2S}$$

It will be assumed that the equilibrium constant, K_g, for components all in the gas phase is equal to the equilibrium constant K_l for components all in the liquid phase, or

$$K_g = \frac{y_{19}\, y_{34}}{y_{18}\, y_{35}} = K_l = \frac{x_{19}\, x_{34}}{x_{18}\, x_{35}} = K = \frac{x_{19}\, S'}{x_{35}}$$

For consistency it follows that a_s, the relative volatility for the system HDS/H_2S, which is given by

$$a_s = \left(\frac{x_{35}}{x_{34}}\right) \Big/ \left(\frac{y_{35}}{y_{34}}\right)$$

is equal to a_0, the relative volatility for the system HDO/H_2O, which is given by

$$a_0 = \left(\frac{x_{19}}{x_{18}}\right) \Big/ \left(\frac{y_{19}}{y_{18}}\right)$$

Thus a common relative volatility, a, can be used for either case: that is,

$$a \simeq a_s \simeq a_0.$$

Further,

$$H'K = \left(\frac{y_{18}}{y_{34}}\right) \cdot \left(\frac{y_{19}\, y_{34}}{y_{18}\, y_{35}}\right) = \frac{y_{19}}{y_{35}} \text{ and } K/S' = \frac{x_{19}}{x_{35}}$$

Substituting in $\beta = \dfrac{x_{35} + x_{19}}{y_{35} + y_{19}}$:

$$\beta = \frac{\left(1 + \dfrac{x_{19}}{x_{35}}\right) x_{35}}{\left(1 + \dfrac{y_{19}}{y_{35}}\right) y_{35}}$$

$$= \frac{\left(1 + \dfrac{K}{S'}\right) x_{35}}{(1 + H'K)\, y_{35}}$$

$$= \frac{(S' + K)}{(1 + H'K)} \frac{x_{35}\, x_{18}}{y_{35}\, x_{34}}$$

or

$$\beta = \frac{(S' + K)}{(1 + H'K)} \left[\left(\frac{x_{35}}{x_{34}}\right) \Big/ \left(\frac{y_{35}}{y_{34}}\right)\right] \frac{x_{18}}{y_{34}}$$

or

$$\beta = \frac{a(S' + K)(1 + H')}{(1 + H'K)(1 + S')}$$

It is now necessary to establish compositions and flowrates for the various gas and liquid streams leaving and entering the exchange towers, the humidifier, the gas saturator and the condenser-cooler (see Fig. 4 A and B). It is not considered necessary to give the straightforward but tedious derivations of these equations—which are based on mass and material balances—or of the expressions giving the number of theoretical stages required in the hot and cold towers, which are finite difference expressions of a familiar form.

Application to Computer and Results

Typical values of the various parameters used in the work are given in Table II.

TABLE II.—*Typical Values Used in Calculating a Flowsheet*

Working pressure 20 atm.
Cold tower temperature 35° C. Degree of enrichment, $E = 10$.
Hot tower temperature 110° C.
Approach to equilibrium for both towers $= 0{\cdot}98 = \varepsilon_c = \varepsilon_h$.
x_f (natural concentration) $= 0{\cdot}000145$.

Table III lists the dependent variables which are related to the parameters given in Table II.

TABLE III.—*Flowsheet Values Dependent on the Selected Values in Table II*

$H_h = 0{\cdot}088$ $H_c = 0{\cdot}004$	$S_h = 0{\cdot}0112$ $S_c = 0{\cdot}0268$	$H_hS_c = 0{\cdot}0099$ (Other combinations of H' and S' are negligible)
	$P = 1$	
$a_h = \dfrac{1}{0{\cdot}9785}$	$K_h = 1{\cdot}8847$	$\beta_h = 1{\cdot}788$
$a_c = \dfrac{1}{0{\cdot}9416}$	$K_c = 2{\cdot}1452$	$\beta_c = 2{\cdot}236$

A trial value of $G/L = 2{\cdot}1$ is chosen. The computing scheme is given in Table IV, and refers to the single-stage flowsheet shown in Fig. 4B:

The calculations given in Table IV are simple and straightforward, but are tedious if they have to be done in any quantity. The use of automatic computing machinery then becomes desirable, and a programme to calculate all the quantities required was written in Autocode[4] by Dr. J. Howlett. Because this is merely a matter of evaluating formulae, the programme was simple and straightforward,

TABLE IV.—*Scheme for Computing the Single-stage Flowsheet Shown in Fig.* 4B, *With Values Taken from Tables II and III and a Trial Value of* $G/L = 2\cdot 1$

(1) Calculate $$Y^0 = \frac{x_f(1 + S_h)}{\beta_c \varepsilon_c(1 + S_c) - (1 + H_c)(S_c - S_h)} = 0\cdot 4525\ C_0$$

(where C_0 is natural concentration)

(2) Calculate $X_f = \beta_c \varepsilon_c Y^0 = 0\cdot 9917\ C_0$

(3) Calculate $$X_w = \frac{\beta_h Y_0(1 + H_c)}{\varepsilon_h(1 + H_h) - \beta_h(1 + S_h)(H_h - H_c)} = 0\cdot 8885\ C_0$$

$(\simeq x_w = 0\cdot 8931\ C_0)$

(4) Recovery $$\simeq 100\left(1 - \frac{x_w}{x_f}\right) = 11\cdot 1\%$$

(5) Calculate $$L = \frac{P(x_p - x_w)}{(x_f - x_w)} = 64\ 393$$

$(= L_1)$

(6) Since $G/L = 2\cdot 1, \qquad G = 135\ 225 \simeq G_1$

(7) Calculate M_c, slope of cold tower operating line, from

$$M_c = \frac{L(1 + S_c)}{G_1(1 + H_c)} = 0\cdot 483\ 413$$

(8) Calculate b_c, intercept on Y-axis of cold tower operating line, from

$$b_c = \frac{G Y^0(1 + H_c) - L x_f(1 + S_h)}{G_1(1 + H_c)} = -\ 0\cdot 026\ 896\ C_0$$

(9) Calculate $$X_p^1 = \frac{E x_f}{1 + S_c}\left[1 + \frac{S_c}{K_c - E x_f(K_c - 1)}\right] = 9\cdot 861\ C_0$$

(10) Calculate $Y_b = X_p^1/\beta_c = 4\cdot 409\ 4\ C_0$

(11) Calculate X_b from

$$X_b = \frac{Y_b + b_c}{M_c} \simeq 8\cdot 3\ C_0$$

(12) Calculate N_c, number of theoretical stages in cold tower:

$$N_c = \log\left\{\frac{(\beta_c M_c - 1)\ X_b + \beta_c b_c}{(\beta_c M_c - 1)\ X_f + \beta_c b_c}\right\} \Big/ \log \beta_c M_c = 45\cdot 162$$

(13) Calculate $L_2 = L + G(H_h - H_c) = 75\ 752$

(14) Take $G_2 = G = 135\ 225$

(15) Calculate $$M_h = \frac{L_2(1 + S_h)}{G_2(1 + H_h)} = 0\cdot 520\ 647$$

(16) Calculate $$b_h = \frac{(G/L)\ Y^0(1 + H_c) - X_w(1 + S_h)}{(G/L)(1 + H_h)} = 0\cdot 024\ 350\ C_0$$

(17) Calculate $$X_t^1 = \frac{L_1 X_b(1 - H_h S_c) + G_1 X_p^1(H_h - H_c) - P x_p(1 - H_h S_c)}{L_1(1 - H_h S_c) + G_1(H_h - H_c) - P(1 - H_h S_c)}$$

$\simeq 8\cdot 58\ C_0$

(18) Calculate $$X_t = \frac{X_t^1\ \beta_h(1 + S_c)}{\beta_h(1 + S_h)(1 - H_h S_c) + (S_c - S_h)(1 + H_h)}$$

$\simeq 8\cdot 65\ C_0$

(19) Calculate N_h, the number of theoretical stages in the hot tower from:

$$N_h = \log\left\{\frac{(1 - \beta_h M_h)\ X_t - \beta_h b_n}{(1 - \beta_h M_h)\ X_w - \beta_n b_n}\right\} \Big/ \log\left(\frac{1}{\beta_h M_n}\right) = 50\cdot 547$$

(20) Calculate $G_2 H_h = G H_h$, the flow rate of steam for humidification

$= 11\ 899$

For any given set of physical conditions there are upper and lower limits for the value of the gas-to-liquid ratio, G/L, at which the number of theoretical stages in the "cold" and "hot" towers respectively become infinite; this is caused by the vanishing of the denominator in the logarithmic term either in equation (12) or equation (19). Between these limits there is a minimum value for the total number of stages in the two towers. Each computer run gave a table of values of N_c, N_h and $N = N_c + N_h$, for values of G/L between the permissible limits, the limits themselves being found by the programme by trial and error. The minimum number of total stages and the corresponding values of G/L were found by interpolation in this table.

It was assumed, as a first approximation, that the height of a theoretical stage in each tower was the same, and values of G/L corresponding to the minimum number of total stages were taken as optimum values. Fig. 5 indicates the type of graph obtained when the numbers of theoretical stages

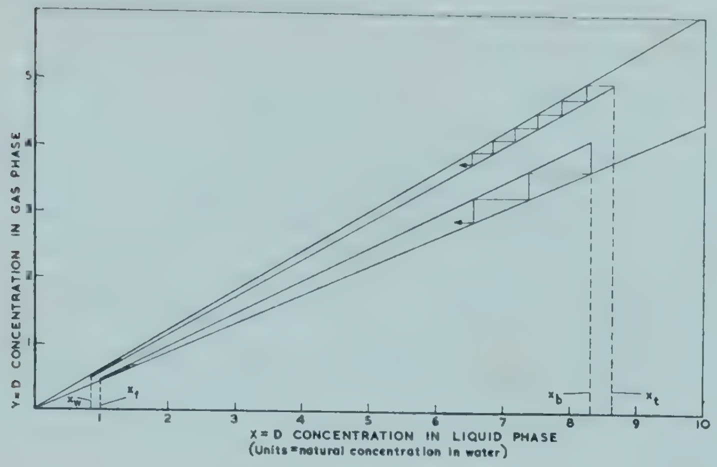

Fig. 4.A.—*McCabe-Thiele diagram for H_2S–H_2O process (specimen example, allowing for humidity and solubility)*

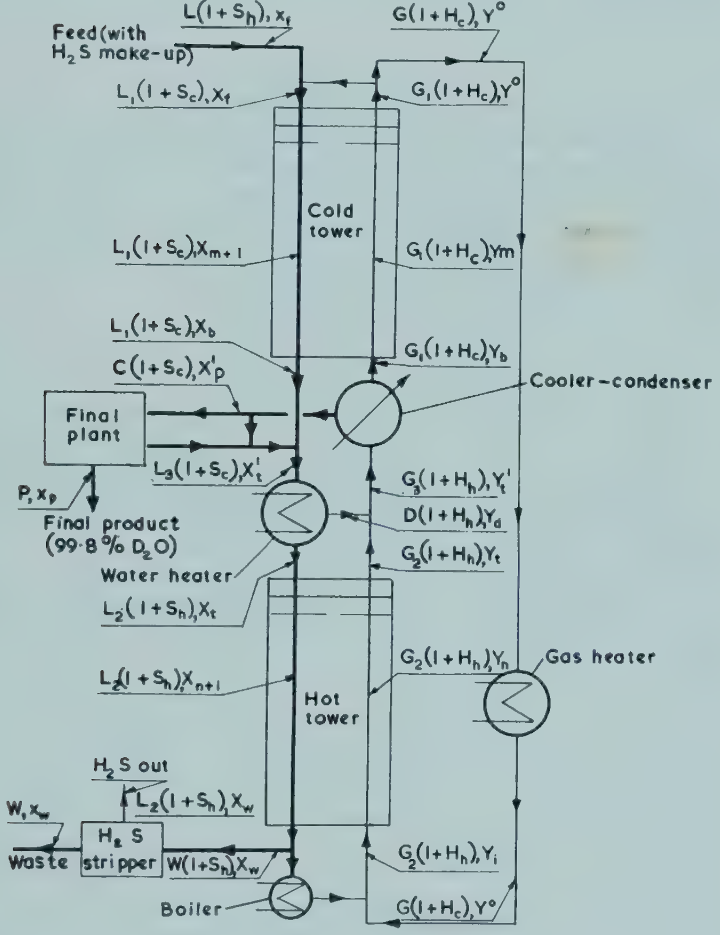

Fig. 4.B.—*Arrangement of plant, and notation, for single-stage calculations (from A.E.R.E. CE/R 2348)*

(transfer units) for the hot tower, the cold tower, and both are plotted against *G/L*. In most cases the minimum combined number is very clearly defined and there is a different optimum value of *G/L* for each combination of pressure, hot and cold tower temperatures, enrichment, and degrees of approach to equilibrium.

Sixteen runs were made in the first set of calculations, corresponding to combinations of four pressures with four hot tower temperatures (the temperature of the cold tower and the degree of approach to equilibrium at the "pinches" was fixed), the limitation being introduced that the humidity, H^1, should not exceed unity, since humidification of the hydrogen sulphide gas from the cold tower before it enters the hot tower is the main energy requirement of the process. As the hot, saturated H_2S leaves the hot tower this water vapour, now enriched in D_2O, has to be condensed out, and equipment to do this, ranging from condenser coolers to ingenious heat exchangers, is bulky and costly.

But for saturation of the hydrogen sulphide with water vapour, recovery would be highest, that is, tower cross-sections for a given output would decrease, at higher hot-tower temperatures. However, the amount of humidifying steam increases as the temperature increases, and an optimum hot-tower temperature was expected as a result of this programme.

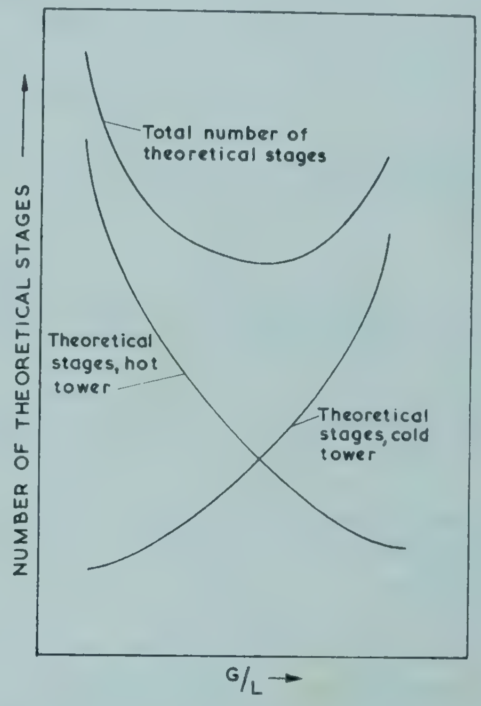

Fig. 5.—*Selection of optimum G/L ratio*

In a second set of computations the more promising first-series results were re-calculated with varying degrees of approach to equilibrium. There were 32 such cases. A third set of computations explored the effect of varying the cold-tower temperature and the enrichment factor in further cases of interest.

In all, determination of the optimum operating parameters involved the calculation of about 70 flowsheets, the later ones being more complex, firstly because other stages were brought in, and secondly because allowance had to be made for curvature of the equilibrium line. As the work proceeded, the value of computer methods in optimisation calculations became abundantly clear to those who undertook the desk calculations at the outset.

Comments and Conclusions

At the time of writing, information on optimum values must be withheld, but a fairly obvious procedure for deriving the optimum hot-tower temperature can be outlined. The first step is the selection of the optimum values of *G/L* based on the minimum number of total theoretical stages, for different hot tower temperatures, other parameters being the same. The minimum number of total stages, which can be taken as a measure of the combined height of the hot and cold towers, is plotted against the flowrate of humidified gas in the hot tower and the gas flowrate for the cold tower. These flowrates are taken as proportional to the respective

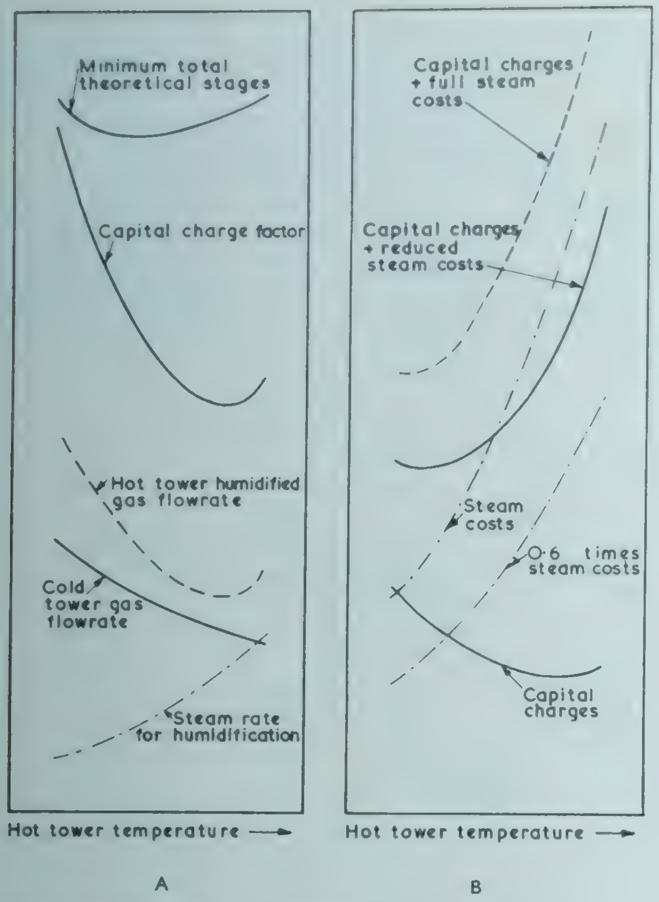

Fig. 6.A.—*Quantities used in compilation of Fig. 6(B)*

Fig. 6.B.—*Estimation of optimum hot tower temperature based on minimum costs*

tower cross-sectional areas. Such plots are shown in Fig. 6A, which includes the steam rate for humidification per unit of product. A rough estimate is now made of the average capital cost per ton D_2O capacity per year, and capital charges worked out for interest on capital, amortisation, maintenance, materials, and labour. Assuming the capital cost to be directly proportional to tower volume, the capital charge is multiplied by the factor:

(minimum number of total stages) (cold + hot flowrates)

for the selected hot-tower temperature. To this capital charge is added the appropriate cost of steam for humidification, leading to plots similar to those in Fig. 6B. The upper curve is typical of such combined costs and shows no minimum value. The full curve with a minimum value shows the effect of reduction, by means of heat recovery, of the unit cost of steam for humidification, by an arbitrary factor of 0·6. This reduction in steam costs must of course also carry capital charges on the necessary heat exchange equipment in which large gas throughputs are alternately humidified and dehumidified. These costs may well be decisive in comparisons between the present process and other processes.

In the final design, of course, engineering considerations, such as the number of first-stage towers in parallel and their shell diameters and thickness, and the choice of materials to withstand the corrosive action of hydrogen sulphide become of importance.

Acknowledgments

The author wishes particularly to thank Dr. J. Howlett, who was responsible for the development and success of the computing programme. Acknowledgment must also be made to Dr. H. R. C. Pratt and Dr. P. T. Walker, who are closely associated with this work, for helpful discussion and criticism.

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A SYSTEM FOR THE CONTROL OF VOLUME BALANCE AND CHEMICAL COMPOSITION OF A PROCESS SOLUTION AND A CHECK ON THE DESIGN OF THE EQUIPMENT USING AN ANALOGUE COMPUTER

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SUMMARY

This paper is the "case history" of the design and operation of a section of a chemical plant concerned with the maintenance of the volume and concentration of a circulating caustic soda solution. The paper describes:

(*a*) The process insofar as it affects this section of plant,
(*b*) The control problem as it appeared during design,
(*c*) The design of the control system and the supporting arguments,
(*d*) A check on the design before start up by means of an analogue computer,
(*e*) Some notes on start-up performance,
(*f*) Proposed design for next plant,
(*g*) An assessment of the value of the computer exercise.

Introduction

The plant considered is a section of the "steep soda" system of a viscose staple-fibre unit. In the course of development work on this process, with the object of making it continuous, it was found necessary to provide a continuous soda make up and correction system.

The process requires that caustic soda (46% concentration) and water be fed into the system with a view to maintaining the level and concentration of caustic soda in a stock tank. The aqueous caustic soda is consumed by the process in varying quantities. Circulating streams of liquor pass into and out of the tank, one of these streams being used for temperature control. Density measurement of the temperature controlled liquor stream is used to deduce the composition of the liquor.

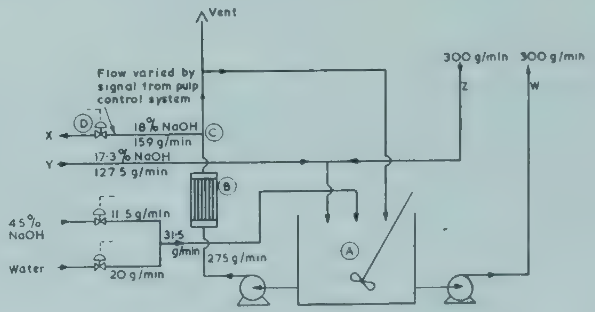
The accuracy of control required is 1/200 of the desired concentration of caustic soda. The control problem divided itself into three loops: (*a*) the liquid-level control loop, (*b*) the analysis control loop, and (*c*) the ratio control loop. These were designed so that the primary (water) flow was set by the level control and the secondary flow was kept in constant ratio to it. The analysis loop had a monitoring function, maintaining the liquor analysis by resetting the ratio (*i.e.* cascading) between the primary and secondary controllers.

The arrangement of the process and its associated controllers, as well as control stability, were checked by simulating the system on an electrical analogue computer. It was necessary to work out the transfer lags and determine the transfer functions of the individual parts of the system to do this.

The results obtained on the production unit confirm in general the correct fundamental design of the process.

The Process

Fig. 1 is a flow sheet of the process. Aqueous caustic soda is fed from a stock tank, A, at approximately 18% concentration: the solution also contains some dissolved hemicellulose and cellulose fibres in suspension. The stream is maintained at a specified temperature in the region of 50°C by means of heat exchanger B and then splits into two, stream X passing to the "process" and the remainder returning to the tank through a standpipe which controls the pressure at point C.



Flows in gallons per minute

Fig. 1.—*Diagrammatic flowsheet of section of plant before instrumentation*

The solution and the caustic soda in it are consumed in varying quantities. As a result, stream Y, which is the combined return stream from the process, is lower in total quantity, lower in caustic soda concentration, and at a different temperature compared with stream X.

Streams W and Z, leaving and returning to the tank, have been included in the diagram for completeness, but they do not significantly affect the control problem since they are equal in quantity and composition.

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* Courtauld's Ltd., Coventry.

The estimated quantities and soda concentrations of the various streams are indicated in Fig. 1. These are average quantities, stream X in particular being very variable since it was proposed that it would be automatically proportioned by means of a flow ratio controller (acting through control valve D) to a flow of solid wood pulp which would itself be far from steady.

The stock tank had to be stirred to prevent the fibres settling out. It was decided to stir it sufficiently to ensure that the contents were thoroughly mixed so that the concentration of caustic soda would be constant throughout the tank and at the outlet.

It was decided that the tank capacity should be 10 000 gal, but that the level would usually be controlled to maintain the tank about one-third full. Two thirds would then be available for receiving liquor drained from the process when shutting down. It was appreciated that the volume of liquor in the tank and the shape of the tank could affect the control characteristics of the process, but the size and shape of the tank were decided on general grounds and the result checked later.

The Control Problem

Water and caustic soda (46%) are added to the stock tank to maintain the level in the tank and the concentration of soda in the liquor.

The main variable load imposed on the system would be the fluctuating flow of the outgoing stream X, the fluctuation being of rapid frequency, the flow varying every few seconds. The return stream Y would also vary. The combination of tank capacity and type of level controller had therefore to cope with this, and an automatic method of analysis had to be used which would be sufficiently rapid and accurate in action.

Control Equipment Installed

Liquid level control loop

The level measurement in the steep soda tank was achieved by means of an air purge system which comprised a dip pipe, a level transmitter of the differential pressure type, a proportional stack type controller, and a diaphragm-operated control valve. Signal transmission was by pneumatic means.

Ratio control loop

The water flow set by the liquid level control loop was measured by an orifice plate coupled to a differential pressure transmitter. The output from the transmitter was cascaded into a pneumatic-set ratio flow controller having proportional and integral control terms. The special feature of this controller was that the ratio setting could be pneumatically adjusted by means of a 3–15 lb/in^2 bellows unit allowing a flow ratio variation from 0·7–1·2. The output from this ratio controller was used to set the position of the 46% soda flow-control valve. The resulting flow was monitored by means of a differential pressure (flow) transmitter receiving its primary information from an orifice plate and transmitting its output signal as a feed back to the ratio controller. The function and use of the pneumatic-set ratio arrangement will now be described by consideration of the analysis control loop.

Analysis control loop

The liquor had a concentration of about 18% caustic soda and contained hemicellulose, suspended fibres, and organic colouring matter. A method was required for continuously determining the concentration of caustic soda with an accuracy of ± one part in 200. A method of feeding this signal into the ratio loop as a correcting factor was also required. The main analytical methods considered were:—

(*a*) continuous titration,
(*b*) electrical conductivity,
(*c*) refractive index, and
(*d*) density.

Continuous titration appeared to be the most desirable method but no satisfactory commercial instrument was available at the time.

Electrical conductivity was not used because of lack of sensitivity over the required range and the necessity of using an electro-pneumatic converter to supply the signal to the pneumatic ratio loop.

The colour of the liquid made refractive index measurement difficult and development work on such a method would have been necessary.

Density measurement seemed practicable and accurate and could be readily fitted into the system. The fibres in the liquid narrowed the choice of the instrument to a U-tube weighing-type primary mechanism, but even in this case, considerable sampling difficulties had to be overcome: in particular the entrainment of air had to be avoided. An instrument capable of measuring density to ±1/2000 had to be obtained and the liquor temperature had to be controlled prior to sampling.

The sample was required to be taken from the steep-soda stream after it had left the tank. The density transmitter sent its output signal to a two-term, stack-type controller. The output signal from this controller was utilized to set the ratio index of the flow ratio controller referred to in the previous section.

Because of the importance of accurate steep-soda concentration measurements, it was decided to check the stability of the system and to learn other relevant lessons by simulating the scheme on an analogue computer.

Simulation of the Steeping Control System

The physical quantities appearing on the flowsheet, *e.g* soda flow, were simulated on a computer operated by Metropolitan Vickers Electrical Co. Ltd., to whom we are indebted for this analysis. Flows were represented by voltages and more complicated component parts of the control system were represented by combinations of electrical elements such as resistances, capacitances, etc. Once the control loops had been simulated completely it was possible to inject a disturbance, chosen to be equivalent to that which might occur on the plant. By this means any design deficiencies, both in the plant and in the instruments, would be revealed, *e.g.* inadequate storage capacity in the main vessel, insufficient terms in the controllers, or other design faults.

The computer used was Short Brothers and Harland's General Purpose Machine. At the time at which the work was carried out, (August, 1956), the machine consisted essentially of eighteen function units designed for repeated computation with drift stabilisation of the amplifiers during the non-computing periods. No "non-linear units" were available: this limitation also applied to the multipliers.

These restrictions precluded consideration of non-linearities such as valve-flow limitation, and demanded the reduction of the problem to one linearised for small departures. In particular, lack of a multiplier to represent the soda: water ratio control restricted the validity of the results to those applying to small deviations from the desired concentration. Under these conditions, the effect of the ratio control is that an increment δW in water flow produces an increment $R\delta W$

of soda flow, ($R =$ constant), and that a change in ratio demanded by the concentration controller produces a change in soda flow, δS.*

The change in ratio of soda to water in the flow to the tank was simulated by considering that if $R = S/W$, then for small changes:

$$\delta R/R = \delta S/S - \delta W/W$$

or

$$\delta R = (\delta S - R\delta W) \, . \, W^{-1}$$

and deriving the change by subtraction.

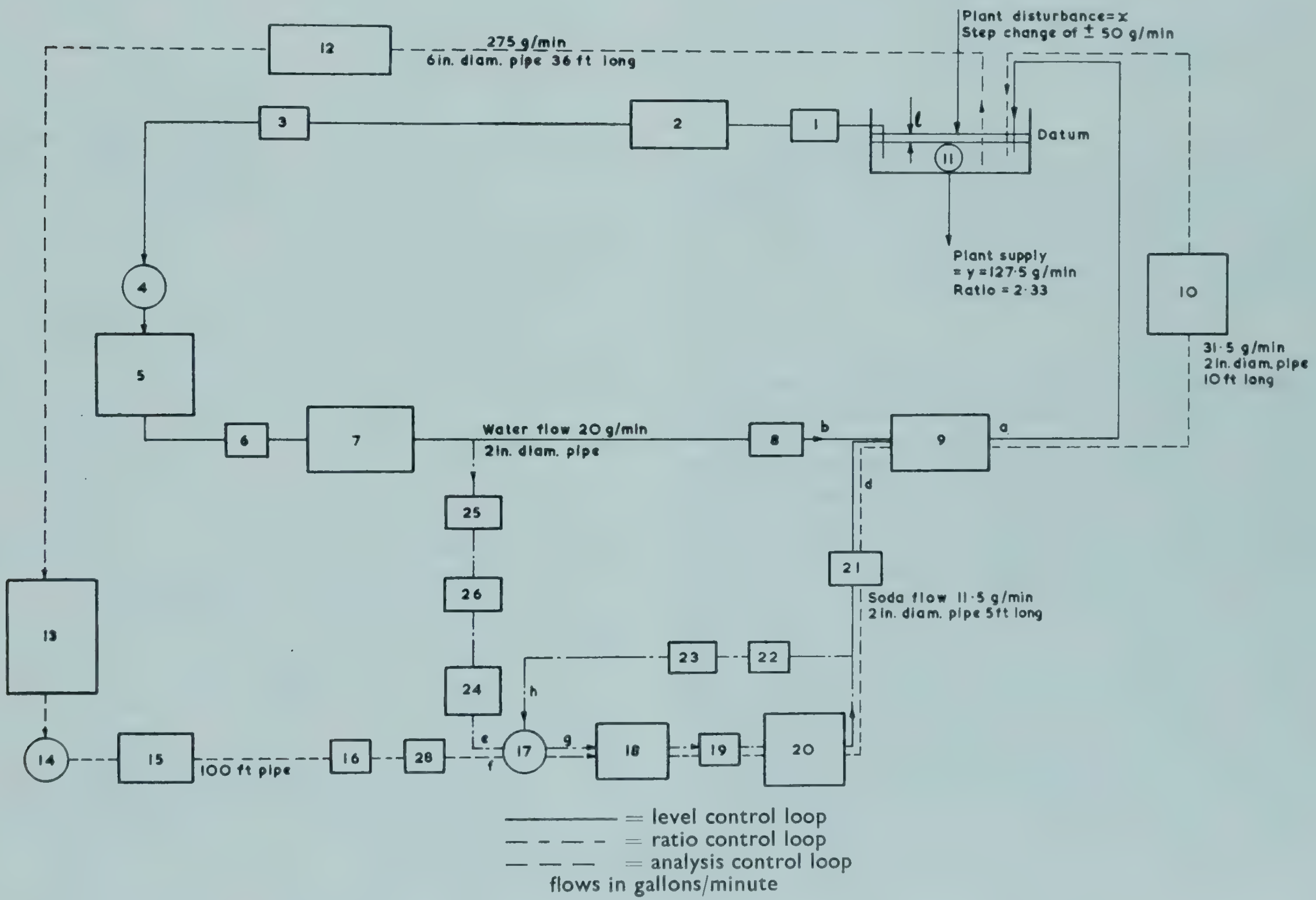
As already mentioned, the control was divided into three subsidiary loops, each having "elements" receiving signals from the adjoining elements, and each element representing a particular component or instrument. At times it was found necessary to represent an operation occurring in an instrument as a separate element, so that a number of elements might represent one particular instrument. Each element was assigned a transfer function which was regarded as an operator operating on the input signal to the element and converting it to the output. The symbol "p" appearing in any transfer function in this paper represents the time derivative of the input function.

It was appreciated that not all the elements would necessarily be of significance in the analysis, but the breakdown allowed the determination of those elements which were in fact significant, and it was thought that the data thus accumulated would be of value in further investigations.

Determination of significance of transfer lags

The following method was used for determining whether transmission lags were of significance to closed loop stability.

All the transmission lags were assumed to be delays with zero attenuation and having phase shifts proportional to frequency. The only other lags in the loops were contributed by the transfer function of the tank. In the level loop this is represented by an integration and in the concentration loop, by a time constant. At the frequencies above which the time


Plant disturbance = x
Step change of ± 50 g/min
275 g/min
6 in. diam. pipe 36 ft long
12
3
2
1
l
11
Datum
Plant supply
= y = 127·5 g/min
Ratio = 2·33
10
31·5 g/min
2 in. diam. pipe
10 ft long
4
5
6
7
Water flow 20 g/min
2 in. diam. pipe
8
b
9
a
d
25
21
Soda flow 11·5 g/min
2 in. diam. pipe 5 ft long
26
13
23
22
24
h
e
g
f
14
15
100 ft pipe
16
28
17
18
19
20
= level control loop
= ratio control loop
= analysis control loop
flows in gallons/minute

1 = pipe
2 = air purge level transmitter
3 = air transmission line
4 = error detector
5 = stack type level controller
6 = air transmission line
7 = control valve
8 = pipe
9 = mixer
10 = process pipe
11 = steep soda tank
12 = sampling pipe
13 = analysis transmitter
14 = error detector
15 = analysis controller
16 = air transmission line
17 = error detector
18 = ratio controller
19 = air transmission line
20 = control valve
21 = process pipe
22 = orifice plate and D.P. transmitter
23 = air transmission line
24 = flow presetter
25 = orifice plate and D.P. transmitter
26 = air transmission line

Fig. 2.—*Diagram of control system for steep soda preparation*

constant produces significant attenuation, its phase shift is near 90°, and the transfer function behaves as an integration.

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* Symbols have the meanings given to them on p. 72.

If the sum of the transmission lags is ΣT_T, where either the time constant or the integral time of the tank is T, and the loop numerical gain factor is K; then if $K/\omega T$ is not small compared with unity at the frequency at which $\omega \Sigma T_T = \pi/2$, the transmission lags are significant. That is, if ΣT_T is small compared with $\pi T/2K$, the transmission lags do not affect loop stability.

Fig. 2 shows the inter-related control loops and Table I gives the transfer function for each element.

Where necessary, control valve characteristics have been reduced to a linear relationship. Two assumptions were made: that the lags in air pipes were exponential [transfer function $1/(1+pT)$], and that the lags in process lines were delays (transfer function e^{-pT}).

The time constants used in the various loops were largely based on experience and must be regarded as empirical.

Results of Analysis

The main results of the analysis and the recommended controller settings are given below.

Loop gains

With the specified gain parameters, the transmission lags in the level and analysis loops are negligible. They are significant in the ratio control loop. This suggests that, theoretically, considerably higher gains could be employed in the level and analysis loops without producing instability.

Inter-action of loops

The response of the ratio control loop is very much faster than those of the level and analysis loops, and therefore the soda flow will rapidly follow demands of either level or analysis. A level change has no effect on analysis and whereas a change in analysis, by varying the soda flow with respect to the water flow, has an effect on the level, this is not a destabilising influence because the level control loop is faster in response than the analysis loop.

Controller setting for ratio control loop

Fig. 3 shows the actual computer traces of three responses of soda flow to a step change in flow demand (due to a step change in the output of the water flow transmitter or analysis controller), for three settings of the ratio controller. The settings of the ratio controller are given in Table II.

TABLE II.—*Settings of Ratio Controller*

	a	*b*	*c*
Proportional band %	66	100	66
Integral time (seconds)	20	20	50

Recommended settings were:

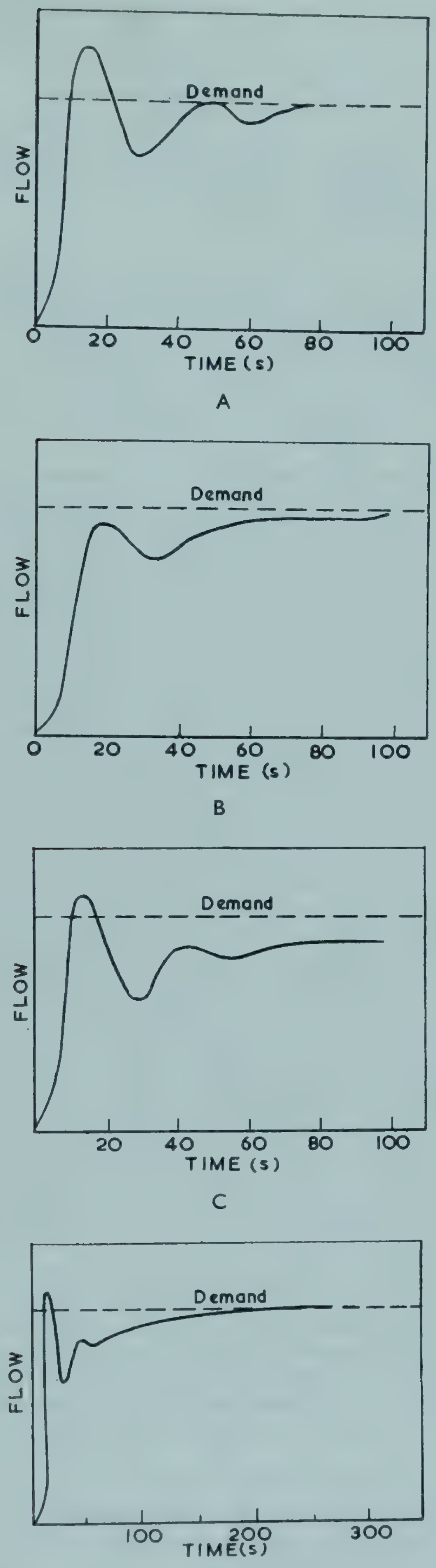
Proportional band	100%
Integral time	20 seconds

Controller setting for level loop

From a curve showing the change in level against time following a step change in flow to the tank, a controller setting of 3% proportional band was recommended.

TABLE I.—*Transfer Functions Used in Analysis*

Element No. (See Fig. 2)	Description	Transfer function (where p = time derivative). Time in seconds
	(a) Liquid Level Loop	
1	Air purge pipe	$\frac{1}{1+p}$
2	Level transmitter	$\frac{1}{1+p}$ lb in^2 ft.$^{-1}$
3	Air transmission line	$\frac{1}{1+5p}$
4	Error detector	-1
5	Level controller	K_1 is $\frac{100}{\text{prop. band}}$
6	Air transmission line	$\frac{1}{1+p}$
7	Valve	$\frac{1 \cdot 97}{1+3p}$ gal min^{-1} lb^{-1} in^2
8	Water supply pipe	$e^{-0 \cdot 027p}$
11	Steep soda tank	$\frac{1}{5 \times 10^4 p}$ ft gal^{-1} min
	(b) Ratio Control Loop	
17	Error detector	Adds presetting pressure signals of Item (24) and (28) and subtracts corrected flow signal of Item 23
18	Controller	$\frac{1}{1+p} K_3 \left(1 + \frac{1}{pT_3}\right)$ K_3 is $\frac{100}{\text{prop. band}}$ T_3 is integral time
19	Air transmission line	$\frac{1}{1+5p}$
20	Control valve	$\frac{100}{1+3p}$ gal h^{-1} lb^{-1} in^2
21	Process pipe	$\frac{1}{1+0 \cdot 06p}$
22	Orifice plate and DP transmitter	$\frac{1}{100(1+0 \cdot 5p)}$ lb in^2 gal^{-1} h
23	Air transmission lines	$\frac{1}{1+5p}$
	(c) Analysis Loop	
10	Soda pipe	$e^{-0 \cdot 043p}$
11	Tank	$\frac{0 \cdot 765}{1+15 \cdot 55 \times 10^3 p}$
12	Sampling pipe	$e^{-9 \cdot 6p}$
13	Analysis transmitter	$20 \cdot 6e^{-4p}$ lb in^{-2} ratio^{-1}
14	Error detector	(-1)
15	Analysis controller	$K_2 \left(1 + \frac{1}{pT_2}\right)$ K_2 is $\frac{100}{\text{prop. band}}$ T_2 is integral time
16	Air transmission line	$\frac{1}{1+4p}$
24	Flow presetter	(1)
25	Orifice plate and DP transmitter	$\frac{1}{190(1+\cdot 05p)}$ lb^{-1} in^{-2} gal^{-1} h
26	Air transmission line	$\frac{1}{1+5p}$
28	Ratio presetter	$0 \cdot 0262$ ratio lb^{-1} in^{-2}

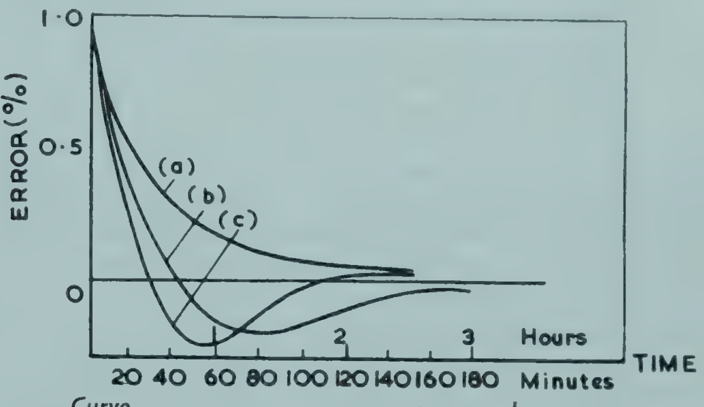


A = Proportional band = 67%, Integral time = 20s
B = Proportional band = 100%, Integral time = 20s
C = Proportional band = 67%, Integral time = 50s
D = As Fig. 3C with contracted time scale

Fig. 3.—*Ratio control loop. Response of soda flow to step in demand for three controller settings*

Controller setting for analysis loop

Fig. 4 shows computer traces of the change in analysis of the steep liquor against time following a step change in analysis, *i.e.* the correcting action of the control. The 3 controller settings are given in Table III.



Curve	*a*	*b*	*c*
Prop. Band (%)	3	3	3
Integral time (min)	∞	45	20

Fig. 4.—*Error in analysis following unit error for three controller settings*

TABLE III.—*Values of Controller Settings*

	a	*b*	*c*
Proportional band %	3	3	3
Integral time (minutes)	Infinite	45	20

Recommended settings were:

Proportional band	3%
Integral time	45 minutes

Tank dimensions

The integral time in the transfer function of the tank in the level loop is proportional to the cross-sectional area of the tank.

The time constant in the transfer function of the tank in the analysis loop is proportional to the volume of the tank. Thus the shape of the tank affects the ratio of these times and hence, with fixed gains in the loops, the ratio of the response times of the two loops.

The volume of the tank may be reduced very considerably without provoking instability of the loops on the preliminary basis of reducing the integral time and time constant until the transmission lags in the system become significant, whilst the response times are decreased proportionally to the reduction of area (level loop) and reduction in volume (analysis loop). The preliminary consideration indicates that the volume of the tank could be reduced by a factor of 30 to 300 gallons and theoretically the tank should be cylindrical, 2 ft diam. × 16 ft high.

It is evident that in these circumstances the contents of the circulating pipes would become significant.

Very roughly, the closed loop responses would have predominant time constants of 20 seconds (level loop) and 40 seconds (analysis loop).

Behaviour of the System on Start-up

The system as described did not operate as required during start-up because of several practical difficulties. The sample taken from a point after the heat exchangers was unreliable because of entrained air, the pneumatic-set ratio controller was unduly complicated mechanically, and the flows of make-up liquor were extremely low at start-up so that the indications were very close to zero on the square root scales.

The difficulties of sampling arose largely because the liquor was frothed easily and frothing took place because of turbulence in the centrifugal pump. A head tank was considered for separating out the air, but the fibres present would also settle. Finally it was decided to take the sample from a point downstream from the filters which were already installed in a side stream. A correction had to be allowed for the lower temperature at this point but the sample was free of fibres and air. The only problem was that sampling, and hence the soda and water make-up, had to be stopped during maintenance on a filter and during this time both level and density fell.

This brought to light a further limitation of the system, which arose from the sizing of the soda and water valves for maximum throughputs in a ratio corresponding to soda of average density. When the level fell, the water valve would be fully opened to restore the level, but the fully open soda valve would only allow the addition of soda of average density. In other words it was impossible to increase density until the level was almost correct.

This condition was not acceptable and a simpler scheme was therefore adopted in which the level controller at high sensitivity was changed to an on-off system in which two electrodes, set to a level difference in the tank of approximately 4 in., controlled the opening and shutting of the water valve. The valve opening was preset by pneumatic pressure.

The constant water flow made it possible to regulate the soda flow by means of a pneumatic signal from the water flow transmitter to the soda valve *via* a fixed-ratio relay. A further relay in the line added the output signal from the existing density controller on a 1 : 1 basis. With a proportional band setting of 15–20% on the density controller the system settled down to straight line control within 2 hours of a density disturbance. It is arranged that the water flow at any given throughput makes up level very slowly so that steady corrections are being made for, say, 2–3 hours. High level is then reached, cutting the feed for the short time required for the level to fall below the lower electrode when the cycle is repeated. Water and soda additions are therefore made nearly all the time.

This modified system works satisfactorily under the conditions experienced at present. The entire system is stable and in agreement with the conclusions of the simulation and analysis, and it has in fact been confirmed that the level controller should be of high sensitivity (and is in practice on-off).

On the other hand, when the throughput of the plant is changed, the feed water control valve has to be readjusted to ensure that the water and soda feed is flowing for nearly all—but not quite all—the time.

When a further project of a similar nature arose, the above points were considered and it was decided that the original system, as confirmed by the simulation, was correct and should be implemented.

Latest Control System being Erected on New Unit

It was decided to use an all-electric system for the new plant because of the facility with which an electrical system could be adapted for this process and also to avoid the pneumatic-adjusted mechanical ratio controller.

An advance has also been made in that it is now possible to make the level and density control systems non-interacting. This is achieved by allowing density to modify both soda and water flows, raising one and reducing the other in such a way as to maintain a constant total flow for a particular output of the level controller, *i.e.* the level controller governs the total flow of soda and water and the density controller the ratio of the two, as will be seen from Fig. 5. This has been made possible by the development of a simple and inexpensive computer relay by an instrument manufacturer.

Start-up data from this unit will shortly be obtained.

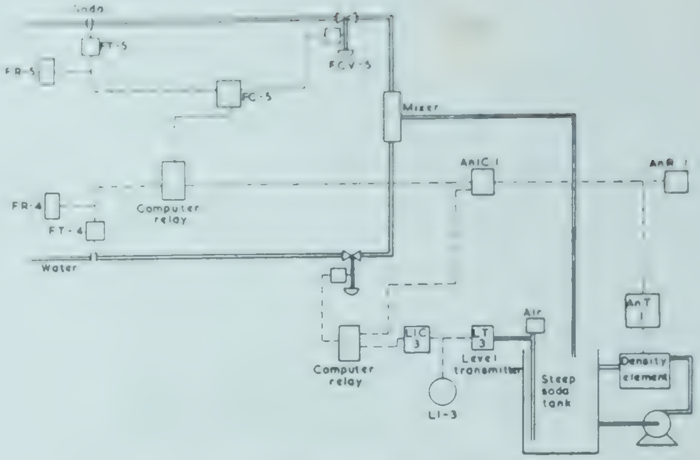


Fig. 5.—*Improved steep soda control system*

Assessment of the Value of the Investigation

It appears that the main principles of the original design were satisfactory and the simulation and analysis merely demonstrated this and showed that the stock tank was unnecessarily large from the point of view of control. General experience of the designers was sufficient to ensure a satisfactory approach, although corrections had to be made to detailed instrumentation. It is thought, however, that the analytical work highlighted both the usefulness and the difficulties of such an analysis. The exercise demonstrated that simulation was practicable reliable, and gave reasonable, results and hence could be confidently used if the complexity of any future plant made past experience an insufficient guide. The present exercise was thus a first step in the evolution of more complex systems, leading eventually to completely integrated control systems for entire plants. In this respect the exercise is considered to have been of value.

The simulation, together with start-up experience, has supported the confident decision to install a similar but improved control system in the next plant of this type.

The main difficulty of the investigation was the absence of basic information on the simple component parts of the system, such as transfer functions, time delays, etc. As a result of this, approximately two months' work by a graduate was necessary, to provide the information for feeding to the computer. The number of staff available might therefore be the limiting factor to the usefulness of the method unless the above data for standard elements can be compiled and published in a similar manner as, for example, heat transfer coefficients.

Acknowledgments

The authors would like to express their thanks to the directors of Courtaulds Limited for permission to publish this paper, to various of their colleagues for help in its preparation, and also to Metropolitan Vickers Electrical Co. Ltd., for the use of their computer service and their co-operation.

Symbols Used

K = loop numerical gain factor (dimensionless).
p = time derivative of an input function.
R = constant (dimensionless).
S = soda flow (gal/min).
W = water flow (gal/min).
T = time constant or integral time (s).
ω = frequency (s^{-1}).

The manuscript of this paper was received on 6 November, 1958.

DISCUSSION OF PAPERS PRESENTED AT THE SECOND SESSION

Mr. D. S. Townend said that Samson's paper was an excellent example of a case history where analogue simulation had been used to provide a worth-while improvement in the control of an item of plant. He would like to plead for such investigations to become much more common instead of being, as he thought was true at the moment, very much the exception.

In a discussion on ergonomics in the daily press, comment was made that the average operator would set to work to make a plant or piece of equipment work as well as he could, in spite of the fact that the plant itself was not really designed to make things easy for him. A large number of control loops on chemical and process plants probably did not work as well as they might if a little extra effort had been put into the initial design and layout of the system.

If he might make one minor criticism of Samson's paper, as a control engineer he would have liked to have more information on the characteristics of the compressors. He found it rather difficult to visualise them from the information given. He asked Samson if there was any alternative way of carrying out control other than by pressure-ratio. In some similar or apparently similar cases, flow measurement had been used as the basic measurement in the system.

Mr. Townend had the impression that Samson had a valve already at the plant and was in a sense stuck with it. Had any of the work shown that he could get better results if he had a different valve, either with different flow characteristics or a different frequency response?

Mr. Townend continued that Idzerda's paper was extremely interesting. As the author had said, there was a mechanical system for accurate ratio blending of materials which had been operating for several years, mainly in the United States of America, with successful results. But as he had also mentioned, it had several disadvantages.

The symposium was not the time to discuss the pros and cons of electronic and pneumatic control, but Idzerda's presentation indicated a certain disappointment with the electronic process controllers. He had described a system which was fundamentally electronic, and it was to be hoped that the electronic engineers would take up what was clearly a challenge and produce something that was rather better than the equipment now available and also cheaper. The two points to which special attention should be given were flexibility in respect of both plant layout and variation of flow rate.

He asked if Idzerda could give some details of the pick-ups on the positive displacement meters. Related to that, had he considered the use in his system of, for instance, the turbine-type of flow-meter, which appeared to hold promise of accurate measurement of large flow rates.

Secondly, could Idzerda clarify the statement on accuracy of blending at the top of page 57 Did the 0·1% quoted relate to the computed ratio from the positive meter readings or was it determined from the measured quality of the blend.

Dr. H. H. Rosenbrock said that Idzerda's device was very interesting. It was extremely elegant and had a very respectable history. The fundamental idea, he believed, went back as far as Clerk Maxwell. He himself had tried to use the principle many years ago for an anemometer and he had found the difficulties which Idzerda had ingeniously overcome. The chief difficulty was the high impedance level at which one had to work.

Dr. Rosenbrock described an alternative method that his company was developing. It had not the elegance of the method used by Idzerda but it avoided the difficulty of high impedance levels. The mechanical device which was currently in use was translated more or less directly into an electronic device. Fig. 1 indicates the method used. One started with a variable frequency pulse generator, generating pulses in the range from about 10 000 to 100 000 per second. The pulses were passed into a common line from which they could go into a number of counting circuits. These circuits would divide by any number up to 9 999 and sent out one pulse for, say, every 4 706 pulses that entered (or whatever number they were set to divide by). After the counters the pulse rates were in the range of about 10 to 100 pulses per second and their ratios were accurately adjusted by the counter settings.

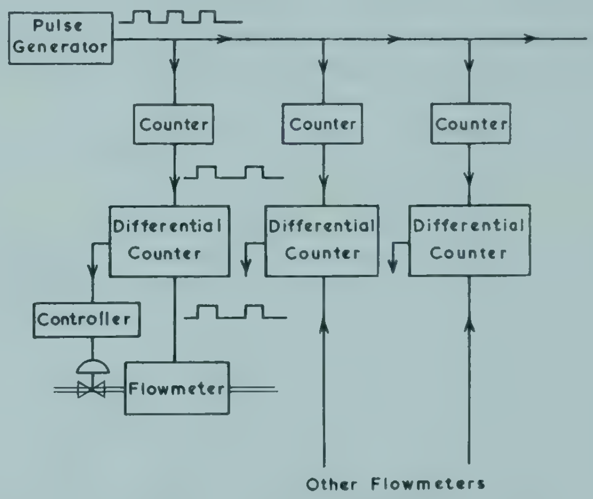


Fig. 1.—*Diagram illustrating differential counter*

For each flow the pulses from the counter went into one side of a differential counter, which counted upwards for pulses entering from that side. The flow meter generated pulses according to the flow and they went into the other side of the differential counter, which counted down for those pulses. The differential counter corresponded to Idzerda's differential integrator, but was a digital device. The count in it at any moment represented the discrepancy between the actual flow and the required flow. A voltage could easily be derived from the count in the differential counter and used to control the flow.

The relative advantages over the mechanical system were probably very much the same as those claimed by Idzerda for his device. The arrangement also offered the advantage that transistors could be used throughout.

Lyon's paper was also very interesting. He had himself been asked to optimise a similar part of a plant for producing deuterium. In that case there were three towers instead of two. Because of the slightly greater complication and the fact that the product cost was optimised throughout there were about twice the number of equations as were given in Lyon's paper. The process of optimisation was completely mechanised. They did not use the computer to evaluate single flow sheets and then compare these by hand, but allowed the computer automatically to search for the best answer.

They felt that since there were five parameters (the same number, he believed, as that considered by Lyon) a factorial analysis would become far too large. If one dealt with only five levels of each with five parameters there would be over 3000 cases to consider.

In effect Lyon had dealt with that problem by taking one or two parameters at a time and that meant that he neglected some of the interactions. He would be interested to know whether Lyon had found that that method of working imposed any limitation.

The method he had used himself was quite straightforward. The chief difficulties arose with the limits. Certain quantities in the calculation must not exceed set limits and, therefore, the curve might be of the type indicated in Fig. 2. The best value obtainable was not at the maximum but on the limit as far as one could go towards the maximum. That situation had been dealt with by multiplying the function to be optimised by another function, which was unity over the permitted range of parameters and dropped to zero, or to a low value, when the limit was reached.

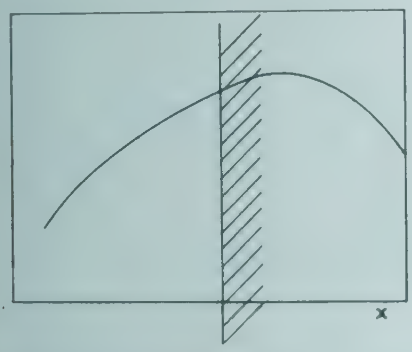


Fig. 2.—Graph illustrating limiting values. The curve shows the function to be maximised. The parameter x is not permitted to go into the shaded region

The computer worked with eight figures but because a curve was horizontal at the maximum or minimum one ordinarily lost about half the figures in locating the optimum. The last four figures could then be used to put in the limits in the way just suggested without any further loss of accuracy. The limits were not always constants. If they were the problem would be simple, but usually they were expressed as inequalities between two quantities, each calculated from the parameters.

After the limits had been put in the remainder of the problem was the same as that referred to by Coutie in his paper (p. 2). Because of the method used for inserting limits, the function usually had very long and narrow ridges. The method used was to make successive steps in each of the parameters until progress slowed down. The axes of the co-ordinates were then rotated until the first lay along the line of greatest advance, the second along a direction at right angles to the first which was the second best direction of advance, and so on. This worked very well and was more stable than the method of steepest descent. In complicated problems he had found it to be about four times quicker than the method of steepest descent.

The programme was organised so that one only had to write in the equations defining the cost in terms of the parameters. The machine than calculated successive flow sheets and continuously reduced the cost until it could make no further progress. This usually required about 200 calculations per parameter: that is to say, for the particular problem with five parameters, 1000 flow sheets had to be calculated. That sounded a great number but it only took about 15 min on the Mercury computer. The time taken did not depend strongly on the complication of the problem since a great deal of the time was needed for the logic and for printing intermediate results.

It would be interesting to learn from Lyon how that number of flow sheets, 1000, compared with the number he had calculated. He gave the number 70 in the paper but it seemed that these were all optimised in respect of G/L, so presumably he had calculated something of the order of 500.

Mr. R. M. J. Withers said that the paper by Idzerda offered a very attractive system for those who were able to use positive displacement meters in their flow lines. It followed fairly well defined electronic paths, and there was no reason to doubt the claims made in the paper. He remembered that at T.R.E. during the war a bucket tachometer was used which worked into a Miller integrator in much the same way.

With regard to the paper by Muller and Pratt, Mr. Withers said that his own company had a variety of controls of the same type at work and had some experience of the problems which could arise.

He was surprised to see an accuracy of one part in 2000 claimed for the U-tube type densitometer, particularly since it was used with a stack type controller and that a controlled accuracy of ± one part in 200 of the concentration only was needed.

His company had had a U-tube type of densitometer developed for their work which had been available commercially for some years but which with a stack type controller would not give a genuine accuracy of densitometric value greater than two per 1000.

The analysis of the system was clear enough, but the process layout was not what his company would employ, and the controls, particularly in the latest proposals, seemed unnecessarily involved.

He suggested more separation of the mixing control from the level control by adding roughly the correct quantities of hydroxide and water from pre-set valves working from the on-off level controller but also ensuring that the stuff in the tank was in all circumstances either heavier or lighter than was needed. The on-off level controller was satisfactory for that particular application and the analysis confirmed that, but it would be equally feasible in other cases to arrange a pair of valves to work sufficiently closely from a simple proportional level controller, if desired. Flow control seemed quite unnecessary.

For the mixing control, the remainder of the water or of the hydroxide would be added direct to the eye of the pump. That gave a fast and tight control of the concentration and minimised interference from throughput changes. Indeed, the tank capacity then helped the mixing control.

It was usual to have a second tank to keep a supply of hydroxide of correct density available and to cater for plant shut-down and start-up procedures; the 300 gallons per minute recirculation could go into that. All the hydroxide to process would go through it. It would have a weir overflow into the first tank. Such a system would employ only two controllers, one of which was a simple on-off one with pre-set valve positions. An extra pump and a tank was required, but it certainly had a fraction of two hours' settling time and was easy to commission and service.

The essential features of the practice in his company were first described by Booth.[1]

Mr. F. P. STAINTHORP said that Parkinson's Law stated that "work expands so as to fill the time available for its completion" and the paper of Muller and Paratt appeared to be ample verification. It was of course, not always obvious which elements in a control loop would have significant time constants; in the case of the liquid level and analysis loops of the system described calculation of the principal delays arising from the tank should immediately suggest that all other constants in these loops would be trivial.

It would be unfortunate if the authors' assessment of the value of the investigation were to be accepted since it would suggest that the evaluation of even a simple system was likely to be laborious and time consuming. The assessment was somewhat misleading since the analogue simulator did not in fact include the very features which became important in systems of that description, *e.g.* the valve-flow limits. Mr. Stainthorp suggested that the investigation would have been of more general value if the relative economic and control merits of various alternative designs of plant and control systems had been assessed. Thus, it would seem that a combination of a small mixing tank (large enough to release the entrained air) with a tight density control followed by a large tank with a loose (averaging) level control might achieve the same object as the author's design and involve much less instrumentation. Mr. Stainthorp thought that several alternative schemes would have suggested themselves if the potential deviations and the necessary corrections had been calculated first, and it would have been interesting to compare their controllability on the analogue computer. Unfortunately the comparison would only be realistic if the non-linear features of the loops were to be included in the model. Sympathy was due to anybody who attempted the problem as so many factors were not known even approximately (*e.g.* the amount and effect of "back-lash" in the measurement to regulator portion of the loop). Might it not be cheaper and simpler to study the control of such systems on a pilot plant? That was, in effect, how the authors arrived at their final solution.

Mr. J. MCMILLAN said that his main point concerned the remarks at the end of the paper by Muller and Pratt, on the assessment of the value of the investigation. Throughout the paper, he thought, there were indications that the authors were a little disappointed in the outcome of their work and they were almost apologetic when they said that to use the analogue computer they had first to work out the system equations.

Mr. McMillan felt rather sorry for the graduate who worked two months to produce the equations, and he thought the authors would agree with him in trying to dispel any impression that the difficulty with analogue methods was that transfer functions had to be worked out. Obviously, to make a quantitative analysis of a problem it was necessary to work out the dynamic behaviour. Analogue computation was just one method of solving the equations, and the fact that an analogue was used was not the reason why the equations had to be derived.

Perhaps the problem might have been slightly unfortunate for the authors, in that the system existed, and systems that existed were hard to change. So far as plant equipment was concerned, the system was fairly simple; there was only one major item of plant in the form of a simple tank. The investigation showed that the tank was unnecessarily large from a control point of view. If that had been a simple level control system and the calculation had been correspondingly simple, he thought everyone would have felt the results well worth while.

In the example, however, perhaps the authors felt a little dissatisfied in that there was nothing sensational in the outcome of the investigation. But many of those present at the symposium could confirm that analogue methods could be applied to more complex systems where the dynamic behaviour of the components was by no means obvious and took several months' time to sort out. The analogue was a very rapid and excellent way of producing the final solution.

Mr. A. H. ISAAC in welcoming the paper by Muller and Pratt emphasised his interest based on a recent experience of the design of an installation very similar to that described. It was for the dilution of sulphuric acid. Mr. Isaac thought that it was important, in the event of any analysis indicating unsatisfactory performance, to consider the design of the plant to improve controllability instead of considering only modifications to the instrumentation. That could be achieved without any sacrifice of other requirements for the plant design.

Referring to the paper, Mr. Isaac pointed out that proportional bands were mentioned in a number of cases with no indication of the ranges of measurement of particular devices. That made interpretation of some of the curves difficult, for example, the 3% proportional band referred to in Fig. 4. The figure showed substantial effects by varying integral times but with the proportional band mentioned it was possibly very misleading. It appeared certain that the effective contribution of integral action to a system where the control valve would be working nearly on/off would be very limited. For similar reasons he suggested that it was not possible to ignore any realistic limits in the performance of the various components of the loop. Whilst appreciating the authors' comments that the simulator would not permit the imposing of finite limits in the variables involved, Mr. Isaac emphasised that the consequences of actual limits in practice must be considered in conjunction with the system simulator results to present a realistic picture of operating characteristics. For example, it was unwise to consider only the effect of a control valve as creating a rate of change of flow since it had a limited region over which it would be effective and control was lost when the valve reached a limit of either full-shut or wide-open. That was obviously important where narrow proportional bands were experienced. Similar considerations applied to the effective measurement range of instruments which must have a finite chart span.

Mr. Isaac thought that the large capacity of the storage tank demanded by process requirements could well be made favourable to control rather than complicate the control problem. It would be achieved by sacrificing precise control of level in the tank, a factor of no great significance, and substituting a controller with a wide proportional band and a slow integral time. That would give the best "averaging" action by absorbing the disturbances in the demand without passing them directly back to the inflow valves. In fact the increase and decrease of demands tended to compensate and to call for no readjustment of the primary inflow. That, therefore, prevented disturbances entering the specific gravity control. The capacity must be used to "smooth" demand conditions without affecting the automatic control of proportioning.

Reverting to the sulphuric acid dilution system which he had mentioned, Mr. Isaac explained that with it he was faced with the opportunity of participating closely in the design of the plant with a view to ensuring the maximum controllability whilst satisfying the various process requirements for adequate storage capacity etc. The measurement of the specific gravity of the 60% acid was made in a section of the main tank in which the acid and water additions were also made. It had a low hold up and it specific gravity was rapidly corrected. From thence the diluted acid overflowed into the main tank which acted as buffer storage. The level measurement was made in the large volume section. The general arrangement is shown in Fig. 3.

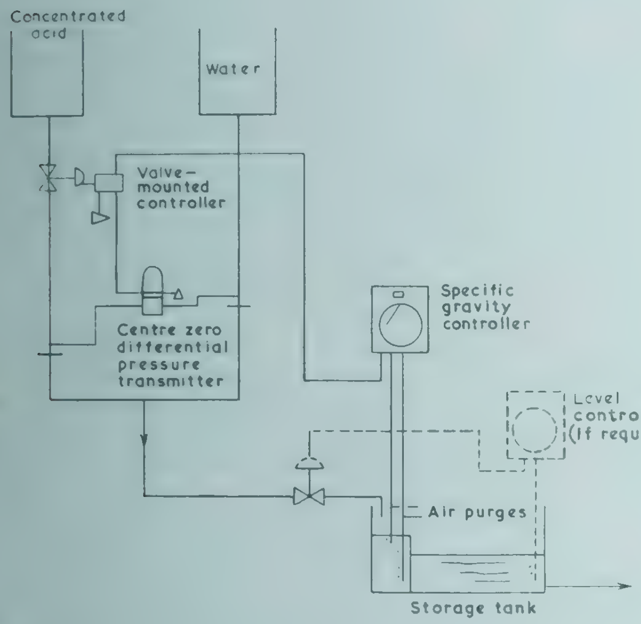

Concentrated acid
Water
Valve-mounted controller
Specific gravity controller
Centre zero differential pressure transmitter
Level controller (If required)
Air purges
Storage tank

Fig. 3.—*Sulphuric acid dilution system. Valve must be open to the effective position represented by the point x to ensure recovery from surge*

Mr. Isaac thought that ratio control was of limited value in an application of the type given. An indication of potential disturbances to the specific gravity arising from the changes in either the acid or water flow could be obtained by measuring the differential pressure across two orifice plates installed as shown in Fig. 3. Since the downstream pressure from both orifice plates could be arranged to be identical, the dilution normally occurred in the ratio of the orifice plate capacities when the differential was zero. Any other requirement could be met by adjusting the required differential value by means of a simple cascade control from the specific gravity controller measuring as described above. A simple valve-mounted controller was used for the differential control.

In practice the system was successful in controlling the dilution of sulphuric acid within ± 0·003 specific gravity (*i.e.* ± 0·27% sulphuric acid) in the case of all the disturbances normally occurring on the plant. It was more than adequate for the duty required.

In the particular case the capacity of the storage tank was sufficient to avoid the necessity for any level controller since the random disturbances averaged out. It was therefore possible to use a manually adjusted primary flow with the large storage capacity. However, had that not been the case automatic control could be applied from the level to the primary valve as shown.

He noted from the paper that two months had been taken solely in evaluating the time constants for the analysis described. He presumed that the complete analysis required some considerable time. Mr. Isaac mentioned that the acid dilution system he had described was designed, engineered, supplied, erected, and commissioned within 12 weeks.

Mr. G. M. E. **Williams** said he had a small point with regard to Fig. 5 of Lyon's paper and he wanted to refer to Fig. 20 of the paper by Keating and Townend discussed at the previous session.

Keating and Townend plotted feed point stage number on the abscissa and in Lyon's paper the corresponding variable, the number of theoretical stages, was plotted along the ordinate. The other variables in the respective cases were distillate composition and gas-to-liquid ratio, G/L, and were therefore qualitatively similar. It was interesting that there was also a marked similarity of shape in the curves shown in the two papers.

The curves each had one minimum with a slowly changing gradient on either side of it. As Mr. N. Ream of Battersea College of Technology had pointed out to him some weeks ago, it was very fortunate that functions of that type were of that form and changed slowly about the minimum as otherwise it would be difficult to bring the process concerned under control.

If he might revert to a different point from the above which came up during the discussion on nomenclature at the previous session, the electrical analogue shown by Idzerda was known to the speaker as a variety of diode pump. There were many good slang terms established in electronics which had been in use for a long time and he would like to make a plea that they should be considered as a basis for a general terminology. It seemed particularly apt to do so in view of the rapidly increasing use of electronic analogue techniques in research and design in the process field.

Mr. E. R. **Hill** wrote:

With reference to Idzerda's paper, I note with interest his remarks on the savings in cost which can be achieved with a successful in-line blending system, compared with the traditional method of tank blending.

In nearly all in-line blending systems using mechanical meters which have been installed to date, it has been decided to provide a fairly complex arrangement of re-cycle lines and tank mixers, in order to give the refiner facilities for correcting the specification of blended products should human or mechanical mal-operation occur at the blender. As greater experience is obtained with this type of equipment it is probable that the extent of these safeguards will be reduced. In the meantime, however, their costs should be included in any economic comparison of the two basic methods of tank and in-line blending.

Since reliability of the in-line blending equipment is the major factor which will dictate the extent of these safeguards, I should be interested to know how Idzerda considers the reliability of the electronic in-line blending system will compare with the mechanical system.

The author states that the electronic system has already undergone field tests and that arrangements are being made for it to be produced commercially. I shall be interested to know when the commercial instrument is likely to be available.

Mr. D. C. F. Pratt wrote:

The paper by Idzerda is of interest and I wonder if he has considered extending his system of ratio control to constitute an analogue computer or simulator. His idea could, for instance, be used to simulate the material balance of a viscose process spin bath system. In such a system, in effect one feeds caustic soda, sulphuric acid, and water. Neutralisation of the acid and alkali takes place, and there are also various "losses" from the system.

The mass balance of the acid could be simulated by means of a condenser as could the mass balances of the sodium sulphate and the water. The total mass balance for the whole system could be simulated by a fourth condenser. This could be of fixed capacity and the charge or voltage on it would represent the total mass in the system. The voltage could be made to adjust the capacity of the other three condensers proportionately, so that the voltage on them would represent concentrations rather than quantities if the charge were adjusted to the mass flow.

It is clear that difficulties could arise because it might be difficult to obtain positive meters for some of the flows and because electrical leakage might be a serious problem if the system were meant to simulate a plant with a time factor of several hours.

Mr. Samson replied that whilst endorsing Mr. Townend's plea for more analyses of the type reported, he did so with reservations, as the majority of processes did not appear to need such work; application engineering, based upon sound experience, would continue to be the method by which control systems were applied to processes except in the atypical case.

It was a regrettable fact that the mathematically-minded control engineer talked a language which was both strange and unhelpful to a practical application engineer, due, principally, to the need to translate mathematical ideas into practicalities. Where that was possible some measure of success in uniting theory and practice could be achieved, but it was unfortunate that a tendency had grown up that mathematics was not to be trusted unless it could be explained in a practical manner.

Until that difficulty has been resolved, control engineers would be doing themselves a dis-service in suggesting that too many problems should be dealt with by computational techniques. Boltzmann once said that there was nothing so practical as a good theory, but it was the misfortune of control engineers in process control work particularly, that good theory was only just beginning to catch up with good practice.

Mr. Townend was interested in alternative methods of controlling this plant. One such method was to control the compressor by a pressure-volume relationship, but that was only satisfactory for a limited range of suction pressures, whereas the pressure ratio measurement of surge point was substantially independent of suction pressure.

The volumetric delivery of the machines was substantially constant at a given suction pressure for a wide range of pressure ratios. (See Fig. 4.) However, it would be noted that when the machine did surge, the characteristic broke down so completely that the by-pass valve had to be let down to a position corresponding to a pressure ratio considerably lower than that at which surge occurred. Between the point of surge and recovery from surge, the characteristic of the compressor was entirely unknown and, consequently, it was hardly practicable to devise an analogue in any form except as a transfer function for the compressor under normal operating conditions. Furthermore, the problem of representing the compressor in a manner other than that illustrated in the paper was accentuated by the behaviour of the compressor if allowed to remain in surge, when the pressure ratio rose to surge at a steady rate and then came out of surge with an exponential decay, but with such a phase lag that the system was immediately driven back into surge; that rise and fall of pressure ratio would continue until the machine failed. Any representation of those conditions, apart from being extremely difficult, was not considered worth while attempting since it went far beyond the bounds of the problem which was to prevent the machine from surging.

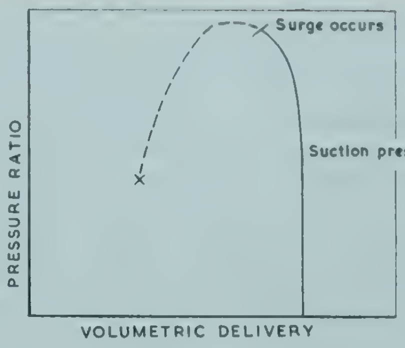


Fig. 4.—*Characteristic curve for compressors*

Mr. Townend was right that the valve was presented together with the plant and that "we were stuck with it". The work on the analogue had clearly indicated that an improved frequency response of the valve would have shown substantial improvements in control. That was because the valve exhibited velocity and acceleration limitations which meant that although its speed of travel for full stroke was fast its movement over small disturbances was not as satisfactory.

Further development on the plant from a control point of view consisted of a velocity feed-back taken from the pressure ratio measurement, which supplemented the normal control action. To do that it was necessary to linearise the valve characteristic so that the pressure ratio versus the valve stem movement had a uniform slope. That was achieved by a pneumatic characteriser which was essential when velocity feed-back was being considered, but could be eliminated in the original control system providing one accepted that the control system was over-damped where the control valve slope was a minimum, and sufficiently damped where the slope increased. Fig. 5 explains the modified loop with the velocity feed-back unit, and it is of interest to note that the same unit as was used in the design of the F.S.A. was used for this additional feature.

This velocity feed-back had an additional advantage that by suitable choice of the feed-back transfer function, the process characteristic at low and intermediate frequencies could be made to approximate the inverse of the feed-back characteristic, with consequently improved stability with the correct choice of feed-back unit.

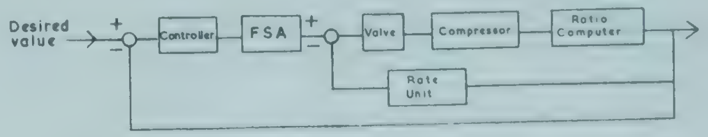


Fig. 5.—*Modified loop with velocity feed back unit*

Mr. **Idzerda** replied to Mr. Townend that the system was able to handle signals in the frequency range of 1–10 c/s. The lower limit of 1 c/s was set by accuracy considerations. The upper limit of 10 c/s was determined by the counters used in the instrument which would work reliably up to 15 c/s; whereas the relays switching the buckets could still follow a frequency of 60 c/s. The pick-ups were simple contacting devices on the positive displacement meters and were therefore able to generate 10 c/s for full load. If, as happened in the field tests, the positive displacement meters rotated more slowly than 10 c/s, either the shaft speed had to be increased by means of a gear or more pulses per shaft revolution could be obtained by using a multiple switch on the shaft. Increasing the shaft speed had the disadvantages that it increased the torque on the positive displacement meter and was therefore never used. A multiple switch was made with a commutator of an ordinary vacuum cleaner and has given reliable performance in practice. In principle it was also possible to generate pulses by photoelectric or magnetic means without any contacting device at all but that tended to complicate the system. It was hoped that in the future manufacturers of positive displacement meters would be willing to supply meters for an integrated system with an output shaft having the desired speed so that the commutator could be omitted.

The application of turbine-type instruments had been considered. However, such instruments supplied frequencies in the range of a few hundred cycles per second, so that the pick-up had to be followed by a frequency divider, which could be obtained as a small transistorised unit. In a discussion with a manufacturer of these turbine meters it also appeared possible to obtain meters rotating at only 10 c/s for full load, but they would have to be specially made.

The accuracy figure of 0·1% was obtained by comparing the ratio as read from the counters of the positive displacement meters with the reading of the ratio setting of the potentiometer in the instrument. The figure therefore did not include any errors of the positive displacement meters themselves.

Dr. Rosenbrock had mentioned an interesting point in speaking of the high impedance level at which the instrument worked. That was perfectly true since the pulsed electric currents flowing into the bathtub capacitor were equivalent to some 3 μA, d.c.; therefore all leakage currents which also could flow into or out of the bathtub should be smaller than 3×10^{-9} amps for the mentioned accuracy of 1 part in a 1000. With the modern components available it was not too great a problem to keep all direct leakage currents below that limit. The most difficult stray current was due to stray capacitances that were discharged into the bathtub capacitor. That could, however, be remedied by screening. In 1½ year's field experience no such trouble had been encountered. Moreover, some checks were made in the laboratory at a relative humidity of 75–80% for several days, in which no influence on the accuracy was observed. It was probably due to the internal heating of the box.

Dr. Rosenbrock also referred to a different system he was developing, using digital counters and a differential mechanism which could count forward and backward, the output of which was translated into an analogue-type signal.

Although such a system must be very accurate it seemed to Mr. Idzerda that the ratio could only be set in discrete steps and that the amount of equipment involved became considerable.

Mr. Williams had referred to a "diode pump". The term was not known to Mr. Idzerda but he thought it was related to one side of the electronic differential integrator, a combination of a diode and a capacitor. The appeal, however, of the differential integrator was that two such circuits were combined in such a way that normally the net result of the combination was zero. Only under those conditions was it possible to maintain high accuracy. The author certainly agreed with Mr. Williams in his remarks concerning the desirability of a general terminology although personally he would in that particular instance prefer "differential integrator" to "differential diode pump".

It was probably true that today an in-line blending system also required facilities for tank mixing afterwards to deal with faulty blends. As Mr. Hill stated it could only be hoped that the extent of the extra facilities could be reduced or omitted if the in-line blender had a very high degree of reliability. It seemed difficult to make a reasonable comparison between the realiabilities of a mechanical and an electronic blender at the present time. The parts of the electronic blender where a breakdown was most likely to occur were the relays and electronic tubes used; the probability of that has to be compared to the breakdown chance of a variable gear a differential gear and possibly of the associated servo system.

In the field-test period the only trouble encountered was in the mechanical pick-ups used. However, both mechanical and electrical circuits could be equipped with alarm circuits which tripped the pumps if the time integral of the deviation of the desired ratio became too great. In the electronic blender it was simply a relay controlled by the level in the bathtub capacitor. The alarm circuit seemed quite safe, the only practical possibility of the wrong blend being obtained would then be because the relays were not correctly following the pulses obtained from the displacement meters, and that Mr. Idzerda believed, would rarely happen.

Mr. Idzerda wrote in reply to Mr. Pratt that he had been thinking of using the electro ratio circuit as an analogue computer; that was exactly the purpose it served in a control circuit for fluid flow, because there the fluid flow streams were replaced by analogue electric currents going into the bath-tub capacitor.

It was gathered that Mr. Pratt did not want to apply a control by means of the electronic differential integrator, but would rather read the concentration in the spin bath from the level in the bathtub capacitor. As he himself stated, trouble could then arise from leakage currents. The idea of such an application had not occurred to Mr. Idzerda and he also wondered whether, in view of this leakage problem, it should be possible to convert Mr. Pratt's concept into the control of one (or more) ingoing (or outgoing) streams to the spin bath, since that would always result in an empty integrating bathtub capacitor, thus reducing the leakage problem considerably.

Finally it appeared that Mr. Pratt wanted to vary some capacitors; for practical reasons it was preferable to work with fixed capacitors and to vary the voltages fed to the bucket capacitors. The active material of one of the streams fed into the spin bath could then be represented by an electric current obtained from a multiplication of a frequency (from a positive displacement meter) and a voltage (representing concentration).

Mr. **Lyon**, replied that his only critic was Dr. Rosenbrock, and most people were aware that Constructors John Brown Ltd. had a very interesting new plan in hand for deuterium production.

He must admit, as a chemical engineer, that some of the methods used must to some people seem naïve in the extreme. The chemical engineer could always plead that he had to work to a fairly tight time-schedule, and in all fairness to Dr. Howlett, who had done the computing, it was a matter for regret to all concerned that he did not have time to apply more sophisticated methods.

This was all he could say on that point, except that making allowance for the different G/L ratios that were tried out for each of the seventy flowsheets that were studied, the number of cases considered would approach the thousand Dr. Rosenbrock had had for his particular problem.

Mr. G. M. E. Williams had compared the curve to determine the minimum number of theoretical stages with the curve in the paper by Keating and Townend. There was one thing these might have in common. In the process he himself had described there was a transfer process between what was virtually a straight operating line and a straight equilibrium line, and the curve was based on two finite difference expressions which gave pairs of lines in the hot and cold columns respectively. In the Keating and Townend problem, one might well be dealing with so narrow a range of concentrations that the same linear relationships applied to the operating and equilibrium lines for the distillation process.

Mr. MULLER said that Mr. Withers had referred to the method of density measurement and to the accuracy mentioned in the paper, *i.e.* 1 in 2000. The actual density measurement was a difficult one as, indeed, was said in the paper, because there were fibres and air entrained in the sample. Therefore, a choice had to be made of the basic detecting element which could be used. There was not very much choice in the available instruments. The accuracy obtained in practice was not as good as 1 in 2000 but probably more like 1 in 1000.

As to level control, the paper pointed out that as a modification it was possible to use on-off level control on a large tank. The important fact to realize in the whole exercise—and it was underlined in the paper—was that the plant reported was only a small part of a bigger plant. The bigger plant could therefore not be ignored in dealing with a small section of it.

Referring to Mr. Stainthorp's comments, Mr. Muller said that there was no intention of elaborating what some speakers had called a simple system. If what was done in practice was examined it would be found that the system as such was basically only a ratio control system—and the ratio system could hardly be called complicated—with a certain feedback connection for the purpose of giving effect to the monitoring of density. Certainly Mr. Pratt and he did not say or think that the method or plant layout used by them was the only way in which their particular problem could be handled. Bearing in mind however the various difficulties and taking, for instance, the point with regard to fitting baffles into the tank, the presence of fibres in the liquor made this undesirable. It could be seen that practical considerations often contradict what might be theoretically desirable.

It had been said that there were certain constants and transfer functions which were mentioned but apparently not used. That was quite true. They were brought in merely for the sake of completeness but it was not claimed that they were all used. A study of the particular system was made and therefore they were listed.

In reply to Mr. McMillan's comments that the authors were disappointed with the outcome of their work and the necessity of determining transfer functions as an essential task in the investigation of any problem, the authors felt no special disappointment. It must be stressed that in the exercise it had been necessary to work from absolutely fundamental data. For furture analyses it would certainly be helpful to all concerned if transfer functions for simple component systems and their constants could be listed so that when a control problem was investigated this data was available. It would be of help to other workers as well as those who did the particular exercise.

As far as progress in the whole field was concerned it was most important that it should be done. There were comparatively few people who had done practical exercises which could be substantiated by the plant or had, in effect, demonstrated on the plant whether the investigation had succeeded or not.

He agreed with Mr. Isaac that such an investigation was probably of greater use in a complex system. Mr. Isaac's method of dealing with a similar problem was most interesting.

It was true to say that modifications could have been made to the stock tank. But as mentioned in the paper, the practical characteristics of the system and the fact that on shut-down the tank would have to take a considerable volume of soda weighed fairly heavily in the balance.

His colleague and he were very grateful for the many comments that had been made. It showed that there were more ways than one of solving such a problem. The final criterion lay however in the fact that both the plant described and another new plant which had recently started to operate worked very satisfactorily in giving soda of constant composition thus ensuring that the rest of the product was up to quality. From the point of view of the authors that was the most important factor.

Reference

[1] Booth, P. M. *Int. Sug. J.*, 1955, **57**, 191.

SESSION 3

APPLICATION OF ON-LINE COMPUTERS

Chairman: **Mr. T. E. Goldup, C.B.E., M.I.E.E.**

Chairman of Group B, British Conference on Automation and Computation

THE APPLICATION OF NEW TECHNIQUES OF CONTROL AND INFORMATION HANDLING TO ROLLING MILLS

By W. N. JENKINS, B.Sc., A.M.I.E.E.*

SUMMARY

The paper surveys the current methods of operation of slabbing mills and strip mills and indicates the present-day applications of automatic control.

The possibilities and scope of future applications of automatic control are also discussed.

Introductory Review

A revolution has occurred in the production of the ordinary sheet steel used for motor-car bodies and tin cans during the last forty years in the United States of America and the last 20 years in this country. The rapid series of changes started when it became the rule for steel to be handled in "lumps" larger than could be handled by a man with a pair of tongs. Once the limitation of manual handling was removed, ingots and their products quickly assumed the most economical size for the type and size of plant involved. Thus a modern strip mill processes ingots of about 15 tons and throughout the plant units of comparable size are handled.

The final products of a strip plant are produced from the ingot as a result of many rolling operations (or "passes"), first in the hot and then in the cold state. The maximum "bite" or "reduction" possible in one pass varies according to temperature and other factors from 10% to 50% of the original dimension.

Slabbing mill

The ingot is first rolled hot in a reversing slabbing mill (Fig. 1), in which 20 or more passes are made on all four sides of the ingot which is rolled back and forth between rolls. The thickness of the slab as produced by this mill is about 6 in. and the length sometimes in excess of 15 ft. The width is usually 30–70 in., and this dimension conforms roughly to the width of the final strip product, since there is very little spread in later rolling operations. It follows that the subsequent reduction of thickness must be accompanied by a proportionate increase in length, and in the case of tinplate, which is only about 0·010 in. thick, a single slab becomes stretched to several miles long. The form of the strip plant is very largely determined by the consequences of the progressive elongation of the strip as it travels through, and the determination not to cut up the material more than necessary.

Hot finishing

All the rolling after the slabbing mill is carried out on the two larger faces of the material with plain cylindrical rolls, and in spite of the lack of support for the sides there is little lateral spread. This is because the width is much greater than the thickness and all but the outer edges are severely restricted in respect of sideways movement. After reheating, the slab enters the first of three or four "roughing" stands (or mills) arranged one after the other and connected by massive roller tables. The material becomes so long after the roughing passes (it is already about 80–120 ft long) that a continuation of the method of separate successive rolling operations would demand excessively long roller tables, and in the following finishing mill (Fig. 2) rolling is carried out for the most part in all five or six constituent stands at the same time. Since the strip enters one stand before leaving the previous one, the distance between the stands is no more than is necessary to allow of adequate access. This compact combination of stands is called a "continuous mill" and it introduces new and interesting control problems in preventing loops and breaks between stands. This aspect will be returned to later.

Cold rolling

After the hot finishing mill, the strip (as it is now called) is already about ¼ mile long and storage before further rolling is necessary for metallurgical reasons and to allow flexibility of operation. A long roller table after the finishing mill is fitted with water sprays to reduce the temperature of the strip to a value suitable for coiling. The coil produced by the hot strip mill is about 72 in. in diameter with a mandrel hole in the centre of about 20 in. The strip is coiled and uncoiled between all the following operations until it is either sold in coiled form or more usually cut up into individual sheets.

Cold rolling down to auto-body gauges is carried out in a three- or four-stand continuous mill and tinplate gauge steel, being a thinner product, is generally produced on a 5-stand continuous cold reduction mill.

The rolling mills in a strip plant are matched in output so that the slabbing mill produces enough material to feed one hot strip mill, but the considerable elongation which occurs at the cold rolling stage makes it impracticable for just one cold mill to maintain the same output. A tinplate mill (Fig. 3) is spectacular in producing strip at more than 50 m.p.h., but in spite of this, one slabbing mill can keep at least two such mills busy, with plenty of spare capacity.

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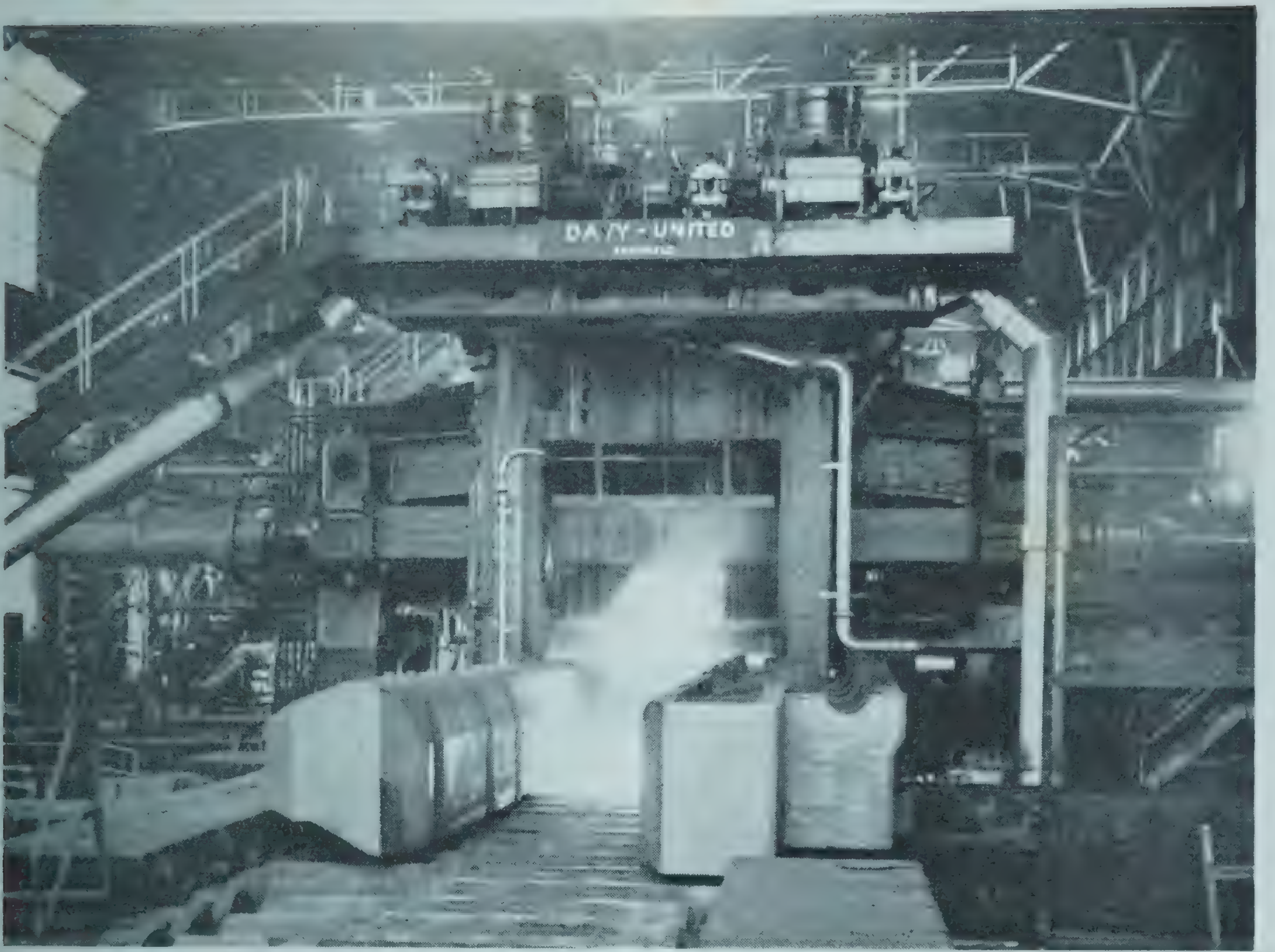


Fig. 1.—*Universal slabbing mill*

(Photo. by courtesy of Steel Company of Wales Ltd.)

Mechanical handling plant

The large scale of the modern strip plant has been made possible by the development of robust and reliable mechanical handling equipment. Notable has been the evolution of the ingot chariot which conveys the ingots to the slabbing mill, the manipulating equipment at the slabbing mill, the heavy duty roller tables which can withstand the shocks of toppling ingots, overhead cranes capable of precise movements with heavy loads, hydraulic coil handling gear, and mobile coil transporters.

Electrical plant

Concurrent with the advances in mechanical engineering have been the remarkable developments in the field of electrical engineering which have made it possible to drive and control the mills and auxiliaries with the required precision. In most rolling mills it is essential to have some means of controlling speed, and for this reason D.C. motors are generally favoured. They take power from variable voltage D.C. supplies derived from works' A.C. supplies *via* motor generator sets (Ward-Leonard systems) or grid controlled mercury arc rectifiers. The Ward-Leonard system is used almost universally for main drives in this country at present, but rectifier systems, having advantages in respect of capital costs and maintenance, are becoming more popular, especially on the continent. It is fortunate that the methods of closed loop control developed just prior to and during the late war are directly applicable to the Ward-Leonard and rectifier systems, and great improvements in accuracy and speed of response have been possible in the control of speed, tension, and voltage.

Application of automatic control

Since the major upheaval caused by the changeover to the manufacture of strip by continuous methods there has been a period of consolidation during which speeds and outputs have been increased and re-matched, and quality has been improved.

The evolution of the modern strip mill can thus be seen as a logical working out of mechanisation, and the industry is now settling down after a transformation.

Although automatic controls are extensively used in modern steel plants, they have been applied incidentally as a means of increasing speed, output, or quality. Only in a few instances have there been attempts to integrate automatic control with information handling systems so as to fulfil the popular conception of automation. This situation is likely to change in the next few years and in what follows it is hoped to give some indication of the progress being made in the field of automatic control and information handling.

The Automatic Rolling Mill

Rolling can be carried out either as a series of successive passes back and forth between the same pair of rolls as in a slabbing mill, or as a concurrent series of passes in separate

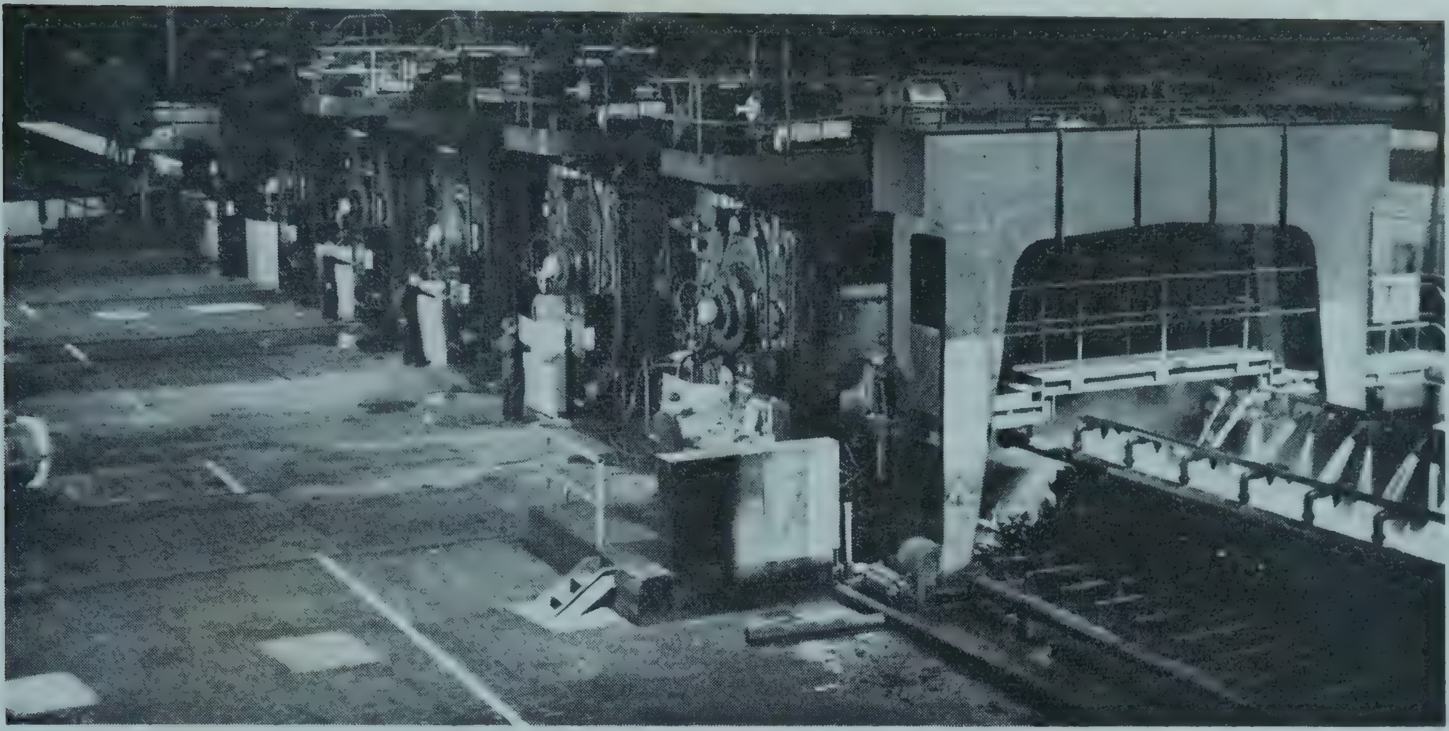


Fig. 2.—*Continuous hot strip finishing mill*
(Photo. by courtesy of Steel Company of Wales Ltd.)

stands as in the continuous mill. For each pass the distance between the rolls (the roll gap) must be set to within fairly close limits to suit the reduction, and the roll speeds must be preset. In the case of the slabbing mill, roll speed is determined so as to ensure the minimum total pass-time (including the time for reversing and re-entering), compatable with driving motor power limitations, and in continuous mills matching of the speed of the separate stands is necessary.

The list of roll-gap and speed settings for a given job together with data on manipulation is called a "rolling programme" and it contains the information which is known in advance of rolling about the settings to be employed. But this information is not usually enough to ensure satisfactory rolling since, in present-day mills, some trimming of settings is necessary during rolling.

The control required of a rolling mill can thus be divided

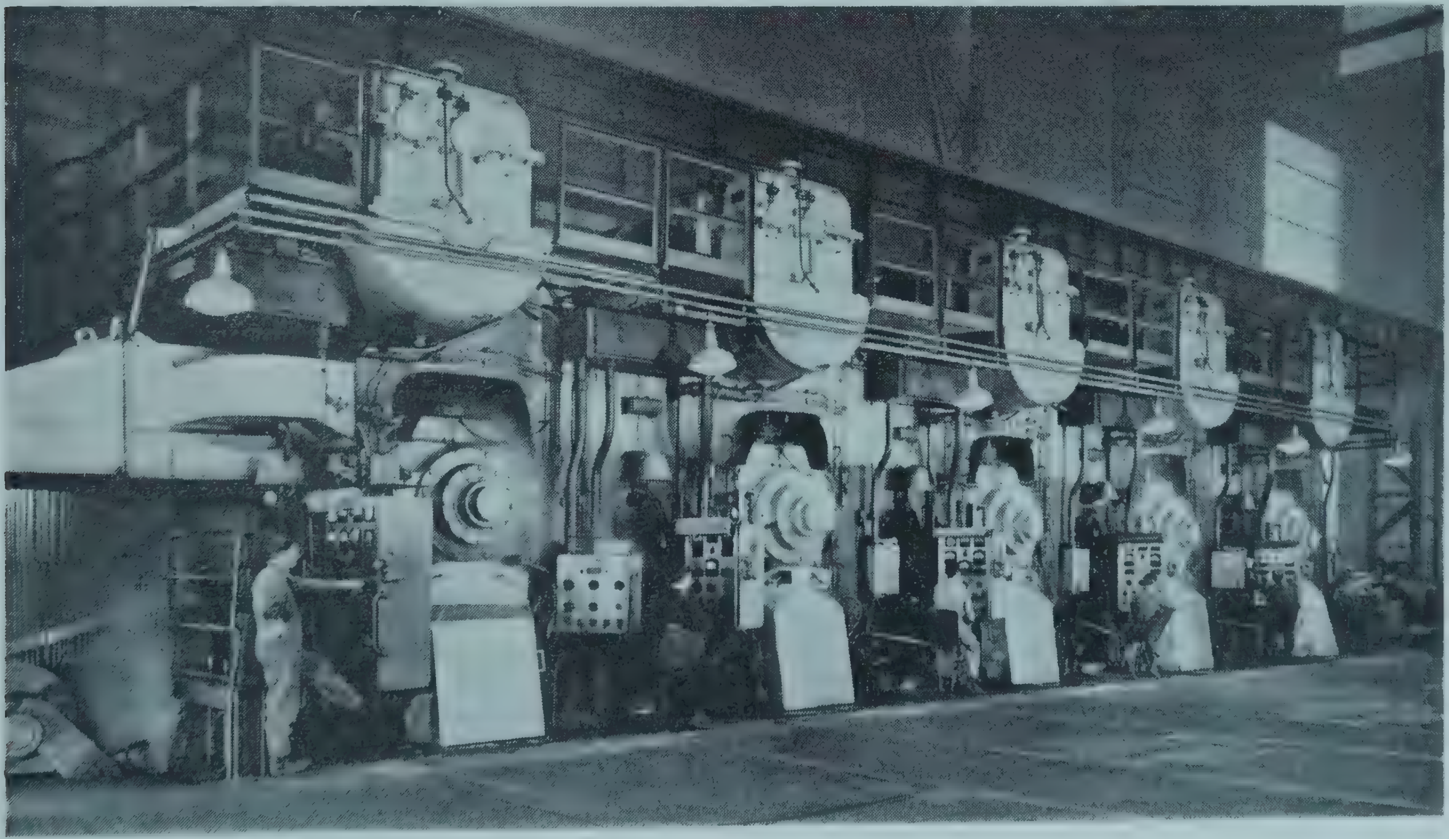


Fig. 3.—*5-stand continuous tinplate mill*
(Photo. by courtesy of Steel Company of Wales Ltd.)

into two categories, "programme control", and "trim control", but the division is for convenience only and is not a rigid one, since trim adjustments sometimes have the magnitude of a programme change.

Programme control

In most present-day rolling mills programme control is carried out manually by an operator who memorizes a series of settings found to be suitable by previous calculation or experience. In some plants, however, automatic programme controllers are now appearing and users claim more consistent products and faster operation with less operator fatigue.

The conventional "tram-car" type of manual position controller is totally inadequate for the setting of rolls and manipulators when automatic programme equipment is employed, and closed-loop position-control is essential. In this field progress was made as a result of the application of techniques used for gun control in the war, but the adaptation was not completely straightforward because of the need for reliability in continuous operation and of accuracy using crude mechanical equipment.[1]

Several types of programme equipment have been evolved. In the U.S.A. punched card storage of programme information is favoured, whereas in Great Britain punched tape and electro-magnetic storage have been used. Several installations covering slabbing mills, and similar primary and roughing mills have been devised,[2, 3, 4] and it is proposed to concentrate the present discussion on the continuous cold reduction mill, which would benefit from automatic operation but which has up to the present received little attention in this respect.

The principles of operation of such mills are generally the same regardless of whether there are three, four, or five stands. The material passes under tension between the stands and it is still under tension when it is coiled after the last stand. The tension between stands simplifies control and helps the reduction of the steel, whereas tension at the coiler helps the reduction at the last stand but it is also necessary in order to produce a rigid self-supporting coil structure.

Coils are rolled at an average rate of about one every 20 minutes, but much of this is not actual rolling time. The periodic changing of rolls is an important factor in reducing mill availability, and some delays due to breakdown and awaiting material are inevitable, but a considerable amount of prime mill-output is lost through delays in setting up. It is believed that further improvements could be made by introducing automatic programme control of the setting-up procedure.

SPEED SETTINGS

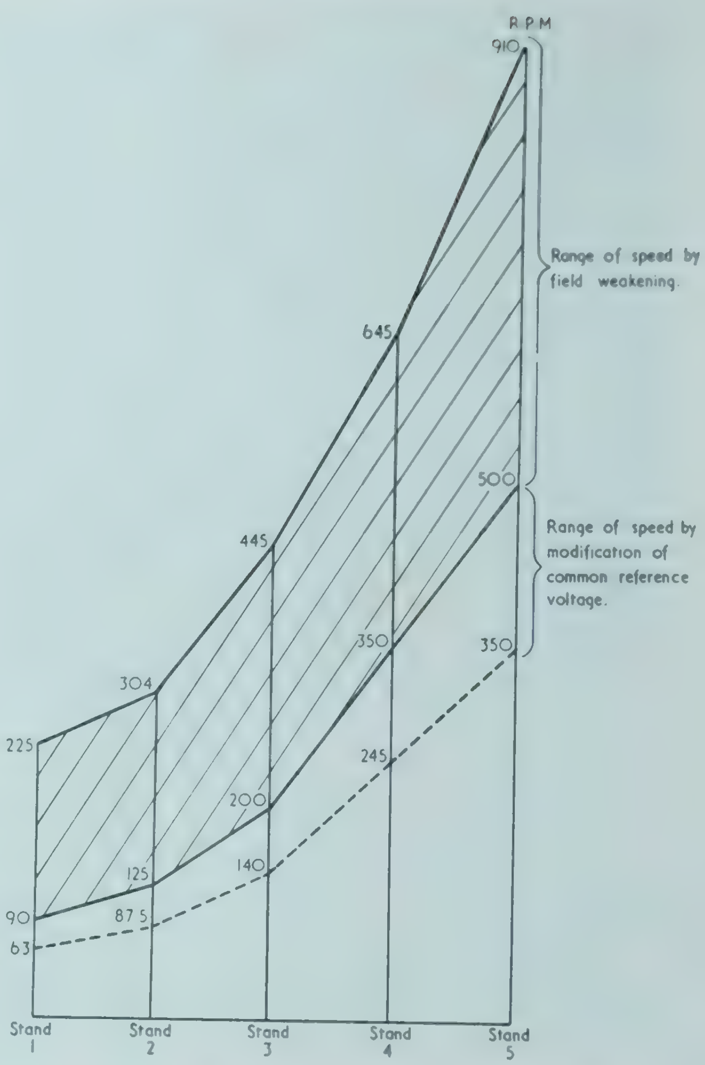


Fig. 4.—*Speed cone of a typical tinplate mill*

Before the first coil of a new order is put through the mill, the programme adjustments must be made to the speeds of the motors driving the individual stands and to the screwdown motors which adjust the roll settings. The decision on the speeds must be made bearing in mind the power and speed limitations of the driving motors. Fig. 4 shows the "speed cone" of a typical 5-stand tinplate mill showing the range of individual speed adjustment of the five driving motors. The unshaded portion represents range of speed available with constant maximum torque, the shaded portion the range of constant available horsepower. Adjustment of speed in the constant torque range is by armature voltage control, and in the constant horsepower range by motor field adjustment. The variation of armature voltage is used for concerted mill control with a reference signal common to all five stands, and the motor fields for individual stand speed adjustment. If a wider range of individual motor control is required the common reference signal is modified for the stand involved. The amount of reduction carried out in any stand under steady conditions is closely related to the ratio of speed of that stand to the speed of the preceding stand. This is because the volume flow from each stand is the same and there is little lateral spread.

Thus with suitable units:

$$U/w = v_{n1}h_{n1} = v_{n2}h_{n2} \qquad . \qquad . \qquad (1)$$

or

$$\frac{v_{n1}}{v_{n2}} = \frac{h_{n2}}{h_{n1}} \qquad . \qquad . \qquad . \qquad (1a)$$

where U = output (tons/h).
w = width of strip.
h_{n1}, v_{n1} = thickness and speed of strip at neutral point of stand 1.
h_{n2}, v_{n2} = thickness and speed of strip at neutral point of stand 2.

The neutral point is the point where the strip speed and roll speed are equal and since little reduction occurs after this point, these speeds and thicknesses approximate to, but are not equal to, the exit speeds and thicknesses.

Within limits imposed by this approximation it appears that the reduction on each stand is determined by the relative stand speeds. There is usually considerable freedom within the speed cone to choose the amount of reduction to be carried out at each stand, but since the amount of reduction also determines, within limits, the motor driving torque required, it must be checked from the speed cone that the required torque is available with the speed setting chosen and the strip width involved. Another factor which influences the choice of relative stand speeds is "strip shape." Strip with bad shape, *i.e.* with loose edges or loose middle, is produced under certain mill conditions, and considerable operational experience is needed in the formulation of satisfactory speed setting programmes.

ROLL SETTINGS

It would appear from equation (1a) that the amount of reduction, being speed-dependent, is independent of the setting of the rolls, and of course this cannot be so. The equation is true only under steady conditions and steady conditions can only be achieved if the roll setting is tolerably near the correct value. A mill with incorrect roll settings would either throw slack between the stands or the tension would build up to breaking point.

The correct roll setting are approximately known from experience, but much trial-and-error trim adjustment is at present necessary during the first minutes of rolling a new order. The traditional method for the setting of the rolls is first to bring the rolls together (to face) and then to screw down (not up) from this position, by a pre-determined amount. The fact that cold rolling is usually carried out with the rolls pressed together before the strip enters, *i.e.* "below face", makes possible a faster and more accurate method of roll setting using roll-force indicators. These instruments (usually load cells) indicate to the operator the force of contact of the rolls on the strip and are used at present mainly during rolling. It is possible, however, to use these instruments before the introduction of strip to set the roll-force corresponding with the correct rolling conditions once strip is introduced.

TRANSLATOR CONTROL

The adjustment of stand speeds and roll load are both eminently suited to operation by programme controller and one of the methods which has potentialities for this field is "translator control" developed by the British Iron and Steel Research Association.

Translator control is similar to other forms of programme controller in that it is a store of plant settings applicable under different conditions. It has the additional feature that the relevant programme is automatically selected on the basis of information fed in from a number of sources.

In the case of the continuous cold mill, the information fed to the translator would be the ingoing gauge and quality, outgoing width and gauge. Significant permutations of these variables would be automatically associated with the corresponding mill motor and roll load settings predetermined by experience. The setting-up procedure for the mill would be reduced to the simple task of setting input conditions on a small desk. Even this could be dispensed with if the mill were integrated in a larger information handling scheme.

It has been found that the equipment used by the G.P.O. in telephone exchanges is the most suitable in the application to rolling mills. In particular the motor-driven high-speed rotary switch known as the motor uniselector has been found useful because of its reliability, cheapness, high speed of operation, and the large number of positions and ways. It is valuable for the "recognition" circuits which are concerned with giving a single output for each significant combination of input conditions.

Such recognition circuits are "tied in" with the programme store for the location of the correct programme. The programme store is also conveniently made from the same motor uniselectors used in conjunction with a rational programme board comprising a multitude of small plugs and sockets by means of which programmes can be built up.

Some preliminary work will be necessary before translator control can be introduced to a cold mill. A particular problem which needs attention is the remote position control of width guides which are at present manually adjusted. Most important also is the need for a thorough investigation of present-day manual practice in programme setting.

Trim control

Equation (1a) can be applied to the mill as a whole. For a tinplate mill:

$$\frac{v_{n5}}{v_{n1}} = \frac{h_{n1}}{h_{n5}} \quad . \quad . \quad . \quad (2)$$

One would expect from this that a mill running at steady speed with constant gauge of ingoing strip would produce material of uniform thickness: in practice several factors combine to make this equation only approximately true. One important factor, particularly in 5-stand mills, is the "speed effect." This phenomenon is mainly observed when rolling thin strip with large rolls. It relates to the decrease of percentage reduction when speed is reduced and other variables are held constant. When relative speed settings are fixed the speed effect causes the interstand tension between stands 4 and 5 to rise to breaking point, and in practice the drives are arranged so that the speed drops with load so as to accommodate the change in reduction. Very stiff drives (*i.e.* drives in which speed does not drop appreciably with load) tend to be unpopular with tinplate mill operators, not only because of the speed effect, but because a mill with soft drives is much easier to set up. The fact remains, however, that for use as a precise reduction device a stiff mill is better [see equation (2)]. It can, of course, be argued that a mill should be stiff at high speed and soft at low speed, but although this compromise is often used in practice it introduces further difficulties in that trim adjustments are then necessary during changes of speed.

It may be that automatic programme control of the setting up procedure would make the adjustment of the mill possible, before entry of strip, very close to the correct operating conditions so that stiffer drives could be used. The amount of trim required during changes of speed would then be reduced but in the tinplate mill the programme controller would then have to include speed adjustments to compensate for the speed effect.

Such steps would not, however, remove the major cause of trim adjustments during rolling in cold reduction mills, which is the variation of ingoing gauge and hardness.

Equation (1) can be written in another form which is particularly suitable when small departures from a steady state are considered:

$$\frac{\Delta U}{U} = \frac{\Delta v_{n5}}{v_{n5}} + \frac{\Delta h_{n5}}{h_{n5}} = \frac{\Delta v_{n1}}{v_{n1}} + \frac{\Delta h_{n1}}{h_{n1}} \quad . \quad (3)$$

where $\Delta U/U$ = fractional change in output (ton/h).

Thus under steady speed conditions one would expect the percentage variations of outgoing gauge to be equal to the percentage variations of ingoing gauge. This is very roughly true in practice.

Many methods have been employed for overcoming these gauge variations. One approach is to improve the consistency of product from the hot strip mill by open loop[5] or closed loop gauge control using an X-ray gaugemeter. The open loop method depends on the fact that the longitudinal profile of hot rolled strip tends to be similar for all coils rolled. A preset amount of screwdown movement is automatically initiated at fixed distances from the start of the strip so as to compensate for known trends. This type of installation is very suited for conversion to a more refined type of closed loop "sampling" system using an X-ray gaugemeter.

Another method is to apply automatic gauge control at the first stand of the cold mill, and already a scheme is operating in this country along these lines. The instrument used for gauge measurement is the B.I.S.R.A. Gaugemeter, which compounds a gauge signal from roll load and screw position,[6] and correction is applied to the screwdown motors. Other methods are also available.[7]

A refinement in automatic gauge control on a cold mill is vernier control of gauge by variation of the speed setting of the last stand. This has been applied in the U.S.A. and an installation is planned for a mill in this country.

Vernier control of gauge is carried out by speed setting adjustment and depends upon the fact that the delivery of a multi-stand mill in tons per hour is practically independent of any control adjustments on the last stand. Thus from equation (3), when $\Delta U/U = 0$:

$$\frac{\Delta v_{n5}}{v_{n5}} = -\frac{\Delta h_{n5}}{h_{n5}}$$

Manual control of gauge is at present carried out by the same principle.

Along with the development of automatic control systems efforts are being directed both in this country[8] and in the U.S.A.[9] to reach a more fundamental understanding of the operation of tandem mills. It is hoped that this work will lead to the design of mills with inherently good characteristics requiring the minimum of superimposed automatic control. Theoretical studies so far made of tandem mills deal with small departures from a stable steady state. The changes of interstand tension, gauge, and speed caused by small changes of speed or screw setting can be related by simultaneous equations equal in number to the number of stands in the mill. The deviations from a steady state are related for a three-stand mill by the following equations:

$$\frac{\Delta U_1}{U_1} = X_1 + a_1 \Delta T_{12}$$

$$\frac{\Delta U_2}{U_2} = X_2 + a_2 \Delta T_{12} + b_2 \Delta T_{23}$$

$$\frac{\Delta U_3}{U_3} = X_3 + a_3 \Delta T_{12} + b_3 \Delta T_{23}$$

where X_1, X_2, X_3 = factors relating incremental changes in roll or speed settings.

$\Delta T_{12}, \Delta T_{23}$ = corresponding changes in interstand tension.

$\left.\begin{matrix} a_1, a_2, a_3 \\ b_2, b_3 \end{matrix}\right\}$ = coefficients determined by characteristics of mill and drive.

For equilibrium,

$$\Delta U_1/U_1 = \Delta U_2/U_2 = \Delta U_3/U_3.$$

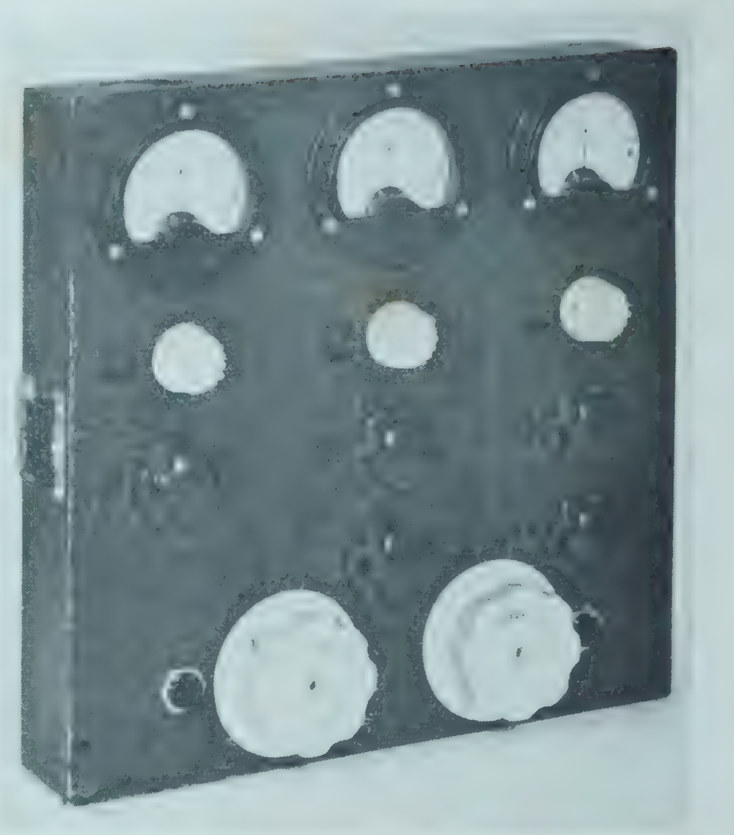


Fig. 5.—*Tandem mill simulator*

Fig. 5 shows a simple simulator built by the author which can be used for solving these equations for any values of the disturbance quantities X_1, X_2, X_3 once the coefficients a_1, a_2, a_3, b_2, b_3 are known. These quantities are set up on the dials. The operation of the simulator entails adjusting by trial-and-error the values of ΔT_{12} and ΔT_{23} on the two dials, until the three indicators of $\Delta U/U$ show the same value. The circuit of the simulator is shown in Fig. 6. The present difficulty limiting the use of the simulator is that the coefficients for production mills are difficult to measure since interruptions of production for measurement cannot be tolerated. The model 3-stand mill shown in Fig. 7 has been built in B.I.S.R.A. and much useful data is being derived from it.[10]

It is probable that advances in the understanding of tandem mills combined with an improvement in automatic control methods will lead eventually to the elimination of manual trim adjustments during rolling.

Central Co-ordination of Rolling Mills

The subject can be conveniently divided into two aspects:

(i) the collection and transmission of information to and from a central office; and

(ii) the automatic sorting, computing, and tabulating, *etc*., at the central office.

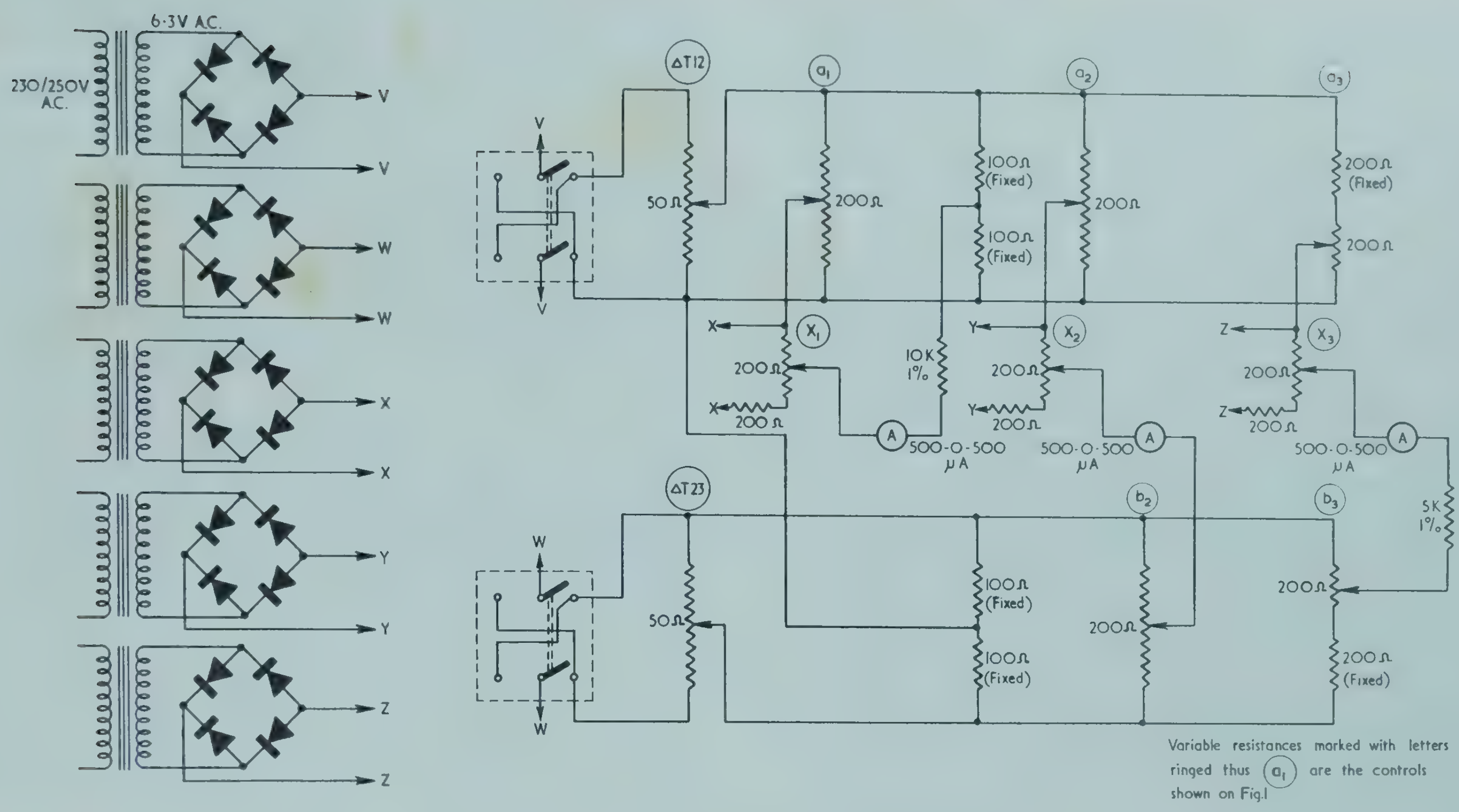
The collection and transmission of information

Information required in the central office for the regulation of the flow of material through the plant includes:

(*a*) news about the arrival and departure of individual ingots, slabs, or coils from rolling mills;

(*b*) news of successful rolling (or otherwise) so that rescheduling can be arranged with the minimum of delay;

(*c*) information on the time taken for rolling as background information for future scheduling and incidentally as data for costing, wages, and maintenance assessment;



Variable resistances marked with letters ringed thus (a_1) are the controls shown on Fig.1

Fig. 6.—*Tandem mill simulator—circuit diagram*

as data for costing, wages and maintenance assessment;

(*d*) weights of ingots, slabs, or coils to give yield;

(*e*) other information, such as rolling temperature, nature and extent of defects, *etc*., for the records or for the customer.

Information is also required to flow from the central office to the rolling mills since the mill operators (if any) need to know the initial and final dimensions required and width, quality, *etc*. In translator-controlled mills this information would be used for the manual or automatic setting of the input switches.

In present day steelworks information is almost always transferred from office to plant and from plant to office on pieces of paper, but this is not likely to continue indefinitely since significant improvements in efficiency are possible by the application of information engineering. Probably the greatest benefit possible from the application of new techniques lies in the reduction in delays in collecting vital data necessary for action. For example, when a cobble occurs in

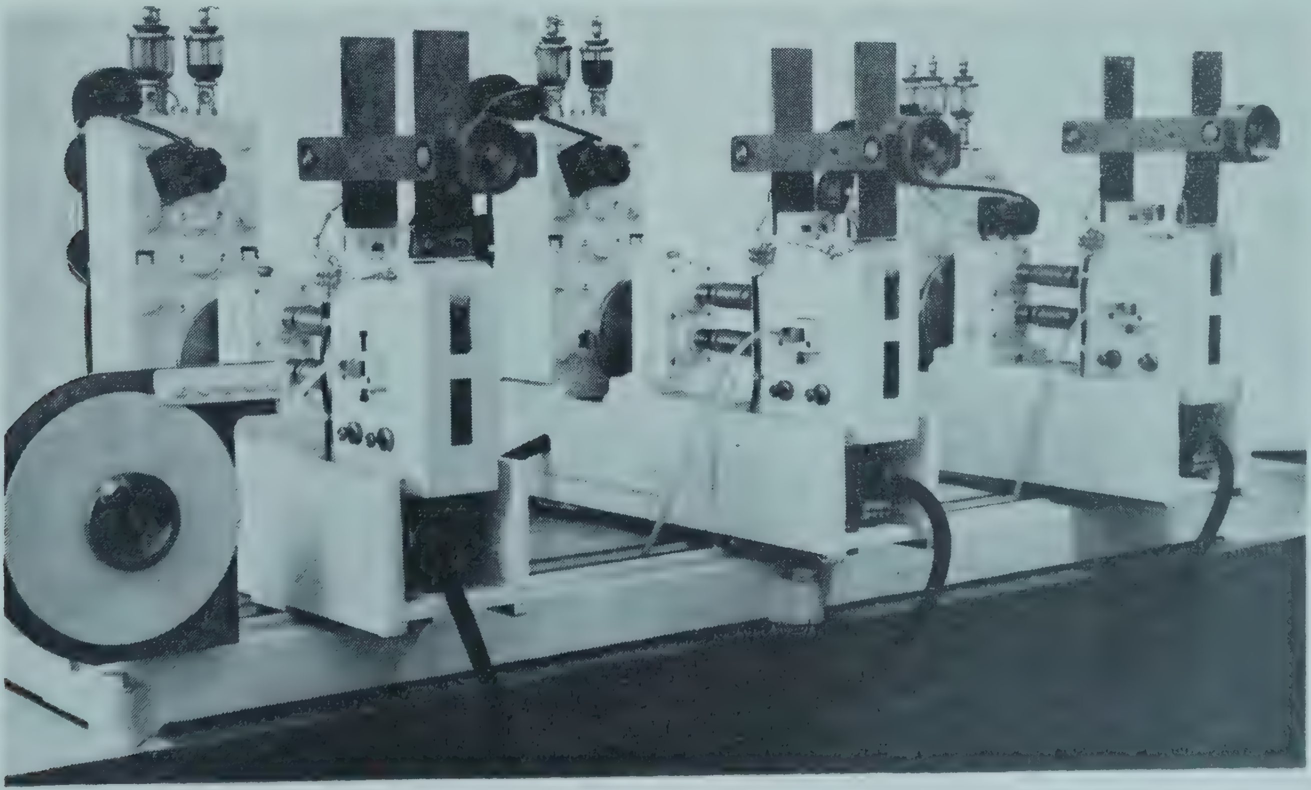


Fig. 7.—*The model tandem mill*

a hot-strip finishing mill several slabs become too cool for rolling and a change of rolling schedule is immediately required. Decisions must be taken for new material to be brought forward to complete the order, and the rejected material must be routed to an earlier stage of production. A concise statement of the latest position would be of great value even with present day methods of production planning: it would be still more important if the production planning itself were mechanised.

Other improvements which are difficult to assess include the reduction in labour force for routine clerical and messenger duties, and the removal of sources of error.

The application of automatic information handling techniques in the steel industry began with the collection of research data on punched tape.[11] Trials on the performance of cranes were carried out in a South Wales steelworks and the significance was appreciated by the management. The result was the appearance of a device for collecting routine production information from mills and processing lines called the "performance recorder".[12] A prototype was applied to a cold reduction mill for recording the running and idle times, order numbers, and reasons for delays on punched tape. The scope of the system was later extended to embrace 20 rolling mills and processing lines and the records were then presented in a central office, both on punched tape and Powers-Samas punched cards. Such integrated data collection systems have been called "CROWD" (Central Recording Of Works Data) and in later versions the scope of the information recorded has been considerably widened.

Another method which is being developed for information handling is the B.I.S.R.A. "Tallimarker", which allows of a 2-way flow of information.[13] With this system the operators will be provided with push-button desks and display screens (or automatic typewriters) at key points in the plant. When a new ingot arrives at a slabbing mill equipped with the Tallimarker the operator presses a button marked "Accept" and immediately on his screen or sheet will appear all the information he requires about the ingot. He would probably see the information given in Table I.

TABLE I.—*Information Displayed on Tallimarker on Arrival of New Ingot at Slabbing Mill*

Slabbing mill display (1)

Order No.	Ingot No.	Ingot weight	Grade	Roll down to
3473	5 of 9	14 tons	122	36 in. × 8 in.

The operator requires some of this information for setting up on the desk of his translator, if such were used, and some of it influences his judgment and actions during the progress of rolling. As soon as the operator finishes with a particular ingot he again presses the "Accept" button and the data for the next ingot appears immediately on his screen. As soon as the previous ingot (*i.e.* No. 5 of order No. 3473) arrives at the slab shears the operator there presses his "Accept" button and information which will be useful to him in shearing and weighing appears on his screen. This is shown in Table II.

TABLE II.—*Information Displayed on Tallimarker on Arrival of New Ingot at Slab Shears*

Slabbing mill display (2)

Order No.	Ingot No.	Ingot wt., tons	Grade	Required no. of slabs	Required slab weight, tons	Actual slab weight		Crop ends
						Tons	Cwt.	
3473	5 of 9	14	122	2	6	5	19	2
						5	18	

The information relating to the actual slab weights is provided automatically by the weigher which is equipped with the necessary analogue-to-digital converter. The operator manually sets in the number of crop ends on his push-button desk.

The storage of information for the Tallimarker is carried out in electromechanical stores. One uniselector is used for registering each numerical (decimal) digit of information. Information is read in and read out using two sets of ten common wires. The uniselector digit stores are arranged in groups according to the ingots to which they refer and supervisory uniselectors are provided for making the appropriate interconnections between stores and push-button and display desks. It is arranged that any information stored on the uniselectors can be read out onto punched tape or punched cards at the central office. Electromechanical stores are used at the present stage in the development of the Tallimarker, since they are inexpensive, reliable, and easy to use, but rapid developments are taking place in the field of magnetic cores and electronics which will probably result in a changeover within the next ten years.

Automatic equipment in the central office

Although it is vital to design office-mechanisation and mill-data collection systems to be mutually compatible, it is often convenient to consider the two separately as far as development is concerned.

The following reasons support this point of view:

(*a*) The skills and background required for office mechanisation are rather different from those connected with data handling in the plant itself. Office equipment is usually designed for operation under supervision during working hours and the standards of reliability expected are rather lower than for plant.

(*b*) Management can make a start in the field of automatic information handling by trying out limited data collection systems in order to assess the extent of benefits to be gained.

(*c*) Separate systems allow more freedom in the development of office mechanisation. For example, it is possible that the purchase of a computer can only be justified if it is shared between several departments.

(*d*) A division between systems facilitates direct intervention by the office staff.

The only major requirement for a data collection system determined by the office mechanisation system is that the information produced should be in a form readily absorbed by the computer or office machinery, and *vice versa*. A suitable buffer stage between data collection equipment and the office system is punched tape, and to a lesser degree, punched cards. Punched card machinery has been used in several steelworks for a number of years for limited computing, sorting, and tabulating work, and there is considerable scope for the integration of such existing systems with data collection systems such as CROWD, which provide a punched tape or punched card output.

The approach of a number of steelworks has been to concentrate on taking advantage of new techniques in office mechanisation before embarking on data collection systems.

An interesting development is taking place in a strip plant. Here punched card equipment is being used for the routine tasks in connection with the scheduling of rolling mills. Several facts have to be considered when assembling a sequence of orders for a rolling mill, and a very important one in continuous mills is a sequence of coil widths. In order to minimise the consequences of roll wear it is necessary for the wide orders to be rolled immediately after a change of rolls and for this to be followed by progressively narrower material until another change is required. The punched card machinery in the strip plant mentioned is used for assembling jobs in order of diminishing width. Another development in the same plant is use of punched card machinery to sort out orders have not been rolled. Also a tape-producing typewriter is being used in conjunction with a tape-to-card converter to put invoice information on punched cards. There is considerable scope along these lines, using current developments in the computer and office machinery field.

Conclusion

Before automation as it is commonly understood can be introduced to the strip plant, rolling mills must be made automatic as units and then integrated into some general information handling scheme. Further work will be necessary for continuous mills to be made completely automatic, particularly in respect of reducing the need for manual trim adjustments during rolling. Although the scope for information handling and data processing is probably not as great as in other industries, a considerable increase in efficiency can be expected from better communications in strip plants, particularly when data can be presented in a convenient form at a central position.

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A DATA PROCESSING MACHINE FOR NUCLEAR POWER REACTOR PROTECTION

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SUMMARY

In a gas-cooled nuclear reactor, it is necessary to measure the radio activity of samples of the coolant gas taken from each of the reactor fuel channels at regular intervals. The output pulse-rate from a scintillation counter is a measure of the activity of the gas sample. If one of the fuel elements becomes ruptured, fission products escape into the coolant stream and manifest themselves as an increase in the activity of the coolant. It is necessary to compare the activity with maximum permitted values and also to check the rate of change of the activity in any fuel channel in order to detect the presence of fission products at as early a stage as possible.

The pulses generated by the nuclear detectors are counted in electronic scaling circuits. The numerical readings thus obtained are then processed in circuits similar to those employed in digital computers incorporating a magnetic drum digital store.

The instrument presents the data in the form of a tabulated log sheet on which the numerical values of the activity for every channel in the gas sampling system are recorded. The recordings are made at intervals, for example every six hours. Continuous numerical checks are performed on each measurement to ensure that it does not exceed a maximum permitted level, and also to measure the rate of change of the activity level and to ensure that this rate lies within the prescribed limits.

In the event of an abnormal change or rate of change of activity being detected, numerical results are printed giving the time of occurrence and the identification code of the faulty cooling channel and a numerical value of the activity measurement.

The Source of Data

Description of nuclear reactor

The core of the reactor consists of a moderator built up of graphite bricks. Vertical channels of circular cross-section pass through this core and the uranium fuel, clad in magnesium cans, is placed in these channels. Each channel carries approximately 13 fuel elements and one reactor contains approximately 3200 channels. The heat generated by the fuel elements is transferred to heat exchangers by means of high pressure carbon dioxide which is circulated in a closed system.

The uranium fuel is sealed in the magnesium cans for two main reasons. Firstly to prevent oxidization of the uranium by the coolant gas and secondly to contain the fission products. If the fission products were allowed to escape they would very soon make the whole coolant circuit and the carbon dioxide highly radioactive and a hazard to personnel.

The fuel cans may, after a time, develop small holes or cracks which allow some fisson products to escape and allow some fuel to oxidise and be carried along by the gas. It is important that these ruptures be detected and the faulty fuel element removed from the system as quickly as possible after they occur. The method of detecting the burst cans is based on measuring the radioactivity of the coolant. Therefore a large escape of fission products or uranium oxide would contaminate the system and reduce the sensitivity of the detection system to further bursts. A brief description of the method of detection will now be given but for more detail the reader is referred to the papers by Long, Laithwaite, and Cunningham[1] and Harlen.[2]

Method of detection of fission products

Fission products in the coolant stream are detected by a method which depends on the fact that many of these products form ionised particles by decay, which then decay further with the emission of β particles. The ionised particles are concentrated by electrostatic precipitation on to a wire which is formed into a continuous loop passing through the precipitation chamber, an adjacent counting chamber containing a scintillation detector, and round a motor-driven pulley as shown in Fig. 1. After a predetermined time the wire moves forward so that the exposed section inside the precipitation chamber is moved into the counting chamber. The number of pulses generated by the scintillation detector

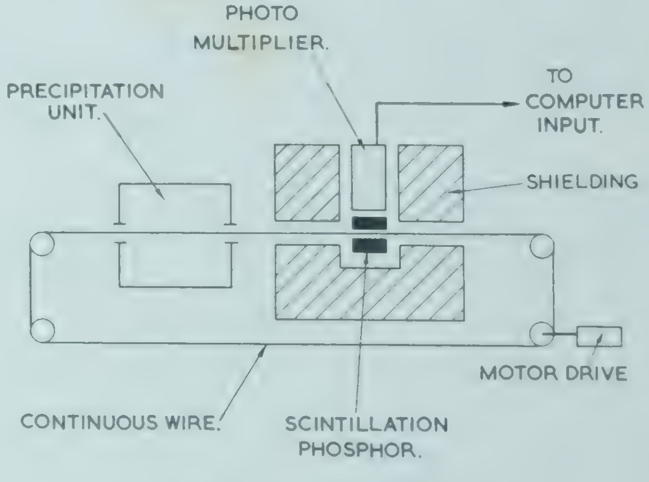

Fig. 1.—*Detection unit*

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* Sunvic Controls Ltd., Harlow, Essex.

per unit time is a measure of the fission product concentration in the fuel channel from which the gas sample was taken.

Use of scanning equipment

The reactor core has about 3200 channels, all of which have to be monitored. The sensitivity of the detection equipment allows these channels to be grouped together in sets of four. Hence there are approximately 800 gas sources to be surveyed. This number is far too great to consider simultaneous monitoring; in practice seventeen sets of detection equipment are shared between these 800 points by a scanning system. A simplified flow diagram for one of the detectors is shown in Fig. 2. Sampling tubes from each of the reactor channels pass through the pressure vessel and the biological shield. They are then coupled together into groups of four at manifolds. From the manifolds the gas passes to 50-way gas selector valves.[3] There are 17 of these valves, one associated with each precipitation unit. The sample of gas selected by the valve then passes in turn through a cooler, a filter, the precipitation chamber, and is returned to the main coolant circuit by a pump. The selector valves dwell on each position for about 36 seconds. During this time a sample of gas is drawn from the reactor and the fission products are condensed onto the precipitator wire. At the end of this time the selector valve moves to its next position and the exposed part of the wire is transferred to the counting chamber. The 17 sets of detection equipment thus scan the whole reactor once every half-hour.

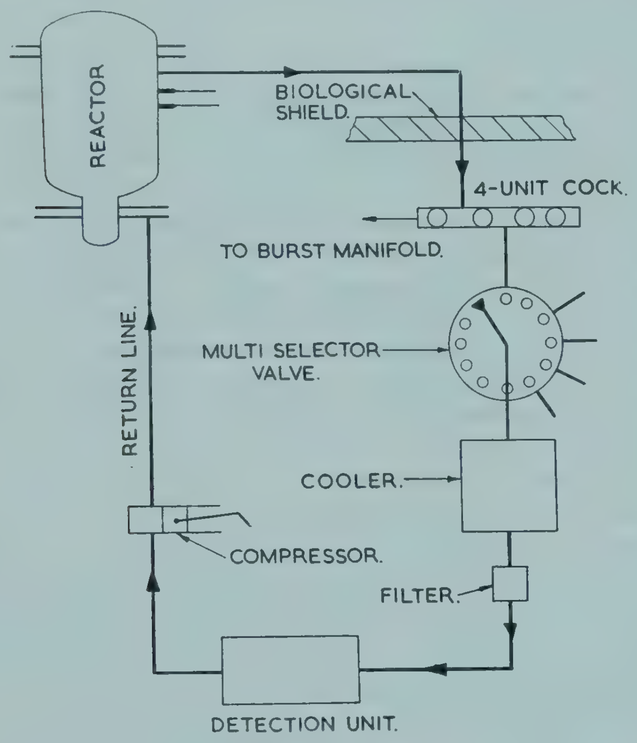

Fig. 2.—*Simplified gas flow diagram*

In the original method of checking the readings of radioactivity, a multi-point recorder was used to plot the total count to a base of time for 53 points separately; each of the graphs was visually examined for abnormality. For values of count measured in thousands, as discussed later, a change of 5% might be significant; on the graphs, which are about 1 in. × 1½ in., this corresponds to a change of deflection equal to one fortieth of an inch at ⅓ full scale. Further, in a power station with two reactors, checking is required for nearly 60 readings per minute to find the occurrence of one abnormal reading in a million standard readings. This examination would give shift employment to eight process operators.

This paper describes a system for automatically recording and checking the counts obtained from the detection system and sounding alarms if the activity in any group of four coolant channels reaches an abnormally high level or starts to increase at an abnormally high rate. When an alarm condition arises the instrument signals to the operator which group of four channels is suspect. The operator then diverts each channel in turn to a spare detection unit by means of manually operated valves in the manifolds. This spare detection unit monitors the selected channels continuously so that the faulty one can be identified quickly from the group of four. The progress of the burst can then be followed on a pen recorder.

Finally, when the existence of a burst has been confirmed, the faulty channel is unloaded until the ruptured fuel element has been removed.

The sensitivity of the system is limited by the fact that some counts are obtained from a completely healthy channel. These counts will be referred to as "background". Background is due to a number of causes. In spite of the utmost care in manufacture, slight traces of uranium are sometimes deposited on the outside of the fuel cans. The fission products from this can escape freely into the gas stream. The gas coolant becomes radio-active due to neutron bombardment, although the detection system does discriminate quite efficiently against this type of activity. A further cause is cosmic ray events which are detected by the scintillation counter.

The background counting rate varies from channel to channel and varies from time to time in any one channel. The latter variations are caused notably by changes in the power level of the reactor but can also arise due to changes in gas pressure and flow rate through the precipitation chamber.

The problem, therefore, is to detect increases in count numbers with background counts which are subject not only to the normal statistical variation of random events but to variation of the mean level.

It will be appreciated that as the sensitivity of the monitoring system is increased the chance of obtaining a false alarm is increased. Hence to assist the operator in deciding whether an alarm is true or false and, indeed, whether a true alarm is an emergency or not, the monitoring system keeps records of the performance of each channel so that the operator can refer to the immediate past history of any measurement point.

Data Handling Requirements

The problem is to devise tests to be applied to the measured counts from the detection equipment such that as early a warning as possible is given of developing bursts, while at the same time maintaining an acceptably low number of false alarms. These tests are to be applied automatically on each measurement without the assistance of the operator except for initially setting up the apparatus. When an alarm condition arises the apparatus must signal the time at which the event occurred, make an identification mark of the suspect channel, and record the value of activity for that channel at the time the alarm conditions arose.

A tabulated log sheet of the latest measurements on each monitored channel group is required at predetermined intervals of a few hours. These records are used to study the long-term behaviour of the plant. The operator must also have access to the measurements on any selected channel group for the previous 24 hours. These records are used to study the behaviour of a channel on which an alarm has been recorded.

Three arithmetic tests for alarm conditions have been devised. These are termed "absolute", "differential", and "average" tests.

Absolute level test

As it is made, each activity measurement is checked against a maximum permitted activity level for that group of channels. Since the background level varies from channel to channel it is necessary to be able to set individual high count levels to each group. These values are individually preset in the apparatus. Each measurement is subtracted from the preset alarm level for that channel and an alarm is sounded if the result is negative.

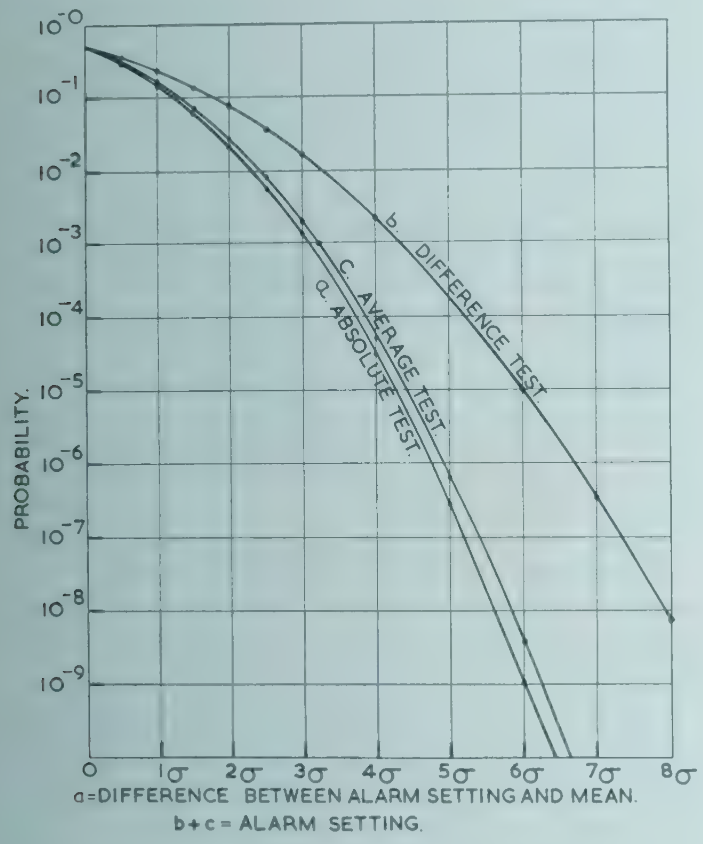

Fig. 3.—*Probability of sounding an alarm*

The measured counts are subject to statistical variations so that there is a finite probability of sounding false alarms no matter how high the alarm level is set. This is illustrated in Fig. 3, where the measured counts from one channel have been assumed to be members of a normal distribution, characterised by a standard deviation σ. In Fig. 3, curve a, the probability of sounding an alarm, is plotted against the difference between the alarm setting and the mean of the measured counts. This difference is plotted in multiples of σ. Hence it will be seen that if the alarm is set at the same level as the mean of the counts then the probability of sounding an alarm is 0·5, *i.e.* 50%. As the difference between the alarm setting and the mean of the counts is increased this probability falls rapidly until a probability of one in a million is achieved at $4\frac{3}{4}\ \sigma$. This level has been arbitrarily accepted as the permitted probability for sounding spurious alarms. This amounts to 3 spurious alarms per month per reactor. If the alarm level is set at the value of $4\frac{3}{4}\ \sigma$ above the mean level of the counts the probability of detecting a burst causing the mean counting rate to rise by $4\frac{3}{4}\ \sigma$ will be 50% for any one measurement.

Alternatively, if the mean level of the count rises due to factors other than a burst, there will be the possibility of an alarm. For example, if a change in reactor power caused the mean counting rate to increase by this same value of $4\frac{3}{4}\ \sigma$ the probability of sounding a spurious alarm will be 50% on every measurement. This means that under conditions of varying background, the alarm levels must be set $4\frac{3}{4}\ \sigma$ above the peak excursion expected in the background. When the background recedes to a minimum this absolute activity alarm test will be relatively insensitive. Other tests, therefore, have been devised to take care of this situation.

Differential test

As each measurement of activity is made it is stored in a digital computer type store. Measurements are erased and replaced by more recent ones when more than 24 hours old. It is therefore possible to check how much the activity of the channel has changed since it was last measured. The previous measurement is extracted from the digital store and subtracted from the latest measurement. This difference is then compared with the maximum permitted difference for that channel. However, this differential measurement is even more markedly affected by statistical variations than the absolute activity test. This is because two measurements, which are statistical in nature, are involved in its calculation. This is illustrated in curve b of Fig. 3 where the probability of sounding an alarm is plotted against the differential alarm level plotted in terms of multiples of σ. It will be seen that if the differential alarm level is set at zero the probability of sounding an alarm on the difference between two successive measurements is 50%. As the differential alarm level is increased the probability of sounding an alarm, falls off rapidly, but to achieve a probability of one in a million the alarm setting must be about $6\frac{3}{4}\ \sigma$. However, in this case the absolute value of the background counts does not affect the sensitivity of the test. In this case the alarm level is chosen so that the rate of change of the background will not give rise to a large number of spurious alarms.

The calculation of the rate of change of activity is not limited to measurements taken on successive scans of the reactor. The difference between the new measurement and any previous measurement taken several scans earlier can be calculated, by extracting the appropriate number from the digital store. This then gives a measure of the change in activity over perhaps a period of several hours. Since the statistical variations of this difference are exactly the same as those of the difference taken between measurements half an hour apart, this latter system is more sensitive to slowly developing bursts, but on the other hand it is also more sensitive to changes of background.

Average test

The "Average Test" is a more elaborate form of the rate of change test and is applied as an alternative to it. In this test, the difference between the latest reading and the average of the previous ten measurements on the same channel is compared with a maximum preset value for this difference. The probability curve for this test is plotted on curve c in Fig. 3. To achieve a spurious alarm rate of one in a million, the alarm setting for this test must be nearly 5 σ. This means that this test can be more sensitive than the simple differential test of curve b and, at the same time has not the severe disadvantage of the absolute test of curve a under conditions of fluctuating background. However, it is limited to conditions where the background counting rate changes very slowly and the total change in background is not important.

Comparison of tests

In the special case where the mean level of the background counts is constant, the absolute level test is the most efficient, *i.e.* it gives the highest ratio of probability of burst detection to probability of spurious alarms. However, with fluctuating background counts the situation is different and depends markedly on the nature of the background fluctuations. It is assumed that there is no difference in kind, of the count-rate variations due to changing background and to bursting fuel-cans, only in magnitude. The absolute level test, which is most sensitive to burst fuel elements, will be the first to be affected by changing background. A burst fuel element is signalled only when a change significantly greater or faster than usual is observed. If the background counting rate shows slow drifts sufficiently large to be noticed in the presence of the random fluctuations, the average alarm is best. However, if the background counting rate shows rapid fluctuations obviously greater than the randoms of the count, the difference alarm test, which compares the previous and current counts, is the best. It is therefore necessary to have some knowledge of the expected background fluctuations before the most efficient tests can be applied to the numerical measurements.

The Data Handling System

The basic measurement of activity is the number of unit pulses occurring during a certain time interval, *i.e.* it is of digital form. Accurate checks are made on the measured values. As numerical values are preferred for recording on a log sheet, a digital system is the most suitable for providing these facilities conveniently. Further, only a digital type of store is feasible for keeping the running record of the results of the past day.

Two separate computers are used to monitor one reactor. One of these monitors the measurements from nine of the detection units and the other from eight, making up the set of 17. Nine (or eight) scaling units accumulate the pulses from the scintillation counters. At the end of the counting period the numbers stored in the scalers are read off in pulse-coded form into the computer and the scalers reset to zero ready to count the pulses from the next activity sample. While this next count is taking place the results of the previous set of measurements are stored and checked.

Fig. 4 shows the main parts of one computer. Prescalers are used to reduce the number of pulses registered on the input scalers to approximately one-fifth of the number capacity of the computer. Manual number inputs are used to record limit values and to send in test readings. Further manual inputs signal a set of limit values common to all channels as an alternative to the stored individual limit values.

These readings are manipulated in the arithmetic unit, the only necessary operations being subtraction and single decimal digit shifting. By far the greater part of the machine is provided for control, number generation, transmission, storage, and display. Hence four-bit decimal coding avoids conversion routines at input and output, a simplification which more than compensates for the small increase in size of the store (15% extra) and the addition of the decimal convertor (5% extra) above that required for a machine operating with pure binary code. For simplicity and, therefore, greater reliability, the machine operates bit-by-bit serially.

The display of alarm readings and routine logs is on teleprinters. A local control is provided for printout and, in conjunction with register 2, enables other operations in the machine to proceed whilst printing takes place.

The magnetic drum stores a day's readings for each gas sample, together with data on alarm limits and channel designations. Position indication signals are sent to the computer as the gas-valve rotates to select each sample. These signals control part of the address-selector of the magnetic-drum. Thus gas-valve positions are identified by angular locations on the drum. Similarly, a counter which registers which of the 48 half-hourly reactor scanning cycles is current controls part of the track-selector of the drum, thus identifying the time of a reading by its track number.

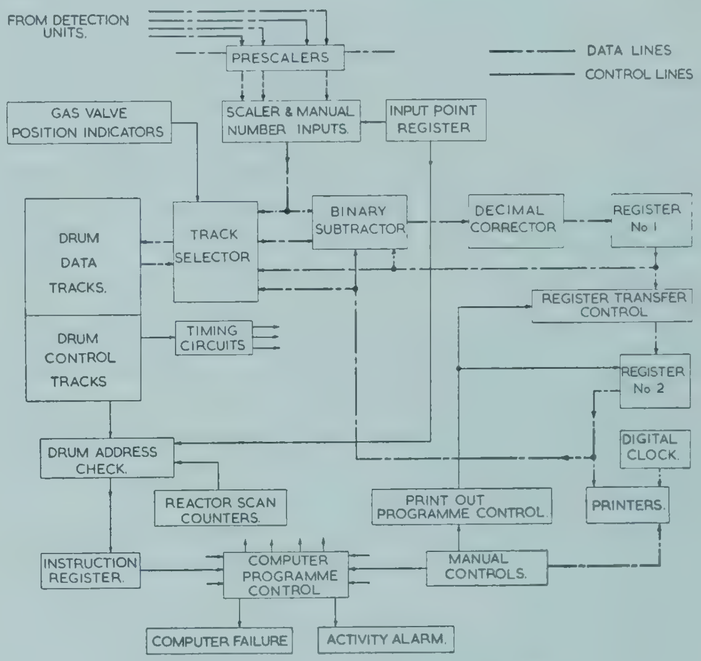

Fig. 4.—*Block diagram of the machine*

The tracks on the magnetic drum are divided into nine equal arcs, one for each input scaler. These nine inputs are all subject to the same programme of operations, and most operations involve the magnetic drum; to reduce the access time a form of multiplexing is used with the drum acting as the scanner. The control circuits are arranged to complete a sequence of operations during the passage of an arc past the reading heads so that a similar sequence can be repeated at the following arc of the track. Several addresses are provided at the end of each arc so that numbers generated during that arc may be deposited, together with a record of the state of certain key control conditions, to be used further on the next drum revolution. The register and controls are then left free to cope with the numbers at the next arc. In this way all nine input channels are subject to a similar sequence during one revolution of the drum.

Programme

The programme of instructions has a succession of up to 16 sub-routines, each of 16 instructions. Ten of the 16 sub-routines are used and each provides one facility (*e.g.* an alarm test, a change to a limit value, a record of measurements) so that each is self-contained. Entry to, or exit from, a sub-routine is solely by way of its "home" instruction. This restriction causes a little clumsiness in programming, but does allow simplification in the conditional transfer control circuits. This simplication is such as to make practical a conditional transfer test as part of every instruction. Once the computer is commissioned only minor changes in the programme are likely. The instructions are, therefore, "wired-in". Each instruction is of four parts which determine the source of number to be processed, the operation on the number and its destination after processing, the possible next instructions, and the conditions to be tested to determine which of the possible next instructions is to be used.

Special Features of the Machine

The location of the computers in a power station where they are "on-line" requires an important change of emphasis in their design compared with business machines or scientific calculating machines which work office hours in office conditions. Although digital computer techniques are used throughout these monitors, the design emphasis is entirely on reliability: speed is of little importance. There are important differences between the two types which result from this approach.

The electronic-type components used are run at a very small fraction of their nominal voltage or power rating, smaller than would be warranted in a pure calculating machine which can be stopped for regular daily maintenance periods.

Circuit configurations are chosen for their tolerance to variations in component parameters and power supply voltages. Nevertheless, a stringent test is made on these components before use. For example, the transistors are checked for stability by measurements made before and after soaking for 200 hours working at the maximum power rating permitted by the manufacturer. Only two power supply voltages are used and these remain within ± 5% of their nominal value.

Although the programme of the computer must never over-run the time allotted between the measurements, the efficient use of this time by the multiplexing technique mentioned before enables a drum rotational speed as slow as 150 r.p.m. and a corresponding digit frequency of 10 kc/s to be used. At this drum speed, even with continuous day and night operation, a bearing life measured in years can be expected. The transistor switches are turned fully on and the resulting delay due to "hole storage" when they are turned off is a small part of the digit time at these frequencies.

Fault finding

On the occasions when faults do occur it is of prime importance that they should not go undetected. Checks are made on all number highways for spurious codes. As they are set up the instructions are checked to verify that one lead in each of the four groups is marked. These four groups correspond to the four parts of each instruction. The magnetic drum store is checked after every writing operation by reading back the number which was recorded and comparing it with the original. Reading operations are each performed twice and the two are compared for equality. The arithmetic processes are checked by performing each calculation twice in different ways if this is possible. The two results are compared.

As well as these checks on the basic units of the machine, a test position is provided on the gas selector valve. No information is taken from the precipitation units for one sampling period per scan of the reactor, but a series of test numbers is inserted in their place. These numbers are subject to the same routine as the normal readings but are arranged to be at limiting values so that an incorrect modification of a single bit will not produce the predetermined result and the machine will be found faulty. Marginal conditions on power supplies and clock waveforms are automatically applied during this routine test period, the magnitude of the margin being set manually. This method of routine testing reduces the amount of additional checking apparatus to a minimum.

The computers will be a small part of the total instrumentation which the maintenance man must service. Special provision is made therefore for fault location and clearance. Fault indicators and counters are fitted at the check points just mentioned. An attempt to defeat the notorious transient fault is incorporated on the drum store and arithmetic checking circuits. The programme allows several tries for correct storage or calculation before giving the "computer faulty" warning. Each incorrect check is registered on fault counters so that the presence of a transient fault is recorded, but otherwise ignored. Fault clearance is mainly by substitution of units known to be good for those in the machine which are suspected of being faulty. To this end, all components, including the magnetic drum but excepting switches, are "plug-in replaceable" and the whole equipment, including drive circuits for the drum and the printers, is constructed from less than 20 different types of unit. The price of about 5% redundancy of hardware is considered to be worth paying.

Conclusions

The labour of repetitive examination of several thousand readings of radio-activity per hour in the power station is a task fit for a machine rather than a human-being. The machine can give exact mathematical discrimination on every single reading and, if the behaviour of the radio-activity is known under conditions of no bursting cartridges and with bursting cartridges, the machine can be set to give the earliest warning that is possible. The attention of the station operator can be concentrated solely on the abnormal behaviour of the fuel elements displayed for him by the computer as the great mass of normal readings are not presented to divert him. Should the operator choose to examine the record of any gas sample, it is readily available. The location of a computer in a power station and the service required from it in these conditions, require special considerations in the design and production of the machine.

Acknowledgments

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The authors wish to acknowledge the contributions made by their many colleagues to this work. They would like to thank Mr. N. R. Davis, the Managing Director of Sunvic Controls Limited, for permission to publish this paper.

Figs. 1 and 2 are reproduced by permission from the *Journal of the British Nuclear Energy Conference*, 1957, Volume 2, Number 2.

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THE USE OF SIMPLE COMPUTERS IN ELECTRONIC PROCESS CONTROL LOOPS

By R. J. REDDING, A.M.I.E.E.*

SUMMARY

A simple computing device employing force balance transducer principles is described. It can add, subtract, multiply, and divide. When this computer is fed with analogue electric signals representing process measurements or reference values, an output representing the solution of an equation is obtained. The following are typical practical uses:

1. **The correction of measurements to standard conditions.**
2. **The calculation or simulation of a parameter which cannot be measured directly.**
3. **The provision of a signal representing a derived quantity such as a ratio.**

The computer and the three-term process controller are complementary, dealing respectively with steady state and transient conditions. Integrated process control schemes show savings in initial costs and indicate the need for fresh features in the process controller.

Introduction

A complex chemical plant is generally controlled by dividing the plant into a number of separate loops in each of which some measured variable is controlled at the required value. This value is usually obtained empirically, *e.g.*, it is found in practice that if certain pressures and temperatures are kept at particular values the required chemical reaction takes place. The choice of loops in any particular instance is determined largely by the availability of suitable measurable parameters and in practice compromises are often necessary. For instance, it might be necessary to employ a rate of flow measurement where weighing is impracticable, or to use a temperature measurement where a measurement of heat input would be better. This applies equally to indicating and recording for supervision as to control.

A further extension to the control scheme is the introduction of quality measurements. Superior plant performance can result if these can be incorporated into the control loops so that the empirical set-point is adjusted automatically. Since the performance of a control system must depend on the quality of the information supplied to it, any improvement in measuring technique is potentially worth while. Thus, a flow measurement which from practical considerations is related to volume flow, would often be much more valuable if it were corrected for changes in temperature and pressure in order to give a true mass flow.

This paper describes a simple analogue computer which receives signals representing easily measured conditions and provides an electrical output which can represent a corrected measurement or a ratio in a form suitable for indicating or control purposes.

Advantages of electric signals

The use of electric as opposed to pneumatic signals for continuous process control was first employed on a large plant in 1951.[1] It can be said that the pneumatic equipment performing any duty has an electrical counterpart although the two may not be economically comparable. However, to substitute electric items directly in a system designed to suit pneumatic control would give few, if any, advantages.

The ease with which electric signals can be associated and computed means that the advantages of electric operation become really significant when these special characteristics are taken into account. In practice this offers a solution to fresh problems and a better co-ordinated, and consequently superior, control arrangement; often with considerable saving in the amount of equipment employed. When a quality control measurement can be introduced into the system the importance of errors in some basic measurements is reduced. On the other hand, the use of the computing device has shown that a need often exists for additional facilities on the process controller. These are easily accommodated in electric circuitry.

A Simple Analogue Computer

The computer is a further extension of the principles used in an electrical measurement transducer system and both have recently been discussed by Putman in relation to boiler control.[2] An abbreviated description is given below.

The electrical measurement transducer

Fig. 1 shows the transducer employed for converting a force due to a measurement into an electric signal. The force is applied to a pivoted beam which carries a pot-coil and magnet. The coil is connected to a thermionic valve circuit in series with the required output equipment (indicating, recording, or controlling mechanisms). The system acts as a continuous force balance between the output current and the input force by means of the balance detector. In this instance, the detector is a contact system modifying the grid potential of the valve and hence its resistance. Other balance detectors, *e.g.*, a flag and oscillator or a light beam and photo cell, may be employed but their cost and complication are considered unwarranted for present purposes.

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The permanent magnet shown may be replaced by an electro magnet connected in series with the pot-coil. The resulting force varies as the square of the current and this multiplying feature is employed to provide a linear signal from a device having a square law characteristic, *e.g.*, an orifice plate.

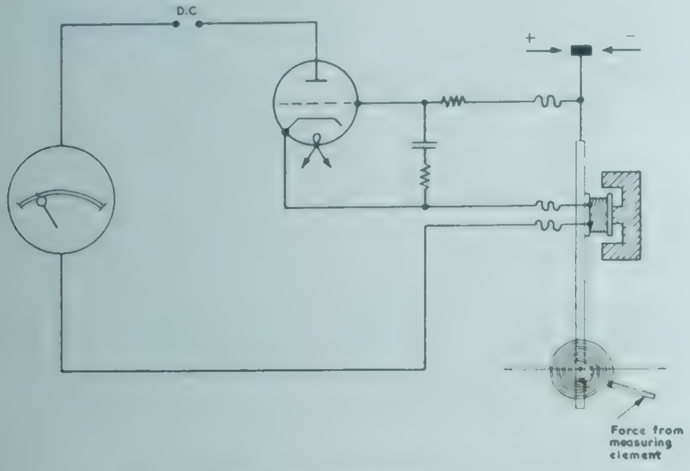
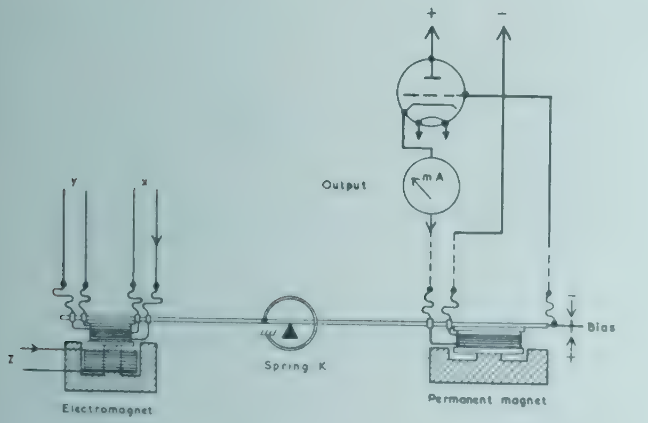


Fig. 1.—*Electrical measurement transducer*

The computer

The computer illustrated in Fig. 2 consists of a pivoted beam carrying a pot-coil at each end. The pot-coils operate with permanent or electro-magnets according to the problem concerned. Signals representing measurements are fed into these force elements and a feedback current provided by the valve circuit rebalances the beam and represents some function of the input signals. By suitable choices of winding the device can be arranged to add, subtract, multiply, and divide, and perform squaring operations and extract square roots. The addition of non-linear elements across the coils to cater for fractional indices has been described by Jawor[3] and equations dealing with powers higher than squares can be met by the use of these computers in series.



Output current $= (x \pm y)z \pm K$
Then if $z = 1$ and $K = 0$
Output current $= x + y$ or $x - y$

Fig. 2.—*Simple computer*

An instrument-type spring is fitted at the pivoted point of the beam. Normally this exerts no torque but it can be offset to introduce a fixed torque representing a constant in an equation. A constant, which is subject to adjustment at various times or is not known initially, is best provided by the use of a coil winding fed with a stabilised d.c. source via a potentiometer.

Practical details of the unit

The unit is referred to as a "simple analogue computer intended for use in process plants" to distinguish it from a mathematical or laboratory tool. Current practice is to tailor each example for the duty required, rather than provide a general-purpose unit which could be programmed to suit a particular problem. However, the standard design has considerable versatility. Provision is made for at least two isolated windings on each coil and considerable modification is possible on site by the external shunting of coils and the interchange of magnets.

The electro-magnets employed for multiplying and dividing require careful design. In particular, the flux from a pot-coil tends to magnetise the mumetal core of the electro magnet. This effect is minimised by a compensating winding on the electro magnet connected in series with the pot-coil.

The preferred value of the input signals is 0–15 mA d.c., but in some instances a signal level as low as 1 mW may suffice. The accuracy of primary measurements is seldom better than ±1% for various practical reasons and for a linear equation the accuracy of the computer is of the same order. When multiplying and dividing are involved, the accuracy depends on the range of variation of the inputs. For variations over a range of 5:1 the accuracy of the computer would not be worse than $\pm 1\frac{1}{2}\%$ of the output range.

Examples of Uses

Correction of flow measurements

The use of the computer to correct an inferential flow measurement to provide a true mass flow signal has been described by Putman.[2] The inverse of this problem occurs in the Venturi pneumatic pyrometer method of measuring flue gas temperature.[4] A sample of the gas is drawn through a Venturi and the differential pressure ΔP_1 is measured. The gas sample is cooled and passed through a second Venturi giving a signal ΔP_2 and the temperature at this point is suitable for measurement by a thermocouple giving a signal T_2. The original temperature T_1 of the flue gas is computed from these three measurements since $T_1 = K \dfrac{\Delta P_1}{\Delta P_2} . T_2$ where K is the constant for the instrument.

Ratio control

It is often necessary to control one quantity in a definite relation to some other quantity. An example of flow ratio control is shown in Fig. 3. It is required that B should be controlled in a definite relationship to manual or automatic changes in A. The two flow measurements are taken to a computer, the output of which is the ratio B/A. This forms the measured value input to the controller whose desired

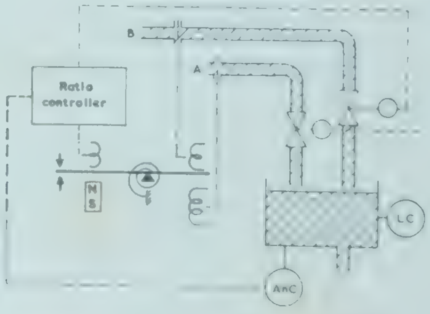


Fig. 3.—*Flow ratio controller*

value is set to the required ratio, causing B to follow changes in A.

If the purpose of the ratio control is to provide a mixture with a definite quality, *e.g.*, density or pH, a measuring device monitoring this parameter can be used to reset or "trim" the ratio as shown in the diagram. The importance of such "end-point" control is that not only is the system self-correcting for variations in the input stream qualities, but it is also self-correcting for errors and slow random drifts in the flow measurements. The difficulties which result from attempting to maintain a constant level in a tank are discussed later.

Calculation of a parameter from other measurements

An early application of the computer concerned a pipe line 20 miles long supplying water to a large industrial organization. It was required to maintain a steady pressure at the far end of the pipe line by controlling the speed of the input pumps.

It is worth noting that the pumping charges are a major cost in the economics of the water supply, and hence it is most important that the input pressure shall be at all times just sufficient to provide the required output pressure. The use of a constant speed pump and a throttling valve in the usual manner would result in excessive power waste due to the pressure drop across the valve. An output pressure signal was not available at the pumping station since the cost of connection over the 20 miles distance was prohibitive. It was necessary, therefore, to compute the required input pressure and to use this as the control signal.

$$\text{The input pressure} = P + A + K(F^2),$$

where P is the required output pressure, A is the hydrostatic pressure corresponding to the difference between the heights of the two sites, and F is the flow in the pipe line.

In practice, the system worked satisfactorily but was influenced by reflected surges in the pipe line due to step action in the speed control system employed. The latter consisted of an induction regulator positioned by a small motor according to the output of the controller. Stepless systems have been employed in later speed control applications, either all electric or by means of a "Dynamatic" or "Fluid-drive" coupling. With the latter, Townend[5] has shown that considerable advantage results from the lower pressure rating in sections of the plant in addition to the power saving.

Water level control

In some instances it has been found possible to combine a measurement transducer with the computing device because of the similar nature of the instruments. One example is the measurement of water level in the drum of a boiler.[6] The indication of level is most easily achieved by measuring the differential pressure across the drum, thus indicating the hydrostatic head. Errors would occur due to the change in density of the water with temperature if this were calibrated in terms of depth of liquid.

This is particularly troublesome when it is required to start up a boiler under control from cold. The level transducer has an electro magnet, enabling a density correction signal to be introduced. However, the density is not easily measured but is related to the temperature or the drum pressure. The latter is readily available for other purposes and the product of head and pressure gives a true level indication over a wide range of operating conditions.

Design Procedure for a Process as a Whole

In addition to the solution of measurement problems, the device can assist the over-all or co-ordinated control of a process. One measurement may feed several computers and a computed output may influence several control loops. The following is the design procedure that has accompanied the evolution of such computing.

Design procedure

Each of the easily measured variables of the plant is given a reference letter, and suitable points for the insertion of regulating units are chosen. The operation of various sections of the plant and later the plant as a whole is considered and an attempt is made to write algebraic equations describing heat balance, mass balance, etc.

There are invariably several ways of achieving the same solution and the final choice rests on the economics of each arrangement for the facilities required. When the most suitable set of equations has been chosen a test can be carried out if necessary using simple electronic simulators to represent the plant. The latter are resistance and capacity networks, giving a choice of several exponential lags, followed by simple amplifiers. Such simulators are invaluable for final testing of a control scheme before despatch to site. For these purposes it is not necessary to simulate the plant accurately and these units merely serve to close the control loops in a convenient manner. It will be realised that a full understanding of the process is essential, hence comprehensive applications have so far been largely confined to comparatively simple processes such as sugar refining[7] and well-understood processes such as steam raising for power generation.[2]

The ranges of the measured variables finally selected must be chosen carefully and the signals weighted to provide the correct over-riding action, during upset conditions and from fail-to-safety considerations.

The output current of a computer can be made bi-directional, *i.e.*, moving from a negative through zero to a positive value. This is particularly advantageous for quality control signals and in trials of new equipment before development to "on-line reliability", since the effect of such a compensating signal can be tried without appreciable disturbance and a failure or a servicing requirement is of minor importance.

The Effect of Simple Computers on Control Schemes

The main effect of introducing such a computing device is that the number of measured variables which can be employed is increased and the number of controllers and regulating units decreased, *i.e.* control loops are made larger and more comprehensive, more information being fed into each control loop. In combustion control a ratio controller may utilise corrected flow signals, the ratio being automatically reset by the calorific value of the fuel and trimmed by the oxygen content of the flue gas. Thus a single controller may be influenced by perhaps six measurement signals which may also be used for other control and supervision purposes.

When a "quality" or efficiency measurement can be introduced into such control loop it will compensate for, or minimise, errors and drifts in the basic measurements. The purpose of the latter is merely to bring the system into the vicinity of the required final controlled condition so that the final end product can exert exact control. The system is a form of cascade control.[8] In such cases there may be little point in attempting to improve the present accuracy (of, say, ±2%) of basic measuring devices for level, pressure, and flow. The most important features are repeatability and freedom from erratic calibration changes: slow drift is of less importance.

The computer has an inherent gain factor according to the constants of the equation being solved and this can be easily changed by varying a shunt resistance across the output coils. This means, in practice, that the range of gain required within the process controller itself can be much less than is usually provided in the general purpose unit. The straightforward design of the simple computers allowed all the process controllers to operate at 100% proportional band in several instances where the range of plant throughput and the size of regulating units were known. There seems no reason why this should not be exploited so that in such circumstances a process controller need only have a variation of, say, 25–100% to cover contingencies instead of the usual range of, perhaps, 3–600%. This would simplify design considerably.

Some difficulties of the simple computer

Difficulties have been experienced when a computed signal representing an abstract or dimensionless quantity has been used with a conventional process controller. There is no physical control loop in the usual sense and hence no fixed plant-constants to which the controller may be tuned. For example, if the ratio of two flows is to be controlled, the corrective action required is determined not only by the error in ratio but also by the magnitude of the flows at that time.

Similarly, the corrective action of an "efficiency controller" must be related to the actual throughput. In such cases the proportional band of the controller must be modified according to the magnitude of one of the quantities concerned. When the range of variation in the quantities is of the order of 3 : 1, the de-tuning necessary to achieve stability at all times is hardly significant, but for larger changes automatic proportional band adjustment is desirable and can be achieved in the latest design of electronic process controller.

A 3-term Process Controller

Fig. 4 is a simplified diagram of a modern 3-term electronic process controller in order to illustrate some new features which become prominent in co-ordinated control systems.

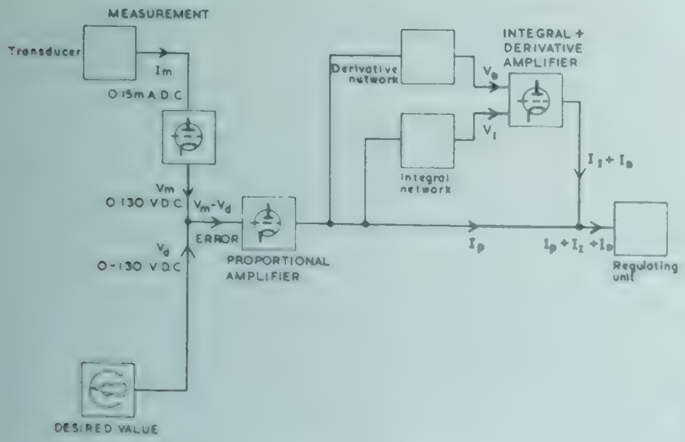


Fig. 4.—*Three-term electronic controller*

The signal representing the control variable results in a voltage across a resistor. This is compared with a manually set signal voltage representing the desired value, the difference between the voltages representing the error in the system. This is applied to an amplifier, the gain of which can be adjusted and is, in fact, calibrated as proportional band width. The output of the proportional amplifier which provides the proportional output signal for the regulating unit is also used as the input to the integral and derivative circuits. These are special R/C networks followed by an amplifier which provide the output signal representing these modes.

It should particularly be noted that the three terms can be pure and non-interacting. As a result, the adjustment of such controllers to suit a plant may be distinctly different from present practice with pneumatic controllers.

Proportional band adjustment is achieved by a variable resistor. If this is shunted by an indirectly heated thermistor, the proportional band can be varied by feeding a signal into the heater and, in the case of flow ratio control, one of the flow signals is employed for this purpose.

Control of non-linear functions

The controller can also be characterised to deal with non-linearities. Some measurements, for example, pH, are inherently non-linear and hence the sensitivity of the controller should ideally vary with the magnitude of the error. This is achieved by the use of non-linear elements across the proportional band adjustment resistor. A rectifier element gives a non-linear response according to the direction of the deviation and two such elements connected back to back give a band pass sensitivity. A more smooth variation over a wider range is achieved by a voltage sensitive device of the Metrosil type in which the resistance falls as the potential across it rises. In practice, a further resistor is provided to permit adjustment of the characterization after the proportional band has been set.

The Relation of Computer and Controller

The basic computing device deals only with algebraic equations and, therefore, with compensations in the steady state of the plant. The controller deals mainly with transient effects, being, in fact, an analogue computer solving a differential equation in real time. The two units are therefore complementary but it is essential to make the correct use of each unit.

As an example, the ratio control system of Fig. 3 could have the further requirement that the level in the surge tank be kept constant in spite of draw-off changes. A level controller could be used to modulate flow A but this would result in a difficult multivariable problem.

A re-arrangement of the system is shown in Fig. 5. Essentially the problem is reduced two loops with differing time

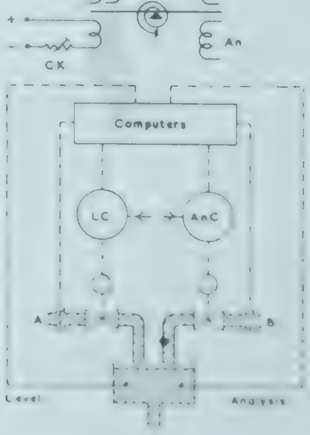


Fig. 5.—*Quality controlled blending*

constants to enhance the dynamic response. Flow A is controlled according to the level in a fast loop and flow B ensures the final quality in a slower loop.

The interaction which occurs within the process is minimised by computers which cross-modulate the level and analysis signals to enhance the steady state performance.

The equations of two simple computers whose outputs form the measured variable signal to the controllers are:

$$\text{Actual level} + CK \text{ flow A} \times \text{Analysis} = \text{M.V. (Level).}$$

$$\text{Analysis} + \frac{1}{CK} \text{ flow B} \times \text{Level} = \text{M.V. (Analysis).}$$

where C is a constant depending on the measurement ranges and K is the ratio of the proportional bands of the controllers. The windings for the former are illustrated in Fig. 5.

In terms of equipment, the first scheme uses one computer and three controllers, and the second two computers and two controllers, the latter being directly set to final desired values. In addition, the second scheme allows a reduction in the size of the surge tank and hence a reduction in the measuring lag of the analysis instrument. In the extreme case there is the possibility of eliminating the tank to give an in-line blending system.

Acknowledgments

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A DIGITAL COMPUTER FOR MULTIPLE SHAFT SPEED REGULATION

By M. W. SAGE, M.A., Grad. I.E.E.*

SUMMARY

A special digital computer has been designed for a process requiring accurate control of the rotational speeds of shafts.

A method of controlling accurately the speeds of four shafts, each being fitted with its own analogue controller, is described. These controllers are linked into a digital servo loop centralising in a computer. This provides an accuracy of control of ±0·25% over a 5 : 1 speed range of all the shafts; a 20 : 1 speed range can be accommodated with a slight fall in the accuracy of control. The computer can be set to perform programmed operations on the shaft speeds so that special effects can be achieved simply.

The reasons for adopting this particular method of speed control are set out together with possible alternatives. The design and construction of the speed control system are outlined. Special attention has been paid to the design of the digital servomechanism and detailed design data for this is given in an Appendix.

Introduction

A need has arisen within the plastics industry for the accurate control of the rotational speed of a number of shafts. The accuracy required is higher than normally encountered and is comparable with that demanded at the wind tunnel installation at the Royal Aeronautical Establishment, Bedford.[1] Complex control programmes are also a design requirement and these have to be performed with the shafts under a very high degree of control. This paper is a description of the design and methods adopted to fulfil such a specification.[4] Experimental results obtained in proving the system are given.

The system adopted incorporates a digital system around an analogue speed control system and this has the effect of greatly increasing the accuracy of control without in any way detracting from the advantages of using an analogue control system. The particular way in which the computer is applied and the method by which a digital servomechanism can be designed to give an optimum response time indicate possibilities of wide application.

The reliability of complex control systems is often open to doubt. This may be true when any failure in a complex system leads to complete failure; if, however, a more complex and more accurate system is designed round an existing system, then a failure of the additional system can cause a return to the less accurate system. This may result in a product of poor quality for the period while the machine is serviced but it does not mean plant failure. The ability to grade failures in a multiple loop control system is an important feature of the speed control system described.

Design specification for control system

The requirements of the speed control system are given below:

(i) Four independent shaft speeds to be controlled to ±0·25% of set speed.

(ii) Speed settings to be repeatable to within ±0·25%.

(iii) Speed control to be accurate over a 5 : 1 range. Control over a range of 20 : 1 in speed must be within ±2·5% over the additional range.

(iv) All shaft speeds to be simultaneously changeable by up to ±10%, the ratios of the speeds during this change being held constant.

(v) All equal-percentage changes in speed to take place in equal intervals of time.

(vi) Certain programmed operations of shaft speed changes to be carried out.

(vii) Ability to switch from automatic to manual control of any shaft with no disturbance to the set speed and with no effect upon the other shafts, which may remain on automatic control.

As a sequel to requirement (iv) it is intended to try and make the plant fully automatic by feeding back to the speed control system information from α-ray or β-ray equipment which is measuring the departure of the end product from the required standard.

Survey of methods

It was assumed that the control of the shaft speeds would be by regulation of electric motors. Previous plants have used either hydraulic drives or a single electric motor driving the shafts through variable-speed gear-boxes. The former proved unsatisfactory due to the relatively small inertia of the drive at low speeds and with the latter it was impossible to change the gear-box ratios by small increments after a long period of operation at a given speed.

Two fundamentally different types of control systems were considered in the initial design stages, analogue and digital. Analogue systems were preferred because of their relative simplicity and because of the familiarity of plant maintenance engineers with them. A wholly analogue system was discarded due to several inherent weaknesses when fulfilling the design specification.

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The accuracy of an analogue system depends on the gain of the system, the accuracy of measurement, and the stability of the reference.

The first factor may be considered as a design variable which, with certain limitations, may be made as large as required. This means that high-accuracy control with an analogue system depends on the second and third factors. Speed measurement for an analogue system is generally by a tachogenerator whose accuracy is governed by the uniformity of its temperature and its freedom from vibration or mechanical shock. The uniformity of temperature can only be obtained on a plant with difficulty: freedom from vibration or mechanical shock can never be guaranteed. Considering the third factor, the stability of reference is largely determined by the choice of reference and the control of its operating conditions. It is possible to achieve stabilities as high as ±0·01% but this demands exceptionally well controlled conditions. A more realistic figure would be ±0·1% with a possible long term drift of ±0·5%. The conclusion was that analogue motor control systems can give the required control accuracy of ±0·25%, but that they are not suitable using existing speed measuring devices.

The digital system considered makes use of a digital tachometer which measures rotational speed by a pulse-counting method. A glass disc with a number of opaque radial lines marked on it is mounted on a shaft and is viewed with a photo-sensitive device. As the disc rotates the opaque lines interrupt light falling on the photo-sensitive element and a train of current pulses is generated. These current pulses are counted over a fixed interval of time; the number counted is directly proportional to the speed of the shaft. The accuracy depends on the number of counts in a timing period and the stability of the timing interval. The former may be made as large as required while the latter is generated from a quartz crystal oscillator which can have a stability of 1 part in 10^6.

This digital method of speed measurement is considered to be ideal, as apart from considerations of accuracy, it is simple, robust, and insensitive to changes in ambient conditions or vibration. The measured speed is indicated as a count which can be compared with another number representing the required speed; the difference between these two numbers is used to apply corrective action. The disadvantage of a digital system is the sampled nature of the error, the period of the sample being governed by the interval during which the pulses are counted. If plant disturbances are slow, this may not matter, but if plant disturbances occur within the sampling period the digital system will not be able to make corrections for them.

The system adopted is shown in Fig. 1. Analogue controllers have been used for direct control of the motors without using an analogue reference. The speed-setting controls of the analogue units are governed by a digital control system outlined above. In this way the fast control action of an analogue system is retained and combined with the high control-stability and accuracy of a digital system: with this combination of techniques the various requirements in the design specification are simply fulfilled. An added advantage is that the analogue units are available as manufacturers' standard items and require very little modification.

Since this system is a two loop servo-mechanism, an analogue loop within a digital feed-back loop, it was considered satisfactory that the control loops should be broken at the juncture of the digital and analogue loops when changing to semi-manual control. This would leave the plant under the control of the analogue units, and, for the periods involved, it is unlikely that the reference or tachogenerator would drift.

Advantages claimed for the digital-analogue control system

This type of control shows many advantages where multiple control of a number of variables is required at high accuracy, and also when it is frequently necessary to achieve certain special effects.

An analogue system with a digital overriding control would appear to have many uses, not only in the control of the special plant for which the control system was designed, but in other places where complex control is required such as paper mills. Further applications are not solely limited to speed control but may be extended to any form of control where the variable can be measured directly to the required accuracy in digital form. If a fast response against external plant influences is not required, it should be possible to dispense with the analogue controller and speed up the response, if needed, by increasing the sampling frequency and making the necessary changes to the parameters of the servo-mechanism.

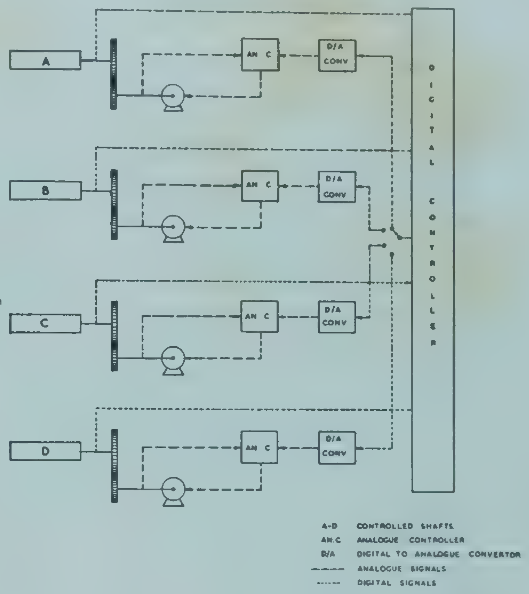


Fig. 1.—*Schematic diagram of control system*

Other implications of this type of control are the possibility of increasing the throughput of a plant where limitations are the speed of manual operation. This can be very important where the process is not started automatically and any increase in plant throughput means a highly controlled speeding-up programme.

It also becomes possible to release several plant operatives where such control and monitoring systems are installed. It has often been said of complex control that one man is replaced by the control system which in turn requires two extra maintenance engineers; this should not be the case with this machine. The system of built-in checking equipment and unit construction should eliminate the need for highly-skilled instrument artificers on first line maintenance. For this reason the digital control system has been designed for reliability and ease of servicing. It is too soon to establish the former, but the care taken in the very early stages of design to establish the latter has been well worth while. It has undoubtedly added to the cost of construction, but this could be saved if it secures an extra half-hour of plant operating time.

Description of Control System

An outline schematic of the control system is shown in Fig. 2.

The control system consists of four analogue control loops governing the speed of the motors within a servo-loop operating in a digital mode. This digital loop, or computer, governs the analogue units in turn on a time-sharing basis. The computer scans motors A–D; motor A being the master motor whose speed governs the speed range of the remaining motors. It is a design requirement that all the drives may be varied over a 5 : 1 speed range keeping the ratios between the shafts constant. To achieve this, the master motor controls a fifth servo-mechanism which is brought into operation when a major change of base-speed is called for.

Only one analogue unit is shown in Fig. 2. The operation of this controller is based upon the well-known Velodyne system. The output from a tacho-generator is compared with a reference voltage and the difference between these is fed to a d.c. amplifier which controls a servo-motor. An input signal causes the servo-motor to drive an induction regulator supplying the rotor volts to the variable speed a.c. motor in such a direction that the motor changes its speed and reduces the difference signal to zero. For stability purposes a derivative term is fed back from the servo-motor to the input of the d.c. amplifier.

All four analogue units share the same reference potential as shown in Fig. 3. The speed of the motors is governed by the proportion of the tachogenerator output which is compared with the reference potential. This variable potentiometer is a ten-turn helical type and is driven by the computer servo-mechanism. The reference potential is not fixed but is variable, the variation being governed by the fifth servo-mechanism mentioned earlier, and this enables a 5 : 1 variation in base speed to be obtained keeping the ratios of the shaft speeds constant during the change.

Operation of the digital computer

The operation of the digital computer is as follows. The required shaft speed numbers are set up on switches. Each number consists of five decimal digits and these digits are read out in parallel into the number storage registers, the switches being so wired that the numbers transferred into the registers are the "nine-complements" of the numbers set up on the switches. The stored numbers are transferred again in a parallel mode, into an error counter during their allotted span of the time-multiplex scanning cycle. When the stored number has been transferred to the error counter, this counter is switched to serial operation, and one is added

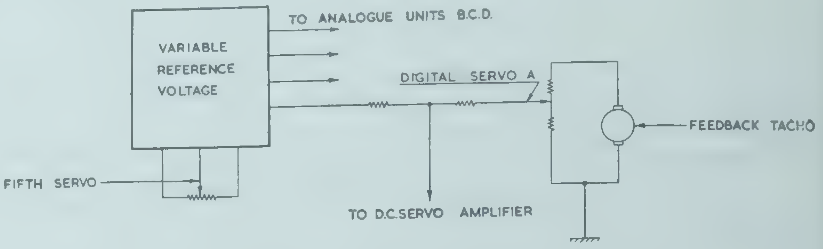

VARIABLE REFERENCE VOLTAGE
TO ANALOGUE UNITS B.C.D.
DIGITAL SERVO A
FEEDBACK TACHO
FIFTH SERVO
TO D.C.SERVO AMPLIFIER

Fig. 3.—*Feedback system for analogue units*

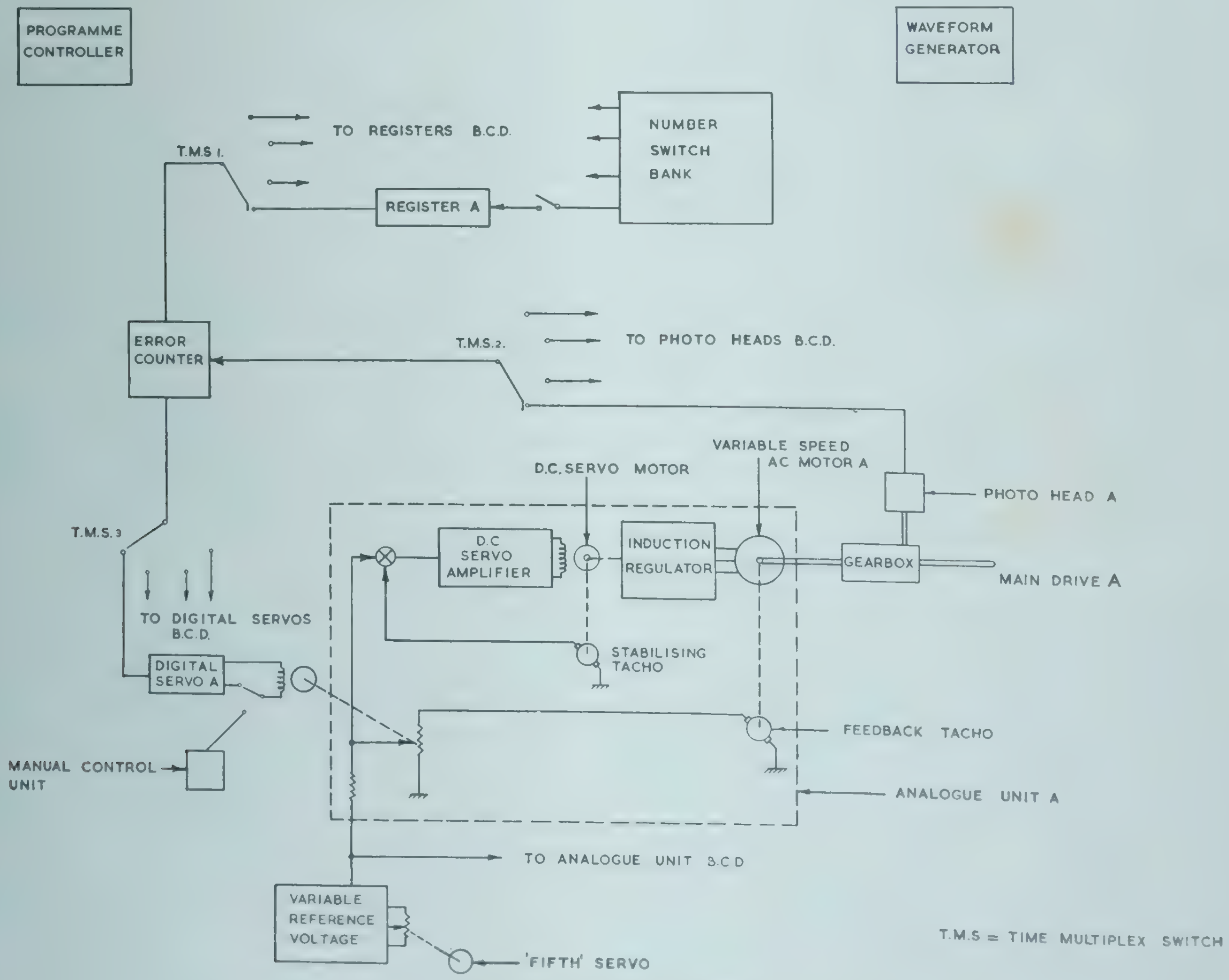

PROGRAMME CONTROLLER
WAVEFORM GENERATOR
T.M.S 1.
TO REGISTERS B.C.D.
NUMBER SWITCH BANK
REGISTER A
ERROR COUNTER
T.M.S.2.
TO PHOTO HEADS B.C.D.
VARIABLE SPEED AC MOTOR A
D.C.SERVO MOTOR
PHOTO HEAD A
T.M.S.3
D.C SERVO AMPLIFIER
INDUCTION REGULATOR
GEARBOX
MAIN DRIVE A
TO DIGITAL SERVOS B.C.D.
STABILISING TACHO
DIGITAL SERVO A
FEEDBACK TACHO
MANUAL CONTROL UNIT
ANALOGUE UNIT A
TO ANALOGUE UNIT B.C.D
VARIABLE REFERENCE VOLTAGE
T.M.S = TIME MULTIPLEX SWITCH
'FIFTH' SERVO

Fig. 2.—*Speed control system*

to it thus forming the true complement of the original number set up on the switches. The counter is now coupled to the appropriate digital tachometer and pulses are fed into it. When the number of pulses fed into the counter is the same as the number set up on the switches the counter will attempt to indicate 10^5. Since it can only count up to 99 999, it will have been reset to zero and also have emitted a carry pulse. The time of occurrence of this pulse is compared with the timing pulse which occurs normally 1 second after the initiation of pulse counting, and an error pulse is generated whose width is proportional to the time error, and of sign dependent on whether the counted down pulse is early or late.

This error pulse is then fed to the appropriate digital servo-mechanism which governs the analogue unit. The digital servo-mechanism is designed so that second order prediction is obtained.[2] It is described more fully later and in the Appendix.

The timing period is increased or decreased by ±10% to achieve a ±10% variation in speed while keeping the speed-ratios constant. This has the advantage that single knob control is obtained for this speed variation. It does mean though that the speed as set up need not correspond to the absolute speed. This has certain repercussions when programmed changes are wanted, and an interlock system is incorporated which prevents any group change occurring unless the timing wave-form is set to 1 second.

Control and monitoring features used are the displays on dekatrons of both the measured speed by the digital tachometer, and the controlling number. This controlling number may be different from the switch positions as the switches do not act as stores but only as a read-in medium. The shaft monitored by the tachometer can be selected at will as can the period for which the measured speed is displayed. This makes it possible for the shaft-speeds to be monitored within the scanning cycle. A faulty channel on the machine can be set to manual control in the case of any failure and the appropriate units replaced without upsetting plant control.

Digital speed-measuring element

The gear-boxes used on the shaft-drives are lorry type gear-boxes which have provision for a tachometer shaft. The photo-heads are mounted directly on these shafts and are thus geared direct to the final drive. The photo-heads are of simple and robust construction as shown in Fig. 4. The main unit is an aluminium casting. This casting carries the bearings for the main shaft, on which the engraved glass disc is mounted accurately concentric with the shaft.

Light from the lamp is passed through the condenser lens, glass disc, and graticule to the focusing lens. The graticule consists of a portion cut from a disc exactly similar to the rotating one. As the shaft rotates alternate light and dark bands cross the focusing lens, which focuses the light on to a photo-transistor and causes the photo-transistor to generate current pulses. The photo-transistor is not used in the conventional manner but is used as a photo junction-diode. This improves the performance with respect to changes in ambient temperature. The output from the photo-transistor is fed to an emitter follower which gives a low output impedance. The least reliable element in the photo-head is the bulb and this is under-run by 30% which improves the life considerably and ensures satisfactory service. The bulbs are standard prefocus bulbs. They are replaced on a time-of-service basis.

Digital Computer

The basis of operation of the digital computer has been described above. The computer consists of a number of plug-in logic units. These logic units are small transistorised printed circuits, two or more of which are mounted on a board which plugs directly into a connector on a main tray assembly. The advantage of this type of construction lies in the ease of replacement of any faulty element and the small number of different boards required. The digital computer is housed in one pedestal of the control console and this enables the transistor logic units to be kept well separated from the thermionic elements, thus overcoming some of the temperature difficulties of a mixed valve and transistor system. A typical computer tray assembly is shown in Fig. 5. The tray carrying the plug-in units can be pulled forward clear of the console and is hinged on one side so that access may be had to both the rear of the tray and the centre of the pedestal where further elements are housed.

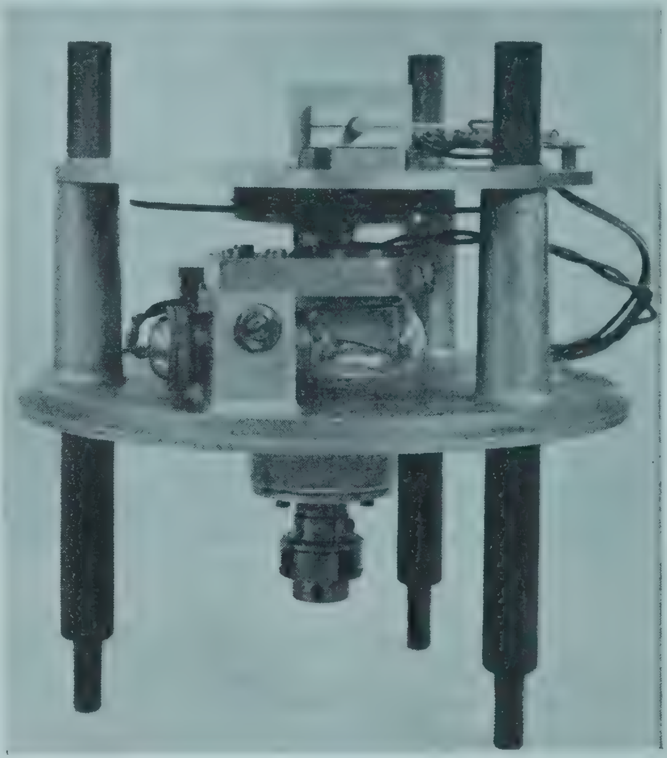


Fig. 4.—*Photohead*

The computer comprises two tray assemblies, one housing the number storage registers and the other the error counter and operational elements. The operation of the computer can be checked by means of a monitor oscilloscope which is built into the console. This enables the various waveforms and the contents of the registers and counters to be checked while the computer is in operation. In all, sixty points are monitored and if a fault occurs it is possible to locate the fault to a few plug-in units, which are then replaced immediately and investigated at leisure to ascertain the faulty component. This means that it becomes possible for trained but comparatively unskilled personnel to carry out maintenance in an emergency.

Machine rhythm

The computer operates as a time-multiplex system in which the four shafts are scanned in turn. In the design, the basic unit of scan-time was taken as one second. A total scan of all four shafts takes 8 seconds to complete, 2 seconds being allotted to each shaft. The total scan is known as the major cycle and the two second periods as the minor cycle. Each minor cycle is divided into two beats, A and B, of unequal length, the A beat taking 1·3 s and the B beat 0·7 s. During the A beat the pulses from the photo-head are counted and the error signal is generated and routed to the appropriate digital servo-mechanism. During the B beat the machine organises itself ready for the next following A beat, *i.e.*, the error counter is cleared to zero and the next number is transferred to it. When a fresh number is required to be read into the machine, this occurs during the A beat prior to the B beat in which it will be transferred to the error counter.

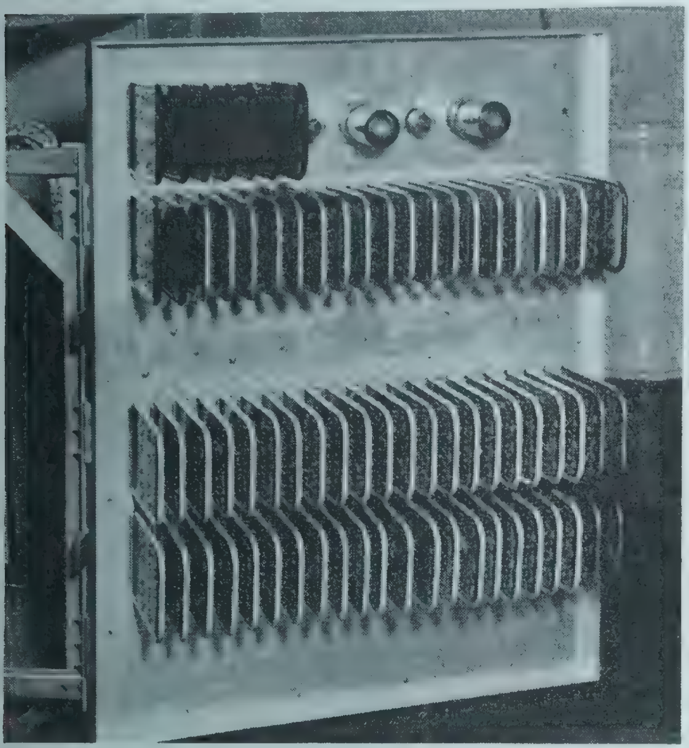


Fig. 5.—*Computer tray assembly*

Logic elements

The functions of a digital computer are generally accomplished by the use of three logic elements. These elements are the electrical equivalents of the logic functions AND, OR, and NOR. In the computer being described, only two of these functions are used—AND and OR. The functions AND and OR are diode gates and the bi-stable element is an Eccles-Jordan type of flip-flop.

The two types of diode gate used are known as α and β gates and are shown in Fig. 6. Either of these is capable of being an AND or an OR gate depending on the polarity of the input pulses. It is common practice nowadays to build computers operating synchronously,[3] *i.e.* the timing of the events is determined by a train of clock pulses: in this case the AND and OR gates cannot be physically identical since the polarity of the clock pulses is fixed. In this computer, owing to its small size and specialised nature, it was decided to design an asynchronous machine in which events occur sequentially. This results in an appreciable saving of components since it is not essential to have pulses of uniform polarity to perform AND/OR functions. In the logic diagrams standard symbols have been retained for AND/OR functions but have the suffix α or β depending upon the physical type of gate.

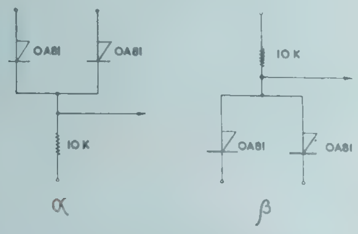


Fig. 6.—*Diode gate*

The flip-flops are of conventional design as shown in Fig. 7. Attention has been paid to obtaining a frequency of operation in excess of 50 kc/s with the ambient temperature elevated to greater than 60° C. This is achieved by keeping the base impedance low and thus minimising the effect of the base collector leakage current. The transistors used are Mullard OC76 and these give a switching time of 3–4 μs with a pulse amplitude of 12 v. An unusual feature of these flip-flops is the relatively high value of the collector base cross coupling resistor. This is kept high and defines the ratio of I_b to I_c and also keeps I_b small compared with I_c. Inputs to the flip-flops are as usual through diodes: there is also a direct input to the base of one side of the flip-flop, which is made use of in feed back arrangements for decade counters. Reset of the flip-flop is normally obtained by lowering either point 5 or point 6 to a negative potential.

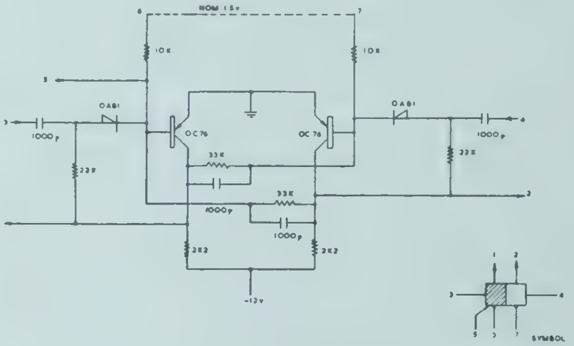


Fig. 7.—*Flip-flop*

There are four other packaged units in the computer. Two of these regenerate pulses or act as buffer stages, and are emitter followers and amplifiers. The remaining two elements are a delay unit which is a monostable multivibrator, and a reset amplifier.

Table I shows the various printed circuit boards used:

Table I.—*Printed Circuit Units*

Printed circuit unit	Function	No. of units per board
Flip-flop	Bistable element	2
α gate	Two input AND/OR gate	4
β gate	Two input OR/AND gate	4
Emitter follower	Buffer stage	4
Amplifier	Pulse inverter and regenerator	4
Delay	Delay of either 30 or 300 μs	2
Z-board	Resetting amplifier for flip-flops	4

Number storage

The most efficient method of operation for a digital computer using binary elements is to use a true binary code.

This however is only a half-truth and does not take into consideration the complexity of transforming from the input number code to the machine code, which in an arithmetical computer can be achieved quite simply by means of a subroutine. This can become uneconomic in a small specialised computer and it is then better for the machine to operate in the same code as the input arithmetic. In the speed control computer this has been done, the input consists of four groups of five decimal digit numbers and the computer operates using decimal arithmetic.

Storage for 20 decimal digits in four static registers each containing a 5 decimal digit number has to be provided. A register consists of five decade counters each acting as a decade storage unit, a decade unit consisting of only four flip-flops. To read a number into a decimal store, the decade is first cleared to zero and a number of pulses equal to the number to be stored is fed into the counter. To read out of the unit, the counter is cycled by a train of 10 pulses. When a pulse had been emitted from the counter, it opens a gate which is also fed with the 10 pulses and allows the remaining pulses of the train of 10 pulses through. The number of these gated pulses is equal to the number stored, *i.e.*,

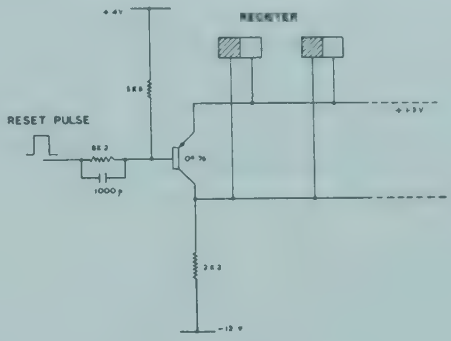
$$10 - (10 - x) = x$$

where x is the number stored.

The method of resetting registers is shown in Fig. 8. The reset amplifier is normally held conducting when the collector potential will be at almost the emitter potential, and the base resistors of both sides of all the flip-flops in the register will be held at +1·3 volts. When the reset amplifier is pulsed positively the collector will fall momentarily to −12 volts and this will force the flip-flops to a zeroed condition.

Number input

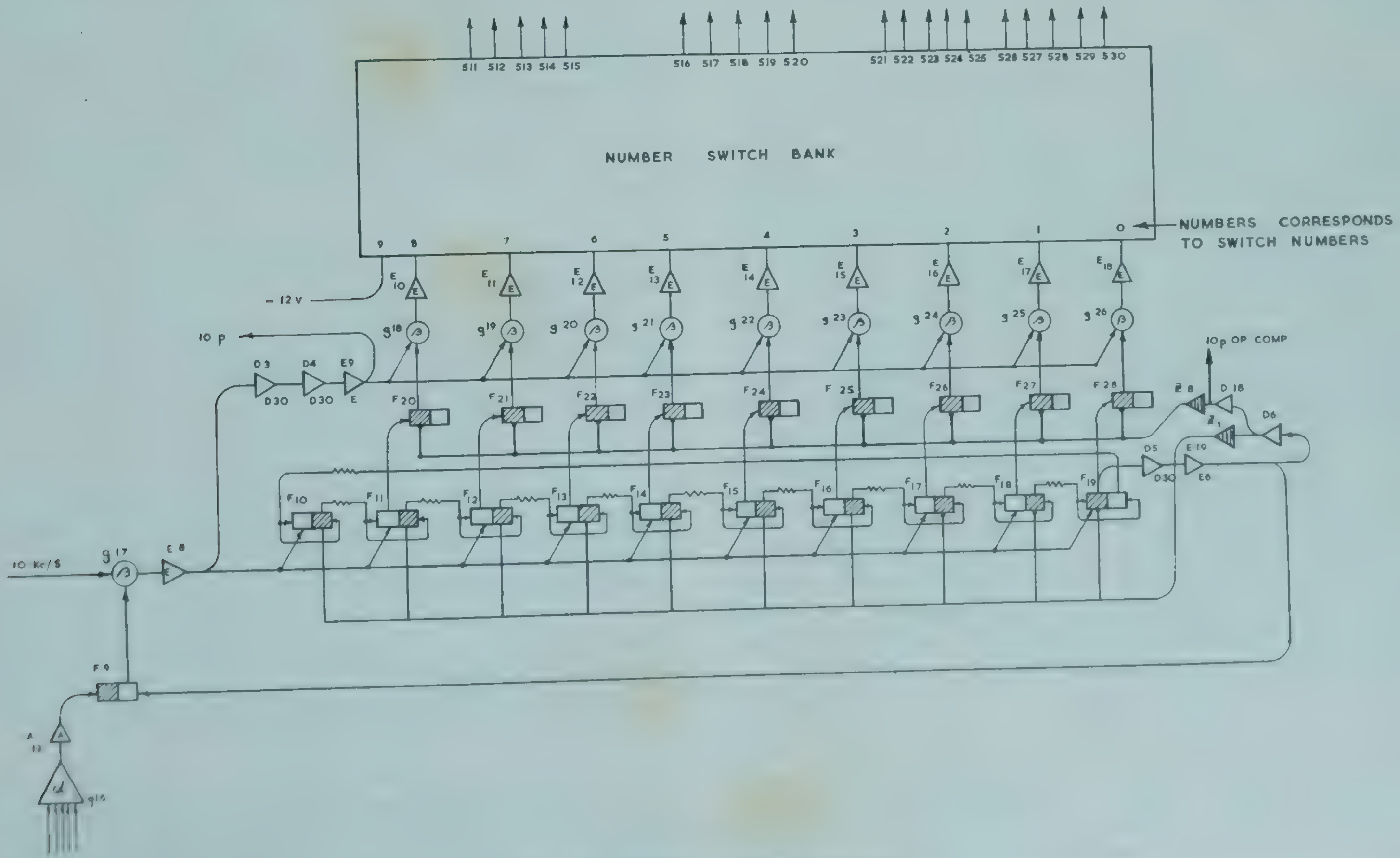
The numbers which represent the desired values of the shaft speeds are set up on four banks of five switches. These switches select from 10 different pulse trains consisting of 9–0 pulses. The switches are so wired that in the 0 position 9 pulses are generated and 0 pulses in the 9 position. Thus the number of pulses selected is the nines complement of the number indicated by the switch. The generation of the ten trains of pulses is as described below and shown in Fig. 9.



+4V
REGISTER
5K6
RESET PULSE
8K2
1000p
OC 76
+1·3V
2K2
−12 V

Fig. 8.—*Resetting of registers*

When a number is to be read in, a pulse is generated which passes through gate G16 to F9 and causes G17 to open. This allows a train of 10 kc/s pulses to pass through to the ring counter F10–F19. The ring counter is cycled and at the 10th pulse emits a pulse to D5. This is delayed and fed to F9, closing F17: only 10 pulses have been allowed to pass through G17. This is the normal method of generating 10 pulses, not only for number input, but also to transfer numbers from the register. As the pulses are fed to the ring counter, the "0" state which is at F19 when the counter is zeroed, is transferred to F10, F11, F12, etc. At the commencement of the second pulse it is transferred to F11 and this causes a trigger to be applied to F20 and closes G18. The other input of G18



S11 S12 S13 S14 S15
S16 S17 S18 S19 S20
S21 S22 S23 S24 S25 S26 S27 S28 S29 S30
NUMBER SWITCH BANK
9 8 7 6 5 4 3 2 1 0
NUMBERS CORRESPONDS TO SWITCH NUMBERS
E10 E11 E12 E13 E14 E15 E16 E17 E18
g18 g19 g20 g21 g22 g23 g24 g25 g26
−12 V
10 P
D3 D4 E9
D30 D30 E
F20 F21 F22 F23 F24 F25 F26 F27 F28
10p OP COMP
D18
D6
D5
E19
D30
E6
F10 F11 F12 F13 F14 F15 F16 F17 F18 F19
10 Kc/S
g17
E8
F9
A12
g16

Fig. 9.—*10-pulse waveform generator*

consists of a delayed 10 pulse train from E9 and thus only one pulse is passed through G18 and is fed to position 8 on the switch. At the completion of the 10 pulses a suitably delayed pulse checks that the ring counter is correctly returned to zero. Another delayed reset pulse resets F20 to F28.

Error Detection

The time of occurrence of the carry pulse C emitted from the error counter is compared with that of the timing pulse τ. A pulse is generated whose width is equal to the time difference between these two pulses. This is fed to the digital servomechanism on one of two channels depending on which occurs first. The method of error detection is shown in Fig. 10. The carry pulse and timing pulse are fed to flip-flops F_1 and F_2 respectively. If the carry pulse occurs before the timing pulse then the gate G_2 is opened and F_1 is closed. The timing pulse closes G_2 and attempts to open G_1 which has been previously closed by F_1. This results in a pulse being emitted from G_2 which is fed to the diode switch DS_2. This pulse is routed by DS_2 to the appropriate digital servo-mechanism. Flip-flops F_1 and F_2 are reset later in the minor cycle. If the timing-pulse occurs before the carry pulse, a pulse is emitted from G_1.

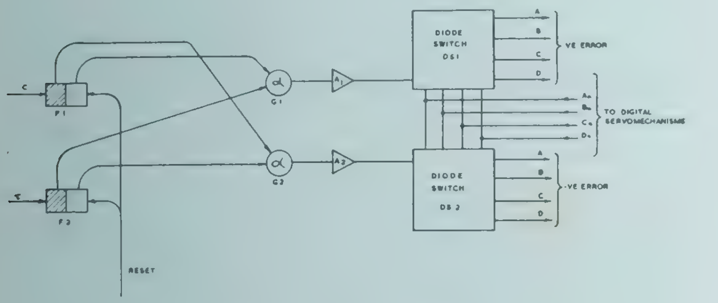


Fig. 10.—*Error detector.*

Digital Servomechanism

The digital servomechanism converts the pulsed error signal into a continuous motion of the speed setting potentiometer. The simplest method of doing this is to pass the error pulses through a low-pass filter. The output from the filter will be approximately a continuous function, but it has the disadvantage that the rate of change of filter output will be extremely slow compared with the frequency of occurrence of the error pulse. This would be very unsatisfactory in the speed control system where the error pulses only occur every 8 seconds. The rate of change of the speed-setting potentiometer can be greatly increased if instead of generating a simple function which indicates the error signal at some time in the past, a "prediction" function is generated which brings the error signal more up to date. The "prediction" function assesses the previous error signals and predicts what is the most likely value for it at the next sampling instant. The speed setting potentiometer is then moved so that the next predicted error should be zero. This technique is used in the digital servomechanism and a detailed description of the servomechanism is given in Appendix I. A block schematic of the unit is shown in Fig. 13.

Equilibrium detection

The programme controller is a relay sequential controller operating on detection of servo-equilibrium. Since the servo-motors are two phase a.c. motors this becomes the detection of the absence of an a.c. signal. There is an inherent over-shoot in the servo-mechanisms rendering it necessary to have an equilibrium detector which comes off fast when equilibrium is disturbed, but does not come on again unless equilibrium has been maintained for at least three samples. The circuit to achieve this is shown in Fig. 15.

The diode V_1 is fed with the amplified signal developed across the control phase winding of the servo-motor. This generates a negative d.c. voltage on the control grid of V_2 and turns off the Servo Equilibrium Relay SE. When the control phase signal is removed, relay SE does not come on immediately as the anode of V_2 falls slowly due to the miller condenser between anode and suppressor. When the anode potential has fallen so that diode V_3 conducts, SE comes on. Due to the slowly falling potential, the magnetic flux in SE builds up slowly and very slow contact action results. The relay switching action is speeded-up by early-opening contacts SE1 which disconnects the Miller condenser and allows the anode potential to fall rapidly.

Programme Controller

The programmed operations required are carried out by relay control. The relay sequences are predetermined and the logic operations carried out sequentially: the timing of changes is determined by detecting equilibrium of the speed control potentiometers. This method of programming has been chosen as only a small number of known programmes are required. A more flexible method would be to use uniselectors or transistors and build up the programme on a patchboard.

It is hoped at some later date to make the plant fully automatic. This will be achieved by measuring the variable parameter of the plant output with either an α-ray or a β-ray equipment. The signal derived is compared with a standard sample and any difference generates an error signal which feeds back into the control computer. As mentioned earlier, if the timing interval is changed, all the speeds are changed in the same proportion. The effect on the plant is to vary the relevant dimension of the plant output. The method of full automatic control is to generate an error signal from the α-ray or β-ray equipment which controls the timing interval.

Conclusion

A combined digital-analogue control system need not become unduly complicated, and need be no less reliable than a well proven analogue system. The superposition of a digital system on an analogue system shows many advantages, foremost being the greatly increased control accuracy and freedom from drift, whether short-term or long-term.

The design of digital servo-mechanisms is not necessarily more complicated than conventional continuous control systems provided care is taken to achieve the optimum type of summating and holding circuit. In the case of this system it was arrived at empirically.

The method of obtaining a "prediction" of the error is described in an Appendix and it is shown that this may bear a strong resemblance to a continuous servo—in which integral plus proportional control is cascaded with integral control.

Acknowledgments

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Symbols Used

C = capacity (μF).
E = emitter follower.
F = flip-flop.
G = gate.
I_b = transistor base current (MA).
I_c = transistor collector (MA).
R = resistance (ohm).
K = kilohm.
▨ = flip-flop.
⊃ = AND gate.
▷ = OR gate.
A = amplifier.
E = emitter follower.
D = delay.
▷ z = board.

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The manuscript of this paper was received on 29 November, 1958.

Appendix

Design of a digital servo-mechanism

Servo-mechanisms operating with a sampled input may be divided into three classes depending on the relation of the sampling period to the natural period of the system. These classes are:

(1) Sampling period < natural period of the system.
(2) Sampling period > natural period of the system.
(3) Sampling period ≃ natural period of the system.

Class 1 behaves as though the input data were continuous since the system smooths the input signal while Class 2 systems are generally unstable unless some very special precautions are taken. Class 3 is the category into which most digital servo-mechanisms fall, including the speed control computer. The main problem in the design of this type of servo-mechanism is to overcome the staleness of the error signal during the interval between samples. The method adopted in this digital computer has been to generate a predicted value of the next input signal from the previous input signals. This reduces the effective staleness of the error signal and improves the response of the servomechanism to a step function input.

There are generalised methods of analysing such servomechanisms by methods similar to continuous servomechanisms using Z-transforms, but these methods do not give a good physical picture of the process, and for simple systems are unduly complicated. The design of this digital servomechanism has been treated as a problem in numerical analysis.

To obtain a predicted value of θ (the output quantity) at time T_0 it is necessary to examine the known values of θ at times T_{-1} to T_{-n}, interpolate a curve through these values which is equivalent to generating an arbitrary function $f(\theta)$, and extrapolate it to obtain the predicted value of θ at T_0. At time T_0 the true value of θ is obtained and this is used to modify the coefficients of $f(\theta)$ to give $f'(\theta)$ which is a function giving the value of θ at times T_0 to T_{-n+1} and is used to predict θ at T_1. The accuracy of the predicted value of θ depends upon the number of points used in the interpolation. This method of predicting θ is equivalent to generating a family of interlaced polynomials.

In the servo-mechanism the object is to reduce the error $(\theta_0 - \theta_i)$ to zero. In a digital servo-mechanism which is said to have linear prediction, the error is assumed to vary linearly and first order differences of error are constant (see Fig. 11).

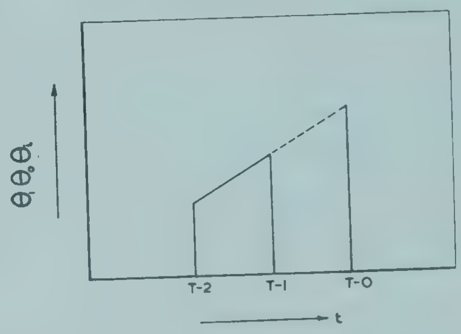

Fig. 11.—*Linear prediction*

This is equivalent to having a quadratic relation between θ_0 and θ_i and $f(\theta)$ is taken as a quadratic. This linear type of prediction system has been used.

Practical design considerations

In a linear prediction system it is necessary to generate function $f(\theta)$ which is a quadratic in time,

i.e.
$$f(\theta) = a_0 + a_1 t + a_2 t^2 \quad . \quad . \quad (1)$$

Now suppose θ increases to $\theta + \delta\theta$, then we generate a second function,

$$f(\theta + \delta\theta) = a_0 + \delta a_0 + (a_1 + \delta a_1)\, t + (a_2 + \delta a_2) t^2 \;. \quad (2)$$

From equations (1) and (2):

$$\delta\theta = \delta a_0 + \delta a_1 t + \delta a_2 t^2 \quad . \quad . \quad (3)$$

Now in a prediction system the value of $\delta\theta$ should be zero at $t = -2T$ and $-T$ and θ at $t = 0$; thus equation (3) must be of the form:

$$\delta\theta = \theta(1 + t/T)(1 + t/2T) \quad . \quad . \quad (4)$$

Equating coefficients between equations (3) and (4).

$$\delta a_0 = \theta$$
$$\delta a_1 = 3/2T \,.\, \theta$$
$$\delta a_2 = 1/2T^2 \,.\, \theta.$$

This gives the relative weights to be attributed to the contributions from θ at each sampling instant.

Two integrator model

Consider the system of integrators shown in Fig. 12. The output from integrator I is:

θ_0 = Value of θ at previous instance plus effect of corrective action.

$$\theta_0 = \theta + \int (a\theta + \int b\theta \mathrm{d}t)\, \mathrm{d}t = \; + a\theta t + \theta\tfrac{1}{2}b\theta t^2 \quad . \quad (5)$$

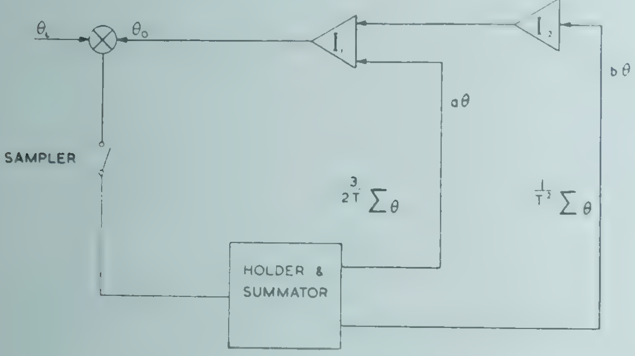

Fig. 12.—*Sampled data mechanism*

The Laplace Transform of equation (5) is:

$$\mathscr{L}(\theta_0/\theta) = \frac{1}{p}\left(a + \frac{b}{p}\right) \quad . \quad . \quad . \quad (6)$$

If now two amplifiers are connected as shown in Fig. 12, then for A_1 the Laplace Transform is—

$$Y_1(p) = \frac{1}{pCR_1} + \frac{R_2}{R_1} \quad . \quad . \quad . \quad (7)$$

and for A_2 the Laplace Transform is—

$$Y_2(p) = \frac{1}{pCR_1} \quad . \quad . \quad . \quad (8)$$

The overall transform for the two amplifiers in cascade is given by:

$$Y(p) = Y_1(p) \,.\, Y_2(p)$$
$$= \frac{1}{p}\left(\frac{R_2}{R_1} \cdot \frac{1}{T} + \frac{1}{pT^2}\right) \quad . \quad . \quad (9)$$

But

$$Y(p) = \mathscr{L}\,\frac{\theta_0}{\theta}$$

and thus equating coefficients of equations (6) and (7).

$$a = \frac{R_2}{R_1} \cdot \frac{1}{T}$$
$$b = \frac{1}{T^2}$$

and comparing equations (3) and (5)

$$a = \frac{3}{2}(1/T)$$
$$b = 1/T^2$$

whence $R_2/R_1 = 3/2$.

The holding circuit should ideally be a circuit similar to a Box-Car detector. This is not necessary in practice and the simple type of diode circuit D_1 shown in Fig. 13 can be used.

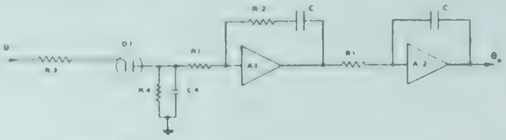

Fig. 13.—*Digital servo amplifiers*

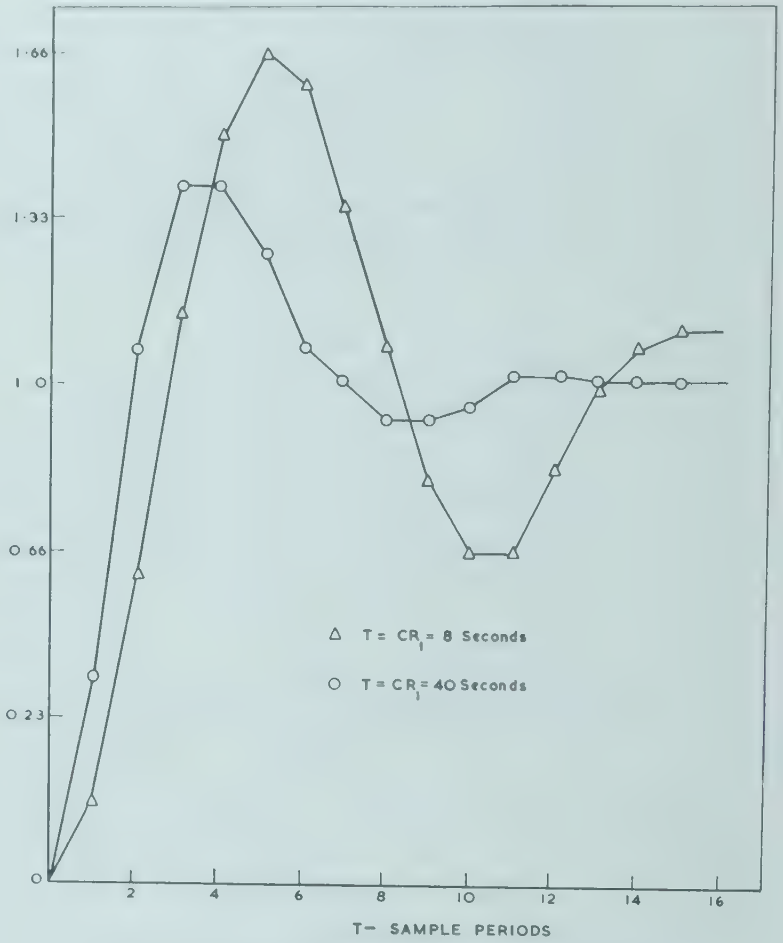

Fig. 14.—*Step function response of digital servo-mechanism*

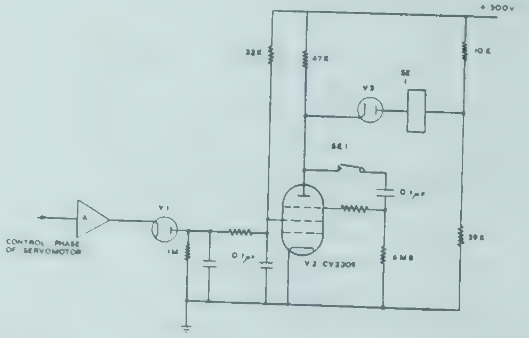

Fig. 15.—*Equilibrium detector*

The use of this type of circuit introduces certain non-linearity and prevents the simple theory above from fully describing the servomechanism. The non-linearity introduced by this type of detector is only significant for large errors and the simple theory holds for small departures.

In practice, the amplifier A_1 is an operational amplifier of the digital servomechanism and amplifier A_2 is the position control servo controlling the speed of the velodynes. Typical response curves of this digital servomechanism are shown in Fig. 14.

DISCUSSION OF PAPERS PRESENTED AT THE THIRD SESSION

Mr. R. M. J. **Withers** said he did not think that Redding could have meant what he had written when he described sugar refining as a comparatively simple process in juxtaposition "to the well understood process of steam raising". His computer could be used for certain simple tasks in the control of quite complicated multi-loop process systems. Fig. 1 showed a typical beet factory in which several computers had been installed. The plant and the automatic process control had been the subject of an earlier article.[1] The squares shown represented various processes, some quite complicated, and the possible interactions were numerous. The plant processes were automatically controlled within the squares.

The most important feature when trying to make such a factory automatic was to maintain steady running conditions throughout the plant, and any adjustments to the throughput rates should be small. To achieve that a number of key flow-controllers had their desired values set manually by the shift superintendent.

If the shift superintendent guessed correctly and there were no breakdowns or process complications none of the levels in the various buffer tanks would become low or high. The simple computers were sensitive to those levels and altered the desired values of the key flow-controller according to preset equations so as to keep the system running smoothly. Feed-forward and feed-back were employed. Thus the factory was automatically controlled from the sugar-boiling back to the beet intake.

The controls had, with slight modifications, been in successful operation for two beet "campaigns". Although the system corrected bad initial settings it did so by maintaining levels which could not be tolerated indefinitely. It might be possible to use a more elaborate computer to prevent the occurrence of abnormal conditions, but the further development did not seem to be justified on economic grounds. In any case prolonged process disturbances needed manual supervision.

It was obvious that the use of simple computers in this way presupposed a high level of conventional control equipment. Simple computers were also used for ratio control and for computation of new variables.

He would like to ask Redding what range of sensitivity change could be achieved in practice by the use of a thermistor in the electronic controller input.

Mr. A. H. **Isaac** commented on the suggestion made by Redding in his paper that it was an advantage for all controllers to operate at 100% proportional band or alternatively to possess a much more limited range of adjustment than was usual by virtue of shunt resistance across the output coils of the computers associated with the measurement of a controller. Such a procedure would be just as applicable by utilising a shunt resistance across the transmitter output or alternatively selection of an appropriate transmitter range. In fact such a result, if it had any value, was in no way a direct consequence of using the computing device described.

The other aspects of Redding's paper on which Mr. Isaac commented was that devoted to the adjustment of the effective controller proportional band in accordance with changes in load. That raised two points. One related to the advantage and desirability of such a system and the other to the performance of the individual constituents of the control loop over a sufficiently wide range to justify or permit such procedure.

Redding had stated that at a load turn-down of 3 : 1 any adjustment would be "hardly significant" but he sug-

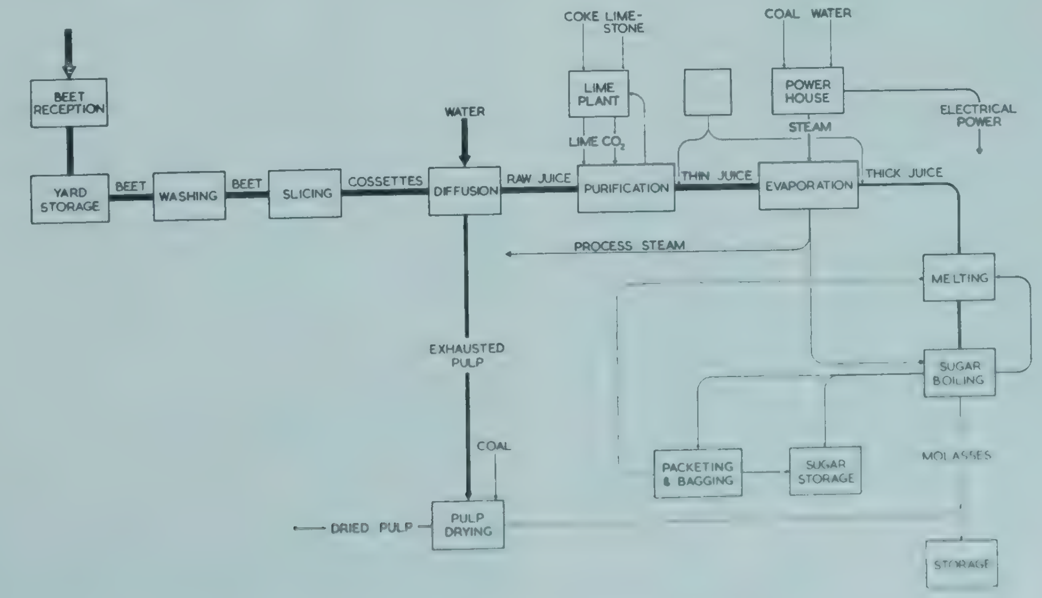

Fig. 1.—*Flow sheet for beet-sugar factory*

gested that for large changes it would be desirable. But he did not give any indication of how large those changes need be to justify the procedure, assuming that this figure is based on plant throughput variations which is measured as flow by a differential device. That would then correspond to a differential pressure variation of 9 : 1.

From Samson's paper presented at an earlier session Mr. Isaac noted that the compressor-control described was complicated by the varying arrangement of duct-work that could be associated with any compressor together with the varying throughputs, static pressures, and other operating conditions. Overall change in load of the order of 300 : 1 were thus encountered. In presenting his paper, Samson had explained that the range of load conditions was satisfactorily catered for with no changes to controller adjustments and substantially satisfactory performance obtained throughout. It seemed therefore that for rangeability of 9 : 1 the system Redding described would not be required and was apparently not called for in at least one specific case with loads varying 300 : 1. It was interesting to note to what type of applications and at what load ratio it would be applicable.

The possibility of rangeabilities of much greater than 20 : 1 was normally remote because of limitations in performance in the measuring systems the accuracy of which would be substantially reduced at this high turndown as would that of the computing device. Redding had mentioned the ability of his electrical computing device to give an accuracy not worse than $\pm 1\frac{1}{2}\%$ over a rangeability of 5 : 1 which suggested that at rangeabilities high enough to justify adjustment of proportional band the accuracy of the computing device would be inadmissable. On the other hand, again referring to Samson's paper, the accuracy figure stated was $\pm 1\%$ over a turn down of 30 : 1 in basic measurements. (He also understood that its accuracy was not worse than $\pm 2\%$ over 250 : 1, the repeatability throughout this range being within $\frac{1}{2}\%$). That was a special design with those requirements in mind (Ref. 1 of the paper). The comparison stresses the necessity for the device to permit a wide range of turndown with reasonable accuracy if any adjustment to the controller sensitivity is to be justified.

The consideration ignored the fact that for at least 25 years it had been the practise to vary the effective proportional band in relation to the load on the system. That was not achieved by any modification to the control instrument but by the use of a percentage-characteristic control valve. It was not sufficient to consider a proportional band simply in terms of a controller output. It must be finally related to the flow within the plant. Where equal percentage valves were used the overall effective proportional band was therefore narrow at high flows (a small change in lift giving a large change in flow) whereas at low flows the effective proportional band was considerably widened. (*i.e.* the same change in lift giving a much smaller change in flow). The effect was thus "built in" and was inherent in the operation of the valve. It was presumed that Redding would normally use such a valve for control. Again, its relevance to earlier comments made by Samson on the poor valve characteristic of the control valve associated with the compressors he described was apparent. Despite them, a wide rangeability could be handled. It therefore appeared difficult to justify any intentional adjustment to the control proportional band by some continuous device within the controller itself in respect of variations in plant load.

Mr. K. K. Schwarz said that from the point of view of an electrical power engineer concerned with control systems he wished to deal with three of the papers and, in particular, to add to Redding's plea for wider co-operation and especially for more overall and integrated design. There was no doubt, in his view, that that was the essential difference between good and bad control and system design. It was a question of economics; one paid for too much equipment in capital cost, one paid for uneconomic equipment in running cost, and one paid for wrong equipment in outages.

The underlying basis, from which one must consider an integrated system, had four aspects: firstly the driven machine and its output and output effect, secondly the driving (power) equipment and its intrinsic behaviour, thirdly the measuring equipment used to determine the variable to be controlled (or one of its effects), and fourthly the automatic control equipment proper—usually consisting of various amplifiers and the like—which linked the power and measuring equipment.

Jenkins had mentioned that the techniques designed for the Ward-Leonard control of rolling mills and the like could be applied to rectifier drives. Mr. Schwarz believed that that was fallacious, because a simple rectifier drive without inversion could not accept reverse power and was therefore uncontrollable in one direction of operation. Apart from effects on the safety of the plant that worsened the dynamic response due to the discontinuity in operation. As there were various means to overcome these deficiencies, it would be interesting to know whether Jenkins had any experience of the way in which this had been done, particularly on the continent, where such drives were very common.

Referring to Redding's paper, Mr. Schwarz commented on the example he gave of a long-distance pipe-line pressure control, which illustrated a number of important points met in practice. The first was measurement. It was not really germane to the question that the pipe-line pressure output could not be measured because of the 20 mile distance, as, even if it could be transmitted back on Post Office lines, which was quite possible, it would still be an unsuitable controlled variable. In fact, the problem of control was the crux, being determined by the hydraulic delay in a pipe 20 mile long. There was a simple way of overcoming this. As the static head would normally be catered for by a substantially constant input pressure obtained from a separate source, the booster pump should be studied separately, *i.e.* the way in which it counteracted the head lost in the pipe. Considering the parabolic characteristic of the pipe head loss against volume, and the volume/head characteristic of the pump at different speeds, it was possible to determine a third relation which was the relation between the desired speed of the pump and the volume passing through the pipe for standard conditions as shown in Fig. 2.* There were errors due to temperature differences, friction variation, and so on, but the computation made of the input pressure was evidently subject to the same errors. When this relation was obtained, if the pump were controlled by the delivery volume with, if necessary, a trimming control for static pressure, there would be a locally completely stable system which, eventually, would have the desired effect once the long delay in the pipeline had been overcome. That was merely a matter of time and nothing could be done about it.

Redding made the comment that the stability of the sys-

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* Equipment for which the above curves are relevant has been designed on the lines suggested above, using the same type of induction regulator controlled variable speed a.c. motor (mentioned by Redding and Sage) with an electro-mechanical (relay) automatic control system. This equipment operates most satisfactorily without any of the disabilities described by Redding.

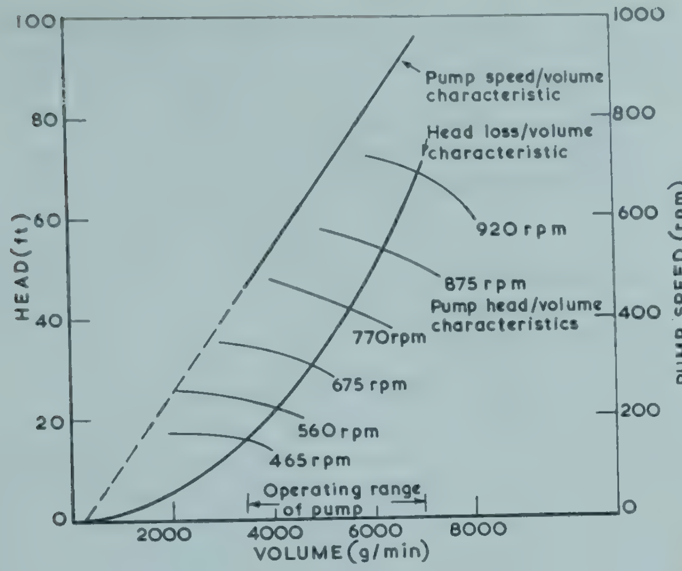

Fig. 2.—*Performance calculation for booster pump system*

tem was determined by the step action of the speed control system. Mr. Schwarz could not accept that because the driving motor, which was of the same type as that which Sage had used for this particular problem with the highest accuracy of control that could be imagined, was in fact one which gave inherently stepless speed variation. The fault must lie in the control system as such, in that the positioning of the induction regulator had not been carried out correctly or in the fact that the system produced fluctuation due to its inherent instabilities.

Redding's point that continuous control had been achieved with slip coupling contravened his earlier statement about economical considerations. While such a coupling had rather lower losses than a throttle valve control, if running costs were a criterion, no slip coupling would be acceptable. Incidentally it suffered from the same discontinuity of control performance as the rectifier drive.

It would be of interest if Sage could say a little more about the beta-ray gauging envisaged. There were two ways in which these beta-ray gauges were being applied; one was with a very tight control and the other with a very soft control. With a paper machine, for instance, where it was necessary to alter the input end conditions from density measurements at the output end, there was a very soft control which would almost certainly need some "computer" giving feed-back and feed-forward information catering for the long delay to obtain a reasonable result. On the other hand, with a single stand mill and tension controlled coiler and uncoiler directly adjacent, the speed of measurement became most important, so that the delay of beta-ray gauges became a difficulty. If a mill operation at 20 ft/s, the time which could be economically spent in correcting errors was very small, posing a very different control problem. As Sage's problem probably fell between these two extremes it would be most interesting to have his more detailed explanation on that point.

Dr. E. Picard expressed great interest in the work of Churchill and Hopkins. He agreed with the authors' statement that the background signal was changing with time. The choice of the test adopted was an immediate consequence of that remark. The paper discussion gave the reasons of that effect. Let G represent the measure: the first solution given by the authors consists in comparing G with a fixed value. That test was affected by statistical and time errors. Their second proposal, the differential test, consisted in comparing G with a previous value, G', of the same measurement and assigning a level to the difference $G - G'$. That process was more significant even though the statistical precision was poorer: however, it did not allow sufficiently for the time-varying background.

The subject had been dealt with in some papers by the French Atomic Energy Commission given at the 2nd Geneva Conference on the Peaceful Uses of Atomic Energy. As shown, it was necessary to introduce an evolution factor, α. That factor represented the setting of the reactor when the actual measurement was made, so that the test adopted in French reactors was now determined from $G - \alpha G'$: that was called the evolution of the measurement.

The third proposal of Churchill and Hopkins was to compare G with its time average value. Such an operation could be carried out only when the reactor was stable, and it did not seem to be valid when the power of the reactors was changing. He thought that the evolution test was the only test which could be adopted when, as was the case in France, all the pneumatic valves of the detection gas-circuit were entirely automatically controlled.

He asked the authors:

(*a*) Was the computer specially developed for the Berkeley power station, and was the programme easy to modify?

(*b*) Could the author give some information about the computer itself, such as the total number of elements, the capacity of the input registers, the capacity of the main memory and the output adopted for routine measurements and alarm measurements?

Mr. F. Steghart confessed that when he had first seen the electrical measurement transducer, described in Redding's paper, he had been to some extent prejudiced against its use of contacts, which were in fact open contacts. He himself had developed a system which was free of contacts, and the resulting transistorised transducer had some advantages.

There was one point which Redding had not mentioned but which deserved attention: his transducer could be damped exceedingly well. One learnt to appreciate the advantage of that particularly when using the output for controlling with a large derivative element. On the other hand, attenuation started at a lower frequency than with other systems. Perhaps the author would elaborate on that problem which was, however, important only for fast processes.

The author described briefly a non-interacting three-term controller with positive feedback to the integrating R/C circuit in the forward loop. Mr. Steghart hoped that they would soon have the pleasure of hearing Redding describe the controller in greater detail. When he did so, it was to be hoped that he would say more about "the adjustment that may be distinctly different from present practice with pneumatic controllers" to which he had referred.

The advantages of non-interacting controllers over interacting controllers were mentioned quite frequently in the literature. Young dealt with them in his graphs.[2] When Mr. Steghart himself described the first non-interacting three-term controller with positive feedback at the Heidelberg Conference in 1955 he had had to defend the use of purely electrical means by emphasizing the advantages of non-interaction.[3] He had mentioned them again in a paper describing various electronic controllers of that type.[4] He hoped that a paper would be written describing the disadvantages of interacting with a wide range of practical examples. If, on the other hand, non-interaction was nothing like so important as Mr. Steghart himself and others who had invented and designed those controllers had come to believe, the point should be proved.

Redding had described the use of indirectly-heated thermistors to change the proportional band of his controller. Mr. Steghart could confirm the effectiveness of that idea, which he had been using for several years for the solution of problems which otherwise gave instability.

Dr. J. A. Vidal said that he could not understand how the device described by Redding could achieve a continuous balance between the output current and the input force by controlling the valve-grid potential with a single on-off contact system, more elaborate detectors such as photo-cells and light beams being rejected as too expensive.

Dr. Vidal would also like to know how a measuring device, such as an orifice plate, should be connected to the transducer.

Mr. R. H. Tizard referred to the method of setting the speeds for the different shafts of the equipment described in Sage's paper. It was done by setting-up the number which corresponded to the number of pulses to be read from the photo-head on the shaft. In order to do this, as shown in Fig. 2 of the paper, it was necessary to have a set of registers and to time-multiplex them as well as the signals from the photo-heads.

An alternative way of making the setting was to arrange that the frequency of the pulses obtained from the shafts was varied according to the setting required, or that the effective count of the pulses was varied. It might be done in two ways. One way was by means of settings in the error counter. Each box in Fig. 3 represented a stage of a binary counter. According to the settings of the switches the arrival of one pulse from the shaft caused the count to increase by any desired amount. An error counter so operated could start from zero in every case and count up to overflow as in the system described. By using diodes the switches could be paralleled for all the shaft counters and multiplexing would then be required only on the single line from each shaft photo-head. The same principle could, of course, be applied to a decimal counter.

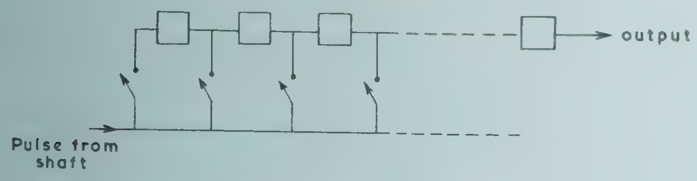

Fig. 3.—*Alternative method of setting-up required number*

An alternative method allowed a simpler counter to be used at the expense of a more complicated shaft counter. It would probably be less suitable for the application discussed but might be better in other cases. Marks on a disc were placed in different rows in the manner shown in Fig. 4, in which the circumferential rows had been developed into straight lines. Each row was provided with a separate phototransistor or other scanning element which could be switched to a common output line. By appropriate switching the number of pulses emitted for a given movement could be adjusted to any value—in the example of Fig. 4 the number of pulses for movement from *a* to *b* could be made anything from 1 to 31. The positions of the marks, as shown, ensured that only one pulse could be received from all the phototransistors at one time.

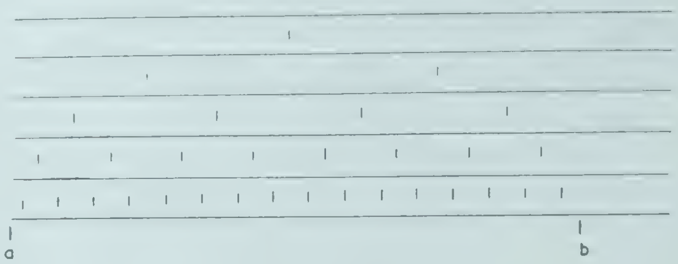

Fig. 4.—*Disc for simplified counter*

Mr. R. D. Trotter said that the paper by Churchill and Hopkins, describing apparatus for data presentation rather than computation, applied to a nuclear power station. Generally speaking it had been found that specially developed apparatus was extremely expensive, and it was to be hoped that the increasing use of the type of equipment described would result in the production of a series of standard units for amplification, differentiation, memory, or display which could be used in many different ways throughout the power station and thus bring down the cost. He seemed to detect the same thoughts in Redding's paper. It seemed obvious that electrical or electronic telemetering techniques were the first step in extending the possibilities of applying data presentation, but pneumatic techniques had a very good record of performance. There was a definite need for more electrical telemetering and it was to be hoped that the performance of such devices would be brought up to the standard set by pneumatic means. Very high standards of availability would be required if extremely expensive primary plant was not to be kept out of operation by malfunctioning of instruments.

Jenkins had pointed out the usefulness of the standard equipment developed by the G.P.O. in providing well-tried components for application to control techniques. The electronic side was not so far advanced in that matter. The money spent on control and instrumentation in a large nuclear station, including the associated equipment for control rods and burst slug detection, was several million pounds, and when another large sum had been added for the wages of the operating staff required to read the indications and operate the controls, it would be seen that a considerable sum of money was involved.

New equipment would be justified if it led to more efficient running, more reliable control, or lower overall costs. So far as that last point was concerned there might be difficulty because the number of operators was possibly fixed by the number who had to be present to control the plant during load changes or starting up, or who had to be available in emergency conditions. New types of equipment, however, might reduce the size and improve the character of the indications given in the central control room. For instance, one boiler panel could be used in multi-boiler stations to give an indication of the conditions on several different boilers, rather than the conventional method, which was to have one panel for each boiler.

Recorders had been used in power stations for many years. They suffered from the disadvantage of using too much paper during normal running, but when anything happened on the plant one wished that the paper speed had been far higher, so as to give a more detailed analysis. There might well be a future, therefore, for data presentation, which included a system whereby it took note of many readings continuously and then forgot them, unless it was required to print out that information because some emergency had occurred.

Referring to the paper by Churchill and Hopkins, Mr. Trotter said that it must be regarded as a very remarkable achievement to detect a leak of the order of a few square millimeters in one out of 40 000 fuel elements in a typical reactor and to enable that faulty element to be detected and discharged within a reasonable time, say one or two hours. There was no doubt that the Calder system had worked extremely well and had given much encouragement to people in industry who were designing equipment for the civil stations. The Calder system employed a single graph for each group of channels, and therefore for the civil stations the system would involve either 800 or 1600 individual graphs for a complete station. Several operators would be required to locate a burst within one hour: they would do nothing but look at the graphs, so that some sort of alarm system was required. Because the ability to think had to be provided, data presentation of the type described had been produced, and, because it was able to think, as a very satisfactory by-product it was able to print out the results of readings obtained in a sensitive and more accurate form than the type of recorder hitherto used.

It had been said that it was difficult to give the right logic routine to the thinking part of the machine: it would have to be found by experience. It was typical of the use which it was hoped to get from that type of equipment that should it be desired to fit in sensitivity changes for certain operating characteristics. A flexible machine would be available which could be altered in characteristics as experience showed was necessary.

It should be mentioned that a big advantage of the recorder system was that it gave a quick picture of many different groups of channels at once. It was a fact that the human eye was able to assess a curve much more easily than statistics, so that some improvement in the display facilities associated with data presentation would be welcome. In the case of data presentation, a three-dimensional type of display would be desirable, when it was necessary to look at the output from several hundred thermocouples inside a reactor pressure vessel.

It was to be hoped that every effort would be made to increase reliability and reduce cost.

Dr. J. H. **Westcott** said that the papers gave two very good examples of what he might call the new trend in instrumentation. It was largely a matter of data handling, of electronics, and of a number of new techniques. Being new, they did not always work quite as they should. There had not been as much experience of them as was desirable, but experience had to be paid for. It also meant that a new relation was being established between makers and users. The present pattern was that the large users had their own advisers who worked out which were the best black boxes to buy, and they then wrote an order and distributed it among the makers. Because they had had a great deal of experience, the arrangement worked. In the future, however, that would no longer be a feasible way of working; there was now need for large-scale facilities for the people who made and an understanding of the power of the new techniques in the people who used.

There were some industries where it was already impossible to avoid dealing with the new techniques. Nuclear power gave a good example. People who were involved in installing nuclear power stations said they were making a financial loss of it, and so many other people said, "We will not go into this until it has sorted itself out". There was a price to pay for experience and it would be to everyone's advantage if it were paid cheerfully.

More seriously, he thought that the relation between user and maker would have to be put on an entirely different basis. On the one side there was experience of use and on the other side all the experience of how to do it, so that it was essential that there should be good relations between the two.

Mr. F. P. **Rout** said that continued reference in the discussion to the desirability of freedom from interaction on multi-term controllers intrigued him. About eight years ago his company had, as chief designer, a particularly colourful character who was rather inclined to ram his ideas down the throats of all and sundry. He argued very logically, however, that if one took a proportional response and integrated it and took a proportional response and differentiated it and then added the two "totals" together one would have proportional + integral + proportional + derivative response. By "adding" a reverse proportional response one then had a pneumatic form of three-term controller free from interaction.

Mr. Rout had no intention of taking sides on the controversial question of whether or not freedom from interaction was essential. It was, however, interesting to recall that when the matter was first raised a few people gave it very careful consideration and argued that it was theoretically desirable to be able to calculate settings in advance and that freedom from interaction was thus beneficial. A number of people quite reasonably commented that although the instruments they used interacted quite "healthily" it did not prevent them from bringing the plant under satisfactory control. The remainder, he presumed, turned to the newly issued glossary of terms, found that interaction was not listed, and concluded that it did not exist. Here, therefore, was a very good example of a man putting forward a cure a number of years before many people were aware of the existence of the malady.

At the previous session it was probably true to say that the greatest benefit had been derived from the most controversial paper and the resultant discussions. Mr. Rout had felt growing sympathy for the person "under attack", and, in common with the bulk of the audience, had been particularly impressed when in the last five minutes the author had made a careful, reasoned and completely convincing statement of what he had done, why he had done it, and the overall effect on the plant. The manner in which these remarks were received signified that the audience felt that the author had taken all aspects of the problem into consideration and acted wisely.

Mr. Rout was inclined to think that this would also be the ultimate verdict on the designer who produced the controller with freedom from interaction some years before the existence of the phenomena was widely accepted and he therefore expressed the opinion that, in the long run, it was the controversial characters who made the greatest contribution to both knowledge and progress. He felt that although much of the mathematical approach of the last two days seemed to have little bearing on current problems some, at least, of the newly expressed thoughts would tomorrow become accepted beliefs and that time spent listening to ideas that appeared to be "right out of this world" was never wasted.

Mr. P. O. **Fuller** said that if pure terms were obtained it was not possible to guarantee that the settings would be non-interacting. On the other hand, oscillating periods were easily discernible with pure terms and damping could be dealt with. If pure terms were used in the form of a quadratic it was possible by using the Laplace Transform technique to produce an under-damped control and cancel out certain forms of open loop response in the plant. On the other hand, if the factor or cascade cancelling form of controller was used it was possible to set it up rather more easily.

If that were clearly understood, he thought that their individual advantages for the different types of plant would be apparent. There was an argument for deciding in the initial stages the sort of plant that was to be controlled, and that included the measurements and the valves that went into the controller, or after it, as well as the plant. For instance, if there was a motor driving a valve with resonance between the motor and the valve it was possible to cancel out that resonance by adjustment of the proportional band or middle term of the quadratic of the non-interacting pure term controller. On the other hand, if it was desired to set up the plant as a cancelling of successive exponential lags the quadratic in the factor form was the more easily manipulated, so that a decision at an early stage about which form of controller was needed in most cases could help in assessing the relative merits of the two systems of adjustment.

Mr. Churchill, in presenting his paper, showed a slide (Fig. 5) giving some information which had come to hand since the paper was written. He said that it gave the history for 13 days of the count rate from a fuel channel containing a ruptured fuel element. The record was not typical; there was in fact no typical way in which fission products built up in the coolant stream. There was, however, a general tendency for the activity of the coolant to increase and sooner or later the measurements became very erratic. It would be seen from the record that between days 4 and 11 there had been a fairly steady build-up and that after day 11 there was a tendency for the activity to build up to even greater levels and the measurements became very erratic.

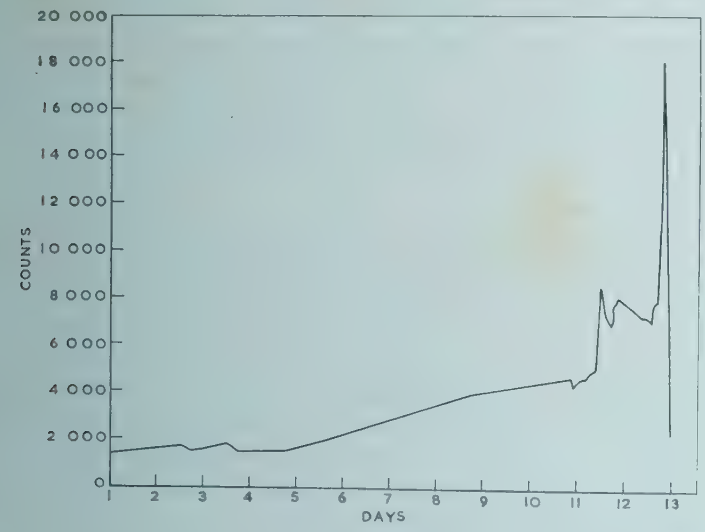

Fig. 5.—*History of count rate from channel containing ruptured element*

In reply to the discussion Mr. Churchill said that the factor α which Dr. Picard proposed in order to take care of the random fluctuations in the performance of a healthy reactor was, he thought, interesting although he foresaw some difficulties in arriving at values for it. Their experience showed that there was no correlation between the fluctuations in the different channels throughout a reactor, so that it would, he thought, be impossible to ascribe a value to α to cover the performance of the whole reactor. A minor point was that in applying a factor of this kind on a digital machine it was necessary to do a full multiplication operation, whereas with the types of tests which the authors proposed one required only an adder-subtractor and a mechanism for shifting the decimal point.

Berkeley power station would be a base-load station, operated at constant power. Under those conditions, the background counting rate depended on the relationship between the neutron flux and the coolent flow rate being maintained constant. If either of those factors changed the background count rate would be influenced. However, whether it was a base-load station or not, at some time or other they would have to start it up, and also, presumably, at some time or other they would have to shut it down. Under those conditions they proposed to abandon the pre-set numerical levels which were held in the computer store and, by throwing a switch, work on limiting values common to all channels, which could be set in manually on hand switches.

Dr. Picard asked whether the computer had been specially constructed. It was difficult to know how to answer that question, but it could be said that the logics and the organization in the machine had been specially developed for this application. The programme was wired in. The general philosophy was that so far as possible permanent or fixed information should be wired rather than stored. To change the programme it was therefore necessary to break and re-make the soldered connections, a job which might take hours or a day or two rather than the few seconds necessary with a universal calculating machine. The size of the drum store was about one-third of a million bits. The input registers from which numbers were read to the machine had a capacity of 999. Each of those registers was preceded by some pre-scaling circuits which had a ratio of 2, 4, or 8. That gave a maximum input capacity of 8000. Each machine had roughly 5000 l.f. transistors. That might appear to be a large number, but it should be pointed out that the machine used almost exclusively transistor logic, in that it had only 800 crystal diodes, which was small compared with the usual ratio between diodes and transistors in such machines.

Mr. Jenkins replied that the mercury arc rectifier applied to reversing mills must operate in all four quadrants of the speed/torque plane. Rectifiers capable of operating in that way had been developed, mainly in Germany, and were in use on a number of the rolling mills there. Rectifier units could directly replace Ward-Leonard generators and the conventional control techniques could be applied to give speed control, current limit, acceleration compensation, etc. Three methods could be used for arranging for the absorption of the regenerated power: by using reversing switches in the armature circuit; by using reversing switches in the field circuit of the motor; or by using back-to-back rectifiers. With reversing mills it was usual to employ reversing switches, but with cold reduction mills, where the demand for regeneration was less, back-to-back rectifiers could be used with the back rectifier of a much lower rating than the forward one. The rectifiers were used for driving motors up to 5000 h.p. and they had great advantages in terms of initial cost, cost of foundations, and cost of associated equipment such as cranes—the whole equipment being lighter. A very important point in the U.K. was that they needed much less maintenance. A report on the use of mercury-arc rectifiers for rolling mills has recently been produced by B.I.S.R.A.

Mr. Trotter had mentioned the high cost of developing electronic equipment. B.I.S.R.A. used uniselectors and relays in producing prototype schemes and had found that they were extremely reliable, easy to use, and very versatile. It was realised that other techniques would probably overtake electromechanical equipment for many purposes within the next ten years, but for the present they provided the only practical solution to a lot of problems in control and information handling.

Post Office techniques were particularly valuable in information handling when digital indication or automatic typewriting was needed, as no amplifier was required.

Mr. REDDING assured Mr. Withers that he had no intention of casting reflections on sugar refining. He was grateful that the full details of the process had been made freely available, as was essential for integrated control system design. There was no difficulty in achieving a sensitivity change of 10 : 1 by means of a thermistor and probably larger ratios could be met.

For the larger turn-down ratios mentioned by Mr. Isaacs, Mr. Redding said that he would endeavour to rearrange the system so that compensation could be achieved in two stages. Probably the reason for the figure of 300 : 1 was that Mr. Isaacs did not correct for the square law characteristic of his flow meters. The characterising of the gain by exploiting the non-linearity of some regulating units mentioned by Mr. Isaacs and Mr. Schwarz was, of course, good engineering where it could be applied, *e.g.*, in simple control loops.

The electrical characterising described had wide scope and flexibility, was cheap and imposed no restrictions, and the need for it particularly rose in complex control loops.

The figures substituted in computer equations to determine the coil windings, etc., must represent normal throughput, not the plant design figures with a large factor of safety; the latter might give a very high gain factor for the computer requiring that the proportional stage of the controller be used as an attenuator; since only two valves were employed to generate the three terms, it was hardly good practice to use one as an attenuator.

In thanking Mr. Steghart and Mr. Rout for their kind remarks, Mr. Redding agreed that the importance of non-interaction needed resolution. However, the discussion had centred on the ease of setting-up. As Mr. Fuller had pointed out, even if the adjustments did not interact, the effect of the terms might still interact in the system as a whole. Mr. Redding felt that the importance lay in the control performance made possible by the combination of non-interaction and pure terms, *i.e.*, when the three components of the output signal were in quadrature, the integral and derivative components being in antiphase at all relevant frequencies with definite calibration.

Such a controller, he thought, would enable an analogue computer study of a plant to be more easily carried into practice, since the controller would be an analogue computer working in real time. Its simplicity might obviate the standardised amplifier and differentiation units envisaged by Dr. Trotter.

The limitations, apart from knowledge of the process, were not on measurement or control so much as those imposed by the current range of regulating units. The difficulty of economically providing a method of controlling the final fluid had sometimes required a fresh approach to the whole control problem. Unfortunately, some examples of computing were on the secret list, but cases where a heating element was employed as both the regulating unit and the detecting element afforded interesting possibilities.

Mr. Redding assured Mr. Schwarz that the step-action mentioned resulted from inadequacy of the on/off motor control system positioning the induction regulator, and agreed that power efficiency of the slip coupling systems was not as high as with motor speed control types.

Mr. Redding stressed that the purpose of the paper was to outline the technique of simple analogue computing and that the examples quoted were illustrative rather than the best that could be achieved in a particular purpose.

The aim of the design of the computer itself was that it should be introduced into process plants and must, therefore be as simple as possible, since, if readily understood, it had better chance of being serviced by local personnel.

In answer to Dr. Vidal, Mr. Redding said that a capacitor between the grid and cathode maintained the current when the contacts were free.

So far, the limitations imposed by the contact detector system had hardly been significant and sometimes the dead zone had an advantage. However, the equipment is capable of considerable further development once the principle is established, if the added complication of other known techniques was justified.

Mr. SAGE replied that Mr. Schwarz was not the only one who wanted to know how the β-ray gauge would behave! There were two main delays involved. The first was the delay of the plant; the β-ray gauge was controlling an effect which occurred at the beginning of the plant. The second delay was the measuring time which was inherent in the β-ray gauge. In their own case the plant delay was very much greater than the measuring delay, so that they could use what Mr. Schwarz termed a soft type of control. They hoped to get a fairly reasonable control and to minimize the effect of plant delay. There was nothing that they could do about the fluctuations which occurred with a period shorter than that of the plant delay, but fluctuations with periods greater than that of the plant delay could be corrected.

Mr. Sage was in complete agreement with Mr. Tizard on the use of the word "digitiser". It had been used as an expedient to bring out more clearly what they were doing with the output from the photo-head. It was nothing more than a multi-pole a.c. generator.

Mr. Tizard had referred to switches and registers and had asked whether there was not a simpler method of doing what was required. There were three reasons for using switches and registers. First, process operators were inveterate knob-twiddlers, and there would be trouble if there were only switches. Secondly, programmes which called for a new set-point had to be read into the machine, and the new programme was not required to take place immediately the new numbers were read in, so that there was need for a buffer store. Thirdly, if there were no buffer stores and the switches were wired in directly from the machine it would be possible to change the switches at any period in the controlling cycle and introduce a very large step transient into the machine. By having switches and registers the number was read into the machine at the correct point in the control cycle.

Referring to the alternative methods mentioned of doing the control functions Mr. Churchill said that they fell into two categories. He asked whether it was not possible to generate the variable frequency and obtain the signal from it, rather than compare two numbers. The question was, which was the more economic method with regard to "hardware"? Mr. Sage did not think that there was more in it than that.

There was very little saving whichever way it was done. He felt that the main disadvantage of Mr. Tizard's second method was that it made the transducer (which was the most important and difficult part of the machinery to maintain, because it was inside the plant) slightly more complicated, and anything which tended to complicate the transducer was not an advantage.

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SESSION 4

DESIGN AND USE OF ANALOGUE COMPUTERS

Chairman: **Dr. M. V. Wilkes, F.R.S.**
President of The British Computer Society

ANALOGUE COMPUTING APPLIED TO PLANT AND PROCESS ECONOMIC ESTIMATION

By D. W. GILLINGS Ph.D., F.Inst.P., F.R.I.C.*

SUMMARY

An analogue computer is described for use in economic estimation and assessment of construction, development, and other projects. The method of computing can be used when alternative timing of capital expenditure, varying rates of operation, variations of costs and realisations, and other changes may be expected and must be taken account of during the life of the project. The design of a prototype computer is outlined and its operation illustrated with examples which are also worked out without its aid. Results are given in the form of a graph of the actual earning or deficit to be ascribed to the project against the time in the project life, corrected for the dependence of money value on time resulting from the provision of interest.

The example given is that of a number of simplified schemes for power station extension. Comparisons are made to show the effect of various times of installation of plant, choices to meet expected demands, and small changes in realisation rates. Results can be expressed as comparative present worth, as "pay-off time" of each scheme, or equivalent rates of return on expenditure can be estimated. Brief reference is made to the scope for development, problems arising from the uncertainty of data and results, and the need for exploration and experience to make adequate use of the equipment.

Introduction and Outline of Approach

The advent of electronic computers has made it practicable to undertake more comprehensive and complete treatment of many types of computation problem than in the past. One of the fields in which simplifying assumptions and short treatment have often to be accepted is that of estimating costs of industrial plant and processes. In particular, methods for estimating and comparing costs of alternative plant designs, modes of operation, and the like are accepted which may conceal assumptions that are far reaching and not borne out in practice as closely as the application of the method might imply.

The present study originated in questions raised by putting forward estimates of the influence on process costs of technical developments which might affect capital requirements. The application of computing described in this paper was suggested to establish that adequate estimation could be carried out when it was not permissible to assume "averaged" production rates for the life of a plant, or that all investment was simultaneous. These are the sorts of limitation usually implicit in annual cost estimates as widely applied. There are established estimation methods which are not subject to such limitations,[1] but these have not hitherto been widely accepted, one of the major reasons for this being the length of calculations required in many real situations. Analogue computing of estimates was first explored for the present stage of work as offering flexibility and rapidity in operation, simplicity of programming for a wide range of examples, and accuracy quite adequate for comparative estimates dealing with future operations.

Estimation Methods

An important feature of all estimates for development, extension, or construction of any sort is the method adopted to deal with capital as opposed to revenue expenditure. There are basically two classes of method. A percentage of capital cost can be taken as an annual cost, and then included with the annual costs actually incurred for materials, power, labour, etc., to form a total annual cost off-setting the total annual realisations. Alternatively, capital cost can be considered as a charge to be met in full, and its influence on operating costs as a whole expressed in several ways. Of these, one of the simplest to estimate, is the "pay-off" time, that is, the number of years for which the process must operate at estimated costs and realisations before the plant cost has been earned. Alternatively, the total earned in the course of a given plant life may be estimated and, when corrected for the dependence of money value on time expressed, as the present worth of the undertaking.

There are, however, situations in which the use of single simplified expressions for the results of estimation may not be applicable. The discussion of such situations with reference to automatically controlled plants has been referred to above, but an example which is more widely met at present is the estimation of alternative costs incurred by installation of different equipments in central services such as electric power, steam, and gas supplies. On large sites it may be essential to expand these facilities continuously, and it is clear that many courses may be open, all of which would enable rising demands for services to be met with varying margins at varying efficiencies and at costs incurred after varying times.

Such situations do not really admit of any simplified treatment on the usual lines. It is possible to deal with varying annual charges for capital on assumed uniform depreciation rates, but if any meaning is to be infused into deductions from such expressions all the original simplification is lost. It thus seems worth adopting a more comprehensive method, even with some apparent loss of simplicity. Instead of using devices to convert commitments to annual costs, or searching for a single index like "pay-off time", examples have been worked for typical, (though hypothetical), costs on the following basis.

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TABLE 1—*Grouping of Costs and Realisations for Comprehesive Estimation*

Item	Fixed capital (– Installation or removal)	Item	Working capital (Increase or decrease)	Item	"Time dependent costs"	Item	"Production dependent costs"	Item	Realisations
	1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10		1 2 3 4 5 6 7 8 9 10
	$\frac{I_a}{(1+i)^n}$		$\frac{\Delta W}{(1+i)^n}$		$\frac{(R_c)_\alpha (D_\alpha)}{(1+i)^n}$		$\frac{[(R_p)x]_1[(M_p)x]_1 + [(R_p)x]_2[(M_p)x]_2}{(1+i)^n}$		$\frac{[(R_r)A]_1[(M_r)A]_1 + [(R_r)A]_2[(M_r)A]_2}{(1+i)^n}$
An entry is made for each item in each of the five categories for each period: the form of each entry is designated by the specimens set out: the make-up and derivation of the entries is described below.									
Any item installed to get plant going, but not charging with materials—these appear in working capital. I_a = cost of any item at time of installation. Insert as negative if removed, at disposal value (net). Tabular form permits easy entry with renewals and addition of capacity at different periods. Divisor corrects each entry to present value: at year . . . . n with interest rate . . $100i\%$ factor is: $\frac{1}{(1+i)^n}$		Charging of plant with raw materials, product hold up, reserve for wage and periodical payments (may be 25% of annual realisation—or may be pooled—enter proportion). $\pm \Delta W$ = cost of change (rise or fall) at time of occurrence.		Lighting, heating, proportion of site "overheads", "care and maintenance" and other *time-only dependent* running and maintenance charges, including labour, general administration. $(R_c)_\alpha$ = rate per unit time for continuous charge item α. D_α = duration of charges—in time units/annum. Check whether "production dependent" or "time dependent" by "zero production" check: what remains is not "Production-dependent"		Raw materials, energy, services, labour where related to production rate. Rate for increments of above may not be same as for basic quantities: hence 1st, 2nd, etc., rates. $[(R_p)x]_1$ = cost rate per unit of production for "$p - d$" charge x, at rate 1, *i.e.*, for first proportion. $[(R_p)x]_2$ = rate, *etc.*, for second proportion, etc. $[(M_p)x]_1$ = number of units annually produced at cost rate $[(R_p)x]_1$.		Main and secondary products. Rate for increments may not be same as for basic quantities: hence 1st, 2nd, etc., rates. $[(R_r)A]_1$ = realisation rate per unit of production for product A, at rate (i), that is for first proportion. $[(R_r)A]_2$ = realisation rate, etc., for second proportion, etc. $[(M_r)A]_1$ = number of units realising rate $[(R_r)A]_1$, etc.	

Basis of estimation

A period is considered which may be the foreseeable life of the plant, an estimated fraction of the life, or even of arbitrary duration either long or short, starting from the present day and extending some time into the future. This is divided into periods, probably years in the first instance, and for each of these periods the estimated figures for costs incurred and revenues realised are set down. These figures are considered under four headings—capital, costs dependent on time (such as some labour), costs dependent on rate of production (such as raw materials, bonus payments to labour), and realisation (also dependent on rate of production). These headings are shown in Table I. For each costs-item a separate figure is shown for each year. As a result of the method of calculation adopted this represents the present worth of the actual forecast figure, using the formula involving the prevailing rates of interest:[1]

$$P_x = \frac{x}{(1+i)^n}$$

where P_x = present worth of £x payable or due after n years, at rate of interest $100i\%$.

Without entering into any possible controversy on the concept of present worth, it is clear that its use permits the comparison of costs incurred at any time, correcting for the variation of money value with time. The estimates can thus be presented in terms of an estimated outstanding cost or positive balance according to the costs, the earning capacity of the plant, and the time for which it has been in operation.

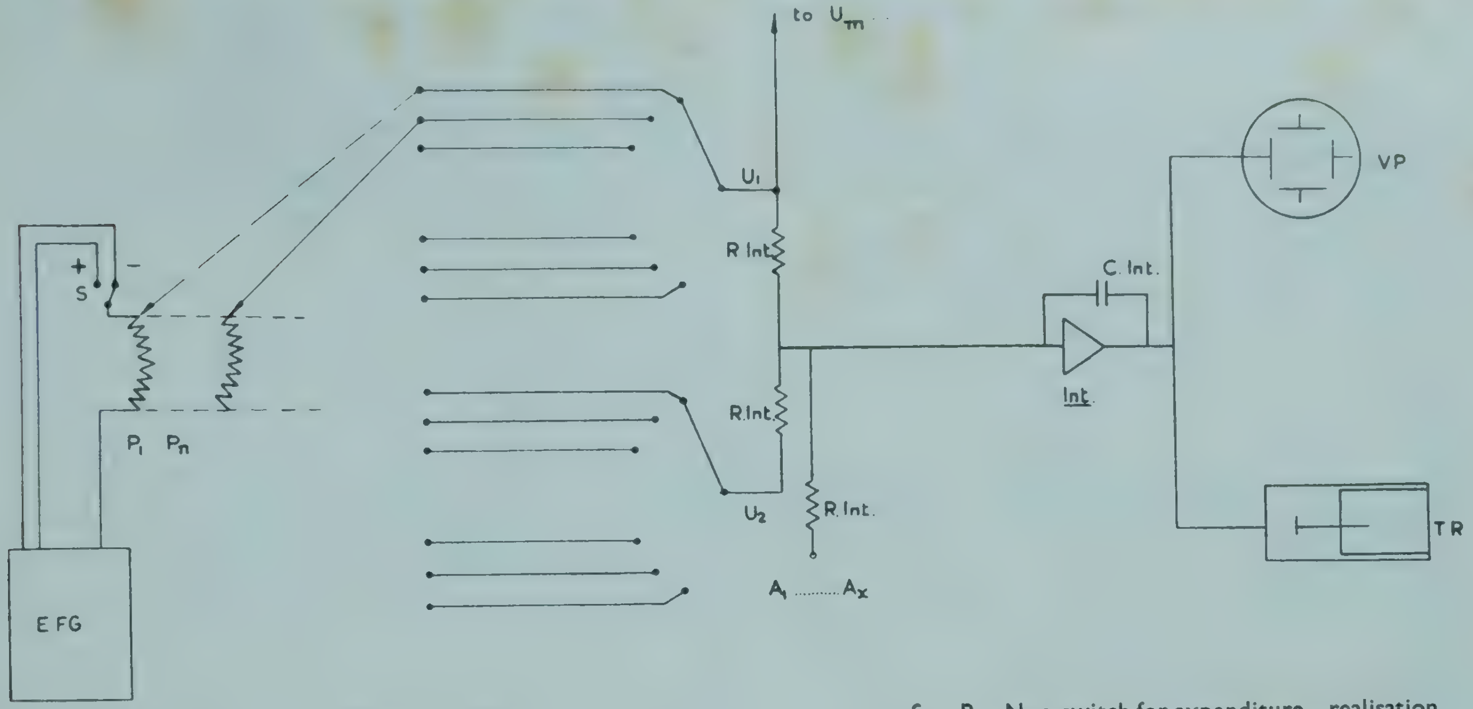
The Estimating Analogue Computer

This method of estimating is comprehensive and its application can be made as thorough and exhaustive as required, but it is clear that the amount of computation, for any situation sufficiently complex to debar simplications, would be excessive without mechanised methods. Moreover, the wide range of information which can be derived from a full treatment is not so readily presented in terms of the single figures appropriate to annual cost estimates.

Choice of computer

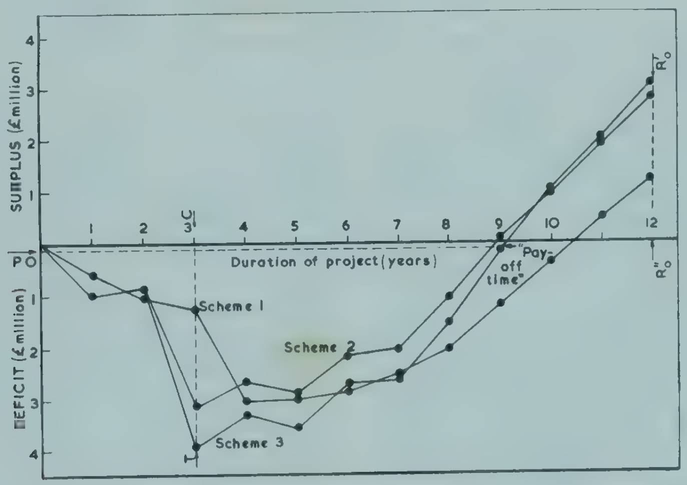
The method can be rendered practicable by adoption of electronic computing, and this may be by either digital or analogue methods. For reasons already given, an analogue computer has been used in this study. The advantages of flexibility, rapidity of programming, and approximate estimation were very real at the introductory stages of this essentially simple application, and the brief investigations needed into actual equipment design were very easily undertaken with standard components. It has been largely expedient, rather than fundamentally significant, that analogue computing has continued to be used. The work is well within the scope of numbers of digital computers.

Two other incidental factors have influenced the choice of computer for the present stage. Firstly, it is clear that the potential accuracy and precision of digital methods cannot be well exploited. Secondly, it has been found more practicable to use a graphical display with analogue machines than with digital machines. Since the clear presentation of results,



Int, C_{int}, R_{int} = Integrating amplifier and components.
U_1, U_2 = "Uniselector" rotary switch banks.
U_m . . . = Additional "Uniselector" rotary switch (as extension).
P_1, . . . P_n = Potentiometers for input setting.
S = Pos-Neg. switch for expenditure—realisation switching.
EFG = Exponential function generator.
A_1 . . . A_x = Point for input not altered with time.
VP = Tube for visual presentation.
TR = Trace recorder for permanent pen record.

Fig. 1.—*Schematic diagram of analogue computer*



Present worth of total capital commitment, that is, amount to be provided at outset just to meet all commitments at time of arising . . . . line TC
"Pay-off time": time required for realisations to amount to total commitment, taking account of prevailing interest rate line PO
Present worth of project at time in life *L* . . . . line $R_0'R_0''$

An *approximate* basis for comparison of rates of return and alternative projects can be derived from TC and R′R″, at a given project life

$$\left(\frac{\text{Return of project 1}}{\text{Return of project 2}}\right) = \frac{(R'R'')_1}{(R'R'')_2} \times \frac{(TC)_2}{(TC)_1}$$

These returns are in addition to interest. To obtain an absolute value for rate of return, including interest, the exponential function generator must be set to give a value for the apparent interest rate so that present worth returns to zero after the lapse of the selected plant life.

Fig. 2.—*Illustration of present worth estimation for three alternative schemes and derivation of comparative investment criteria from plots:*

and derivation of figures from graphical form is advantageous in the present work, there is accordingly further support for the choice of the analogue method.

Operation of computer

The basic sequence of operations which the computer must execute are:

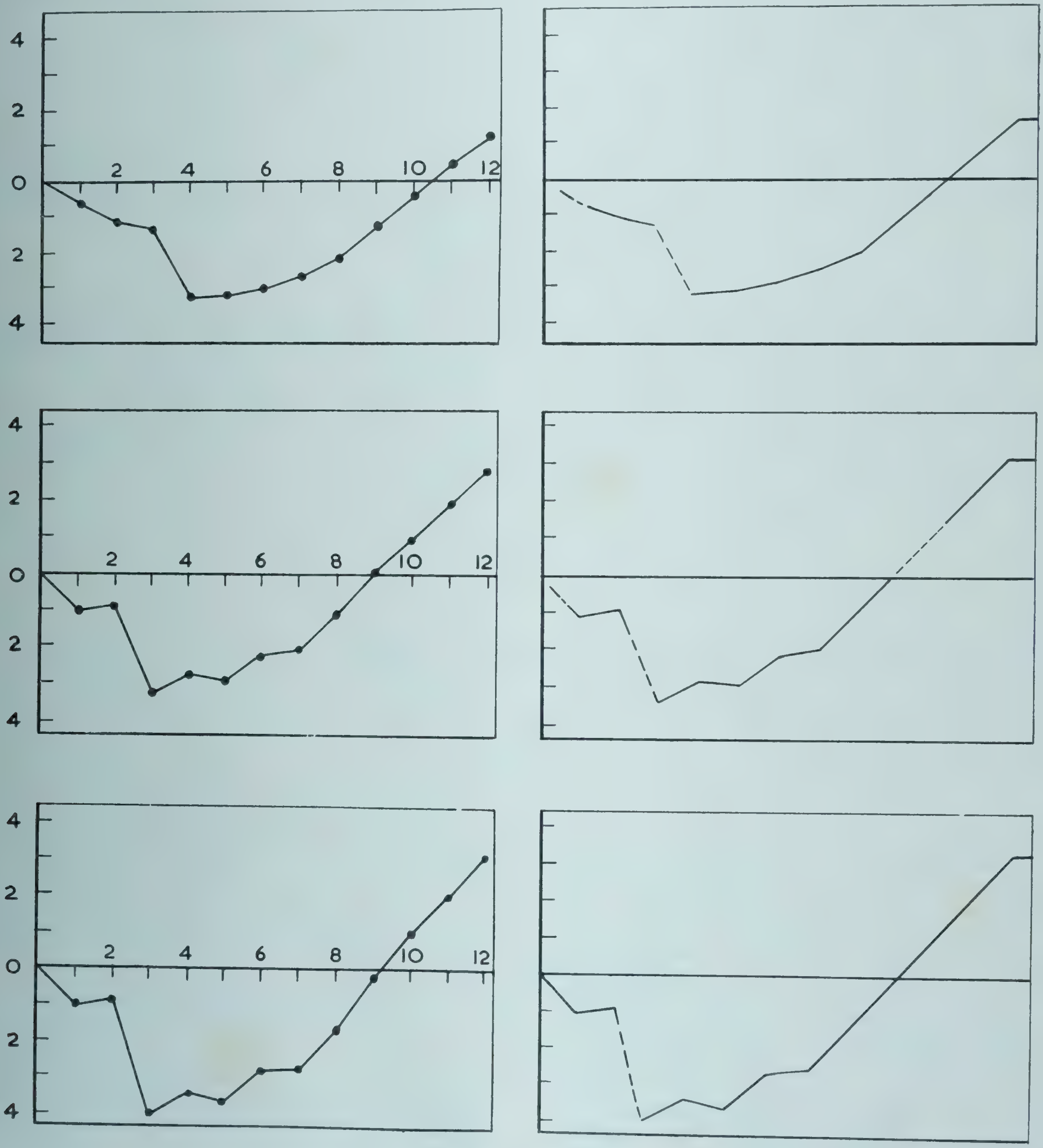
(i) for each annual period, form the sum of all kinds of expenditures and realisations, a net expenditure being expressed negatively.

(ii) correct this sum to its present worth, taking account of the time of its incidence in the future and the prevailing rate of interest.

(iii) sum all the corrected annual sums to give a cumulative total which is potentially amassed for the life of the plant.

The operations can be carried out, respectively, by an adding amplifier, some form of multiplier, and an integrator. Adding and integration present no problem; multiplication is often preferably dealt with by some means other than using a multiplier as such.

The computer operates on a number of voltage signals presented to its input; and the presentation of these voltages is an important operation in itself. For the estimating computation, it is necessary to set the inputs for each annual period and to present these inputs at the correct time during the integration. In the computer set up for first demonstration, a multiple contact director switch assembly—"uni-



Upper pair: Scheme 1.
Middle pair: Scheme 2.
Lower pair: Scheme 3.

Fig. 3.—*Comparison of calculations by long method (left) and display by computer (right)*

selector"—was used in conjunction with any array of potentiometers to set the value of inputs for each year.

Fig. 1 shows the scheme. The requirement for multiplication is met for this specific task by introducing a correct exponential decay into the main operating voltage supplying the inputs. This decay follows the law from which present worth is derived, so that the total inputs are corrected before integrating.

Application

The method can be illustrated by taking an example which, though very considerably simplified, retains realistic features with which the method is adapted to deal. The example taken is the expansion of power generating capacity. It is assumed that a number of choices are open in planning increases in boiler and electrical generation capacity at a large works. These may be between successive installations of small units at a rate matching the rate of increase of demand, or fewer installations of larger, more efficient units which will however spend the early part of their life under-utilised.

Problem

The situation considered may be as follows: a steady increase in annual demand for electrical energy is forecast for one site, say 20 MW/annum. The following alternatives are considered to meet this requirement for eight years' expansion:

(*a*) One 20 MW set initially, thereafter 1 per annum, and an additional boiler after 3 years.

(*b*) One 40 MW set initially, thereafter one every 2 years, and an additional boiler after 2 years.

(*c*) One 40 MW set initially.
One 40 MW high pressure set after 2 years, with higher pressure boiler.
One 40 MW high pressure set after 4 years.

It will also be required to explore the effect of delaying or advancing the time of installation of the sets or boilers possibly by fractions of a year and indeed to maintain the whole programme of investigation flexible.

The example is relatively trivial, but since the probable life of the sets is really indeterminate, and costs for many years after the first eight will enter into the estimation, even this example illustrates the departures from treatment of actual conditions which may be implied by simplified methods.

Basis of estimate

The notes to Table II give the figures assumed for the expenditures on plant and fuel, the realisation from the sale of electrical energy, and the abbreviations used in the table entries. Table II shows the figures used for full calculation of the cash position forecast on the annual basis, as indicated in Table I.

Results

The results of the full calculations are set out in Fig. 2, the comparison of the three alternatives being presented by the three graphs together. In Fig. 3, the traces from the analogue computer records are set out for comparison with the calculations. These were recorded at about 60 in./min. It is evident that the time of recording is very short, and results of estimating for alternatives can be recorded in rapid succession. As indicated in the legend to Fig. 2 several figures can be derived from the graphical presentation which relate this to established indices for estimation, such as capital commitment, rate of return, and "pay-off" time. The potential flexibility of the method of estimation is illustrated by two further plots, one dealing with small changes in the pattern of investment, and the other with changes in realisation from sales under conditions of variable demand.

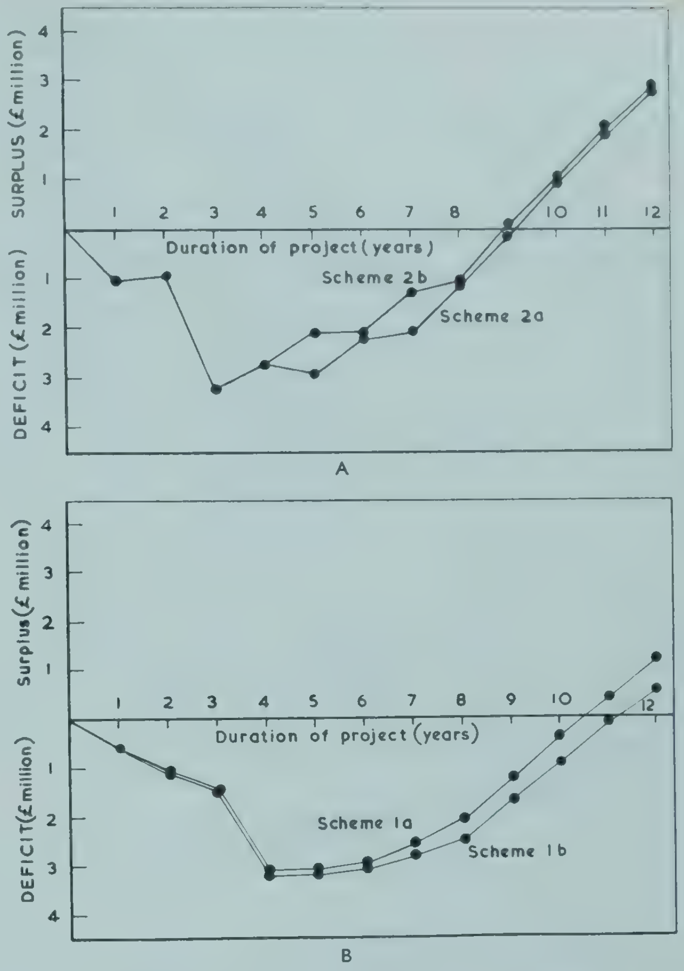


Fig. 4.—*Illustrating comparative assessment of* (A) *alternative capital expenditure rates, and* (B) *alternative realisation rates*

Effect of postponement of investment

It could be assumed that a study of probable demands and fluctuations had been made and that expenditure in periods 5 and 7 in Schemes (ii) and (iii) (see Table II) could be postponed to period 6 and 8. This could be justified on the grounds that risk of insufficient capacity would still be acceptably low.

To obtain new values the computation is either repeated or the present result altered. If new data is inserted from year 5 onward the following changes result for Scheme (ii), and are shown in Fig. 4A.

Annual Period	5	6	7	8	9	10	11	12
Cumulative totals	−1069	−2101	−1244	−1000	+40	+1030	+1973	+2872

Effect of change in realisation rate

Where a power generation plant serves a manufacturing demand and manufacture and power generation are under the same management it may be desired to minimise the effective "profit" from power generation, that is to set the cost to the manufacturing units as consumers so that, taken over the life of the plant, the costs of generation in every form are just met by the charges paid by the manufacturing unit. The effect of changing the rate of realisation can readily

TABLE II.—*Comparison of Three Schemes for Expansion of Generating Capacity on Present Worth Basis*

Scheme 1

Item	Capital Annual period 1	2	3	4	5	6	7	8
Generators Boilers	600	600	600	600 2000	600	600	600	600
Total	600	600	600	2600	600	600	600	600

Item	"Production dependent" costs Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Coal		500	1000	1500	2000	2500	3000	3500	4000	4000	4000	4000
Total		500	1000	1500	2000	2500	3000	3500	4000	4000	4000	4000

Item	Realisations Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Sale of kWh		666	1333	2000	2666	3333	4000	4666	5333	5333	5333	5333

Item	Annual and cumulative totals and present worth. (Interest rate 5%) Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Cap.	600	600	600	2600	600	600	600	600				
"PD"		500	1000	1500	2000	2500	3000	3500	4000	4000	4000	4000
"Outs"	-600	-1100	-1600	-4100	-2600	-3100	-3600	-4100	-4000	-4000	-4000	-4000
"R's"		666	1333	2000	2666	3333	4000	4666	5333	5333	5333	5333
T	-600	-433	-266	-2100	+66	+233	+400	+566	+1333	+1333	+1333	+1333
PWF	1	0·959	0·907	0·864	0·823	0·783	0·746	0·711	0·677	0·645	0·614	0·585
PWT	-600	-410	-240	-1810	+54	+185	+298	+400	+905	+861	+820	+780
CTPWT	-600	1010	-1250	-3060	-3006	-2821	-2523	-2123	-1218	-357	+461	+1241

Scheme 2

Item	Capital Annual period 1	2	3	4	5	6	7	8
Generators Boilers	1000		1000 2000		1000		1000	
Total	1000		3000		1000		1000	

Item	"Production dependent" costs Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Coal		475	950	1425	1900	2375	2850	3325	3800	3800	3800	3800
Total		475	950	1425	1900	2375	2850	3325	3800	3800	3800	3800

Item	Realisations Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Sale of kWh		666	1333	2000	2666	3333	4000	4666	5333	5333	5333	5333

Item	Annual and cumulative totals and present worth. (Interest rate 5%) Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Cap.	1000		3000		1000		1000					
"PD"		475	950	1425	1900	2375	2850	3325	3800	3800	3800	3800
"Outs"	-1000	-475	-3950	-1425	-2900	-2375	-3850	-3325	-3800	-3800	3800	-3800
"R's"		666	1333	2000	2666	3333	4000	4666	5333	5333	5333	5333
T	-1000	+191	-2617	+575	-233	+958	+150	+1341	+1533	+1533	+1533	+1533
PWF	1	0·959	0·907	0·864	0 823	0·783	0·746	0·711	0·677	0·645	0·614	0 585
PWT	-1000	+182	-2370	+489	-193	+750	+112	+955	+1040	+990	+943	+899
CPTWT	-1000	-818	-3188	-2699	-2892	-2142	-2020	-1075	-35	+953	+1898	+2797

Scheme 3

Item	Capital Annual period 1	2	3	4	5	6	7	8
Generators Boilers	1000		1200 2600		1200		1200	
Total	1000		3800		1200		1200	

Item	"Production dependent" costs Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Coal (a)		475	475	425	475	425	475	425	475			
(b)				425		425		425				
(c)				475		475		475				
Total		475	950	1325	1800	2175	2650	3025	3500	3500	3500	3500

Item	Realisations Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Sale of kWh		666	1333	2000	2666	3333	4000	4666	5333	5333	5333	5333

Item	Annual and cumulative totals and present worth. (Interest rate 5%) Annual period 1	2	3	4	5	6	7	8	9	10	11	12
Cap.	1000		3800		1200		1200					
"PD"		475	950	1325	1800	2175	2650	3025	3500	3500	3500	3500
"Outs"	1000	-475	-4750	-1325	-3000	-2175	-3850	-3025	-3500	-3500	-3550	-3500
"R's"		666	1333	2000	2666	3333	4000	4666	5333	5333	5333	5333
T	-1000	+191	-3417	+675	-333	-1158	+150	+1641	+1833	+1833	+1833	+1833
PWF	1	0·959	0·907	0·864	0·823	0·783	0·746	0·711	0·677	0·665	0 614	0·585
PWT	-1000	+182	-3100	+582	-274	+905	+112	+1170	+1245	+1190	+1130	+1080
CTPWT	-1000	+818	-3918	-3336	-3610	-2705	-2593	-1623	-178	+1012	+2042	+3122

NOTES

1. *All figures in table are in units of £000's.*
2. *Assumed costs and other data for schemes proposed in example:*

(i) Costs of plant:

20 MW generating set	£600 000
40 MW generating set	£1 000 000
40 MW high pressure set	£1 200 000
Boiler, installed: capacity as required after increase of 40 MW in load	£2 000 000
Boiler, ditto, high pressure	£2 600 000

(ii) Running costs: Only cost which varies between schemes is cost of coal. At 1·0 lb coal/kWh, 20 MW increase demands 20 000 lb coal in addition per hour. At 8000 h/year total increase 160 000 000 lb/year: If coal price is ·75 pence/lb annual coal cost for 20 MW = $\frac{£120\,000\,000}{240}$ £500 000

It is assumed that efficiency of sets differs for different sizes and pressures:

for 20 MW set, coal at 1·0 lb/kWh, ann. cost = £500 000
40 MW set, coal at ·95 lb/kWh, ann. cost = £475 000
40 MW H.P. set,
coal at ·85 lb/kWh, ann. cost = £425 000

Data on coal consumption for scheme 3 is treated specially. On installation of a high-pressure boiler and set, the full load for the new set is imposed at once, resulting in consumption of coal at the lower rate corresponding to higher efficiency not only for the 20 MW increase over the year of installation, (See entry "Coal (a)" in scheme 3 production dependent cost) but also in consumption of coal (see entry "Coal (b)") to produce energy at low rate in lieu of coal consumed at higher rate (see entry "Coal (c)") for the same duty in one of the older boilers. As the load increases more coal is consumed at the higher rate and the whole generating capacity in the installation is fully loaded. During years 4, 6, 8, net increase of coal consumption is less than indicated simply by an expansion at the new rate; in each of the later years 5, 7, 9, expansion continues but with coal consumption once again at the old rate—new capacity is in fact loaded as heavily as possible as soon as possible.

(iii) Realisation: All energy is assumed sold at a cost representing coal cost + 33% for the least efficient scheme, *i.e.* annual increase = £666 000—a gross simplification acceptable solely as illustrative in the present context.

3. *Abbreviations for Item headings, annual and cumulative totals:*

Cap = capital expenditure, transferred from "Capital total" item
"PD" = production dependent (*i.e.* coal cost in this example).
"Outs" = total outgoings.
"R's" = realisations.
T = total change in cash position for the annual period (*i.e.* realisations for year – payments for year).
PWF = present worth factor – value of 1 unit cash "at this day", incurred or received at time entered in table of future annual periods.
PWT = total × PWF, *i.e.* present worth of annual total.
CTPWT = cumulative total of present worth of annual totals, *i.e.* entry in column of annual period "*n*" = totals of all present worths of annual net expenditure or gain totals "from this day" to year "*n*"

TABLE III.—*Effect of Changing Unit Cost of Electricity*

	Annual period											
	1	2	3	4	5	6	7	8	9	10	11	12
Unit cost 133% . .	−600	−1010	−1250	−3060	−3006	−2821	−2523	−2123	−1218	−357	+461	+1241
Unit cost 130% . .	−600	−1025	−1294	−3145	−3144	−3022	−2775	−2475	−1656	−878	−138	+597

be estimated for the whole life of the plant. Table III sets out the effect of changing the unit cost charged for electrical energy from 133% to 130% of the coal cost. The effect of this small change is to delay the "pay-off time"; this becomes about 8% longer for the 2·3% decrease of realisation, all illustrated in Fig. 4B.

Development for Further Application

It is outside the scope of this paper to discuss the significcance of differences in the results of estimates relating to the alternatives discussed. This can only be done in specific situations. The development of the computer provides a tool by which it may be possible to make the best use of available data in making decisions about real situations, without being bound by the assumptions which may underlie less comprehensive estimating procedures. It is probable that quite extensive application and experience of the method will be required to assess its value and discern any points on which further development would be advantageous.

The work described represents only an interim stage in possible developments. As it is based on a comprehensive use of all relevant data and can thus be considered fundamentally sound as a starting point, a wide range of cost situations could ultimately be dealt with. These might include, for example, simultaneous treatment of costs at steady, or steadily changing rates, and some costs incurred at arbitrary intervals, the former being represented by a constant voltage or a voltage changing at a defined rate introduced directly to the adding amplifier through its own resistance network, as an $A_1 - A_\alpha$, in Fig. 1. This is, of course, only one of many variants of investment and expenditure pattern all of which can, in principle, be explored without serious problems of equipment. Other detail changes in input circuits would permit application to problems in which a large number of inputs have to be used, possibly selectively, to take account of a range of variable and conflicting factors influencing operation and costs.

One more general aspect of estimation method must however be referred to. This is the element of uncertainty met, albeit to a widely varying extent, whenever prediction based on many technical and commercial factors is undertaken.

All the situations and all the examples dealt with so far, have been considered, explicitly or implicitly, as if the figures which could be entered into an economic assessment, and all the results obtained were definite, deterministic figures. In point of fact all such figures are subject to errors. These errors arise either in the technical stages of deriving the figures, or in the uncertainty of figures such as those for estimates of production rates at the end of the life of a plant, which may be in 10 years or more.

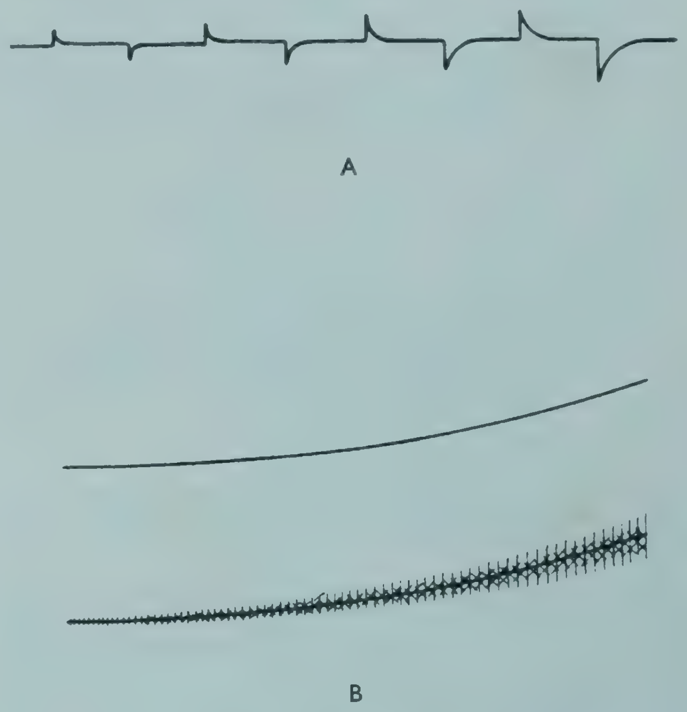


Fig. 5.—*Uncertainty in estimated figures and results:*

Suggestion of a possible method for display:

(A) *fluctuation of increasing amplitude to superpose on input voltages to represent uncertainty increasing with time.*

(B) *indication of general effect on visual display; lower figure shows spreading of line to a diffuse band which must be achieved.*

The representation of such margins of error can be achieved in principle by modulation of the voltages presented to the computer, so that a voltage representing a quantity Q is presented not as f(Q) but f($Q \pm \sigma$), where σ = some representation of error, either precisely derived, such as the standard deviation of a mean value derived from experiment, or an expression of error in subjective judgement as, for instance, of market prospects in the distant future. If a voltage of waveform shown in Fig. 5A is used as the main supply to the input potentiometers instead of the steady voltage and the amplitude of the alternating component is increased as a function of time, the form of trace appearing as a display tube presentation is changed from single valued as in Fig. 5B to one representing fluctuating values, the amplitude of fluctuation increasing as time passes. For the many instances in which it may be considered inadequate to assume only random fluctuations, a more intensive study of the origins of uncertainty must be pursued, and the consequences of a multiplicity of contingencies explored separately.

Reference

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SOME PROBLEMS REGARDING THE DESIGN OF ELECTRONIC ANALOGUE COMPUTERS FOR MANUFACTURING PROCESS CONTROL

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SUMMARY

The recent trend towards the use of computers for the control of complete processes is discussed with particular reference to control analysers.

Two methods of application are considered—"scanning" and "minimisation".

The effect of these methods on the design of computers is assessed, and the outline of a recently built computer is presented.

Introduction

During recent years there has been a clear trend towards the use of mathematical machines for manufacturing process control.

This trend is conditioned on the one hand by the higher speeds adopted in modern manufacturing processes and by the higher demands on the controls in respect of accuracy, quality of products, efficiency and so on: in other words, by conditions and requirements where, in many cases, the human operator may not be able to control the process at the requisite speed or efficiency. On the other hand, this trend may be explained by the tremendous growth of the computer manufacturing industry and by the extension of the capabilities of the machines.

Types of Computer

The mathematical machines which may be used for the automatic control of manufacturing process may be classified in three main categories.

(i) Command mathematical machines for programme control. These machines operate according to preset programmes and are not susceptible to feedback effects from the results of the operations performed. These machines have found wide application for the control of machine tool operations such as milling, drilling and boring.

(ii) Computers and computing relays which form part of the main circuit of an automatic control system. The number of such units may be large and they may perform various functions, their main task being the processing of input data to give the different forms of presentation (correlation functions, extrapolation relations, algebraic relations etc.) needed for automatic control.

(iii) Mathematical machines such as control analysers which are designed to give the optimum control of a process according to the best criteria. The scope of such machines is practically unlimited. They may be employed for the control of power systems to give the most profitable and efficient results; for the control of dynamic systems to follow a preset programme taking into account any disturbing factors; for process control in the iron and steel industry to obtain high quality on a commercially profitable basis; for plant management based on statistical grounds; and so on. The practical design of the computer and its association with the given process may be realised in different ways.

Application of Mathematical Analysers

One of the modes of application of mathematical analysers is by way of parallel process analysis. In this case the analyser is used for the repetitive solving of a set of equations representing the behaviour of the system. The solution is made for a period of time from the moment of measurement of some required characteristic up to a certain time at which either the whole process or one of its stages is terminated. In other words, the computer repetitively solves the problem of determining the given process characteristics on an advanced time scale.

At each solution of the problem, the computer "scans" the different courses of action and selects the best for the given conditions. The controlling parameters to ensure the optimum process conditions are then injected into the process control circuit in some form or other. Thus the computer, operating with the process, is able to correct the process in accordance with the results of the analysis on an advance time scale.

In addition, the computer may be used to exercise "judgement" in such a case.

Mathematical Formulation of the Programme

The process is represented by a set of n differential equations:

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \mathrm{F}_i\,(x_1;\ x_2\ \ldots\ x_n;\ a_{ij};t) \quad . \quad . \quad . \qquad (1)$$

$$i,j = 1\ \ldots\ n$$

The set of conditions given by equation (2) determines the optimum character of the process.

$$\begin{aligned} \mathrm{H}_i[x_1(t_1);\ x_2(t_1);\ \ldots\ (x_n t_1) \\ x_1(t_2);\ (x_2 t_2);\ \ldots\ (x_n t_2) \\ x_1;\ x_2;\ \ldots\ x_n;\ t] = 0 \quad . \qquad (2) \\ i = 1\ \ldots\ n \end{aligned}$$

It is assumed that the character and structure of the relation F_i (which includes certain coefficients a_{ij} as well as the character and structure of the conditions H$_i$) are not dependent on the variables x_i.

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It is required to select such values of some coefficients a_{if} and initial values of some magnitudes $x_i(t_1)$ at which the relations $x_i(t)$ will satisfy the sets of equations (1) and (2).

Types of Computer

It is now a general practice to divide computers into two groups: machines of discrete action (digital computers) and machines of continuous action (analogue computers). Machines in the first group have unlimited possibilities in regard to the accuracy obtainable, but as a rule they are slow-acting and of complicated design.

Continuous-action machines, giving a relatively low accuracy, are simple in design and, in principle, there are no limits to their speed of operation.

The choice of the type of machine to use is difficult, and should only be made on the basis of definite technical requirements. The choice should take account of the required and realisable accuracy, speed, reliability, and simplicity. It should also take into account the type of problem to be solved and other factors. It should be noted that the problem of selecting the particular type of computer for the control of a process which has a high reaction rate can only be solved by bearing in mind that an analogue computer might be adequate for the purpose.

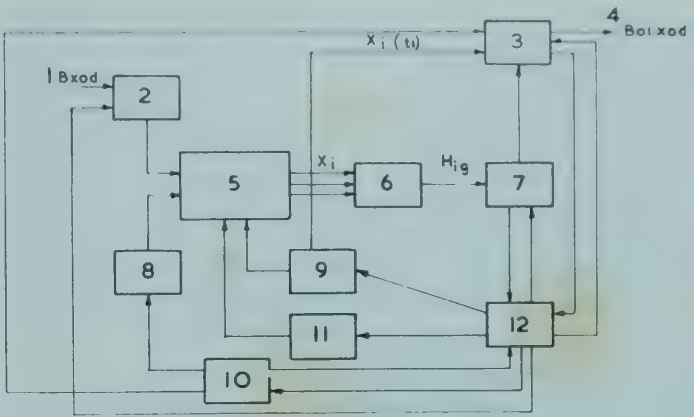
The Method of Scanning

Among the methods used for solving the problem posed by the sets of equations (1) and (2) the most general and practically feasible is the method of scanning. In this method an analogue computer is used to consider a large number of alternative procedures at different values of the variables and to choose solutions satisfying the sets of equations (1) and (2). It goes without saying that the most practical and efficient method of scanning is one in which a quick-acting analogue computer is used which enables the scanning of a large number of alternative solutions to be made in a short interval of time. It should be noted also that the possibility of quickly obtaining alternative solutions brings a new sense into the acceleration of the electric computer operation.

Use of Analogue Computer

The automatic analogue computer, the block diagram of which is presented in Fig. 1, comprises the analogue machine proper; the computer unit; the circuit for the control of logical operations; the circuit for presetting initial conditions and modifying coefficients; the system of presetting time intervals; the system of visual control; and the input and output data system. The analogue machine ensures electrical simulation of a given set of differential equations representing the behaviour of the system. The discrepancy between the obtained solution and the preset solution is given by the value μ, which for the preset solution should be below a certain small value ε; the determination of the value μ for each solution is made by means of the computing device.

In the general case, the logical operations control circuit appraises the effect of a change in the variables to be determined (initial conditions, coefficients) by comparing the values μ, obtained at the preceding and following solutions, and controls the change of these variables. The operation of the whole unit is synchronised by a time-interval presetting system.



1 = Input.
2 = Data input system.
3 = Input and output data system.
4 = Output.
5 = Analogue machine.
6 = Computer unit.
7 = System for determining μ and $\Delta\mu$.
8 = Analogue machine operation control.
9 = Initial conditions presetting system.
10 = Time intervals presetting system.
11 = Coefficients change system.
12 = Logical operations control circuit.

Fig. 1.—*Block-diagram of the automatic electrical analogue computer*

In the design of the analogue computer are combined both elements of the continuous and impulse technique of mathematical machines. The electrical analogue proper and the computer are circuits of continuous action, while some of the control circuits—among them the logical operation control circuit—are systems designed on the basis of impulse-technique elements.

When designing an automatic high-speed electric analogue computer for the analysis of sets of ordinary non-linear differential equations there arise a number of problems to be solved, these being connected not only with basic principles determining the choice of a particular alternative, but also with purely constructional questions.

Major questions which arise in building an electric analogue computer are: the choice of an adequate method for determining the values of variables which satisfy best preset conditions; the design of the logical operations control circuit; the choice of the frequency of repetitive solutions; and the design of the circuit of the electrical analogue computer and of its elements, to ensure operation of the whole system on a high-speed scale.

Scanning and Minimisation

Among the existing methods of seeking solutions the most general and most convenient for practical realisation in the form of machines are the methods of "scanning" and "minimisation" which differ in the sequence in which the variables are modified.

When using the scanning method the change in the variables is brought about according to a preset programme independently of the character of the solution obtained after the preceding change of the variables; in the minimisation method the change in variables is conditioned by the character of the preceding solution.

Fig. 2 illustrates the sequence in which changes are brought about with variables when using the scanning method for obtaining the solution of a set of differential equations. The task is to find out the values of two variables—*i.e.*, initial

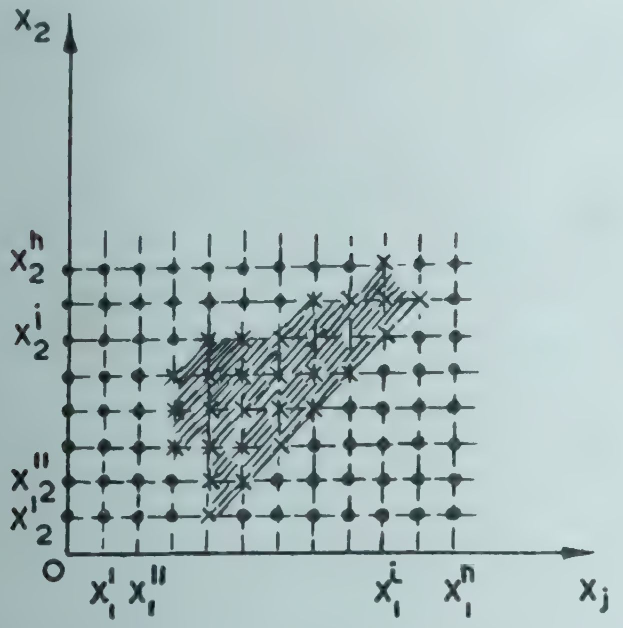


Fig. 2.—*Illustration of the "scanning method", involving two variables. Solutions satisfying the condition $\mu < \varepsilon$ are marked with crosses*

conditions for x_1 and x_2. In this case the problem is first solved by adopting a constant $x_{1(0)}$ for several values of $x_{2(0)}$; then a new value of $x_{1(0)}$ is adopted for the same values of $x_{2(0)}$ and so on. In cases when at certain combinations of $x_{1(0)}$ and $x_{2(0)}$ the value of μ is less than ε it may be accepted that the solution of the problem has been achieved.

In cases where three or more variables are involved, the programme of their modification becomes more complicated and the number of solutions greatly increases.

The scanning method has the inconvenience of the value of ε being dependent on the step-change of the variable adopted by the programme. When this step-change is large, the value of ε should be large enough to avoid skipping a solution. As a result, the solution of the problem is obtained in the form of zones of value of the variables values and not as some definite magnitudes of the variables.

Notwithstanding this, the scanning method is particularly suitable in cases where the region of variable modification in which the desired solution falls is unknown, and in cases where the solution is not single valued. However, if the number of unknowns is increased, the scanning of alternative procedures will become time-consuming. In such cases, and also to eliminate the effect of the inter-relation between the values of ε and the step-change of the variables, the scanning method may be made more complex by making the changes in the variables partly dependent upon previous solutions.

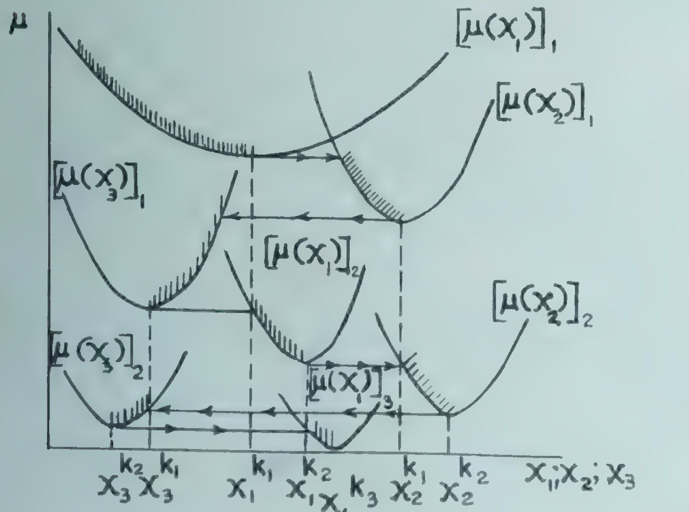


Fig. 3.—*Illustration of the minimisation method. The hatched portions indicate regions of μ modification on curves $\mu(x)$*

Method of minimisation

In the method of "minimisation", the change in variables is brought about by modifying the function of μ so that any change in the variables brings about a reduction of μ.

For example, when seeking the solution of a problem represented by three variables, one of the variables, x_1 is modified until the difference represented by equation (3) becomes negative.

$$\Delta\mu_{j+1} = \mu_{j+1} - \mu_j \quad . \quad . \quad . \quad (3)$$

(j is the number of the trial after the trial at which modification of the unknown started.)

This is illustrated in Fig. 3. This step gives a reduction in the absolute value of μ. At this point $x_1 = x^{k_1}$, say. Subsequently the value of x^{k_1} is maintained unchanged and x_2 is changed: the change in μ will follow the law $[\mu(x_2)]_1$, and x_2 will be modified until $\Delta\mu$ again becomes negative. Now with $x_1{}^{k_1}$ and $x_2{}^{k_1}$ remaining constant, the third variable is modified until a negative difference is obtained which, in Fig. 3 corresponds to the values $x_1{}^{k_3}$, $x_2{}^{k_2}$ and $x_3{}^{k_2}$ with $\mu = 0$.

In spite of its apparent simplicity, the minimisation method is limited by a number of factors. The method is not a new one. It is used for the solution of sets of linear algebraic equations, which have been shown to give convergence by this method.

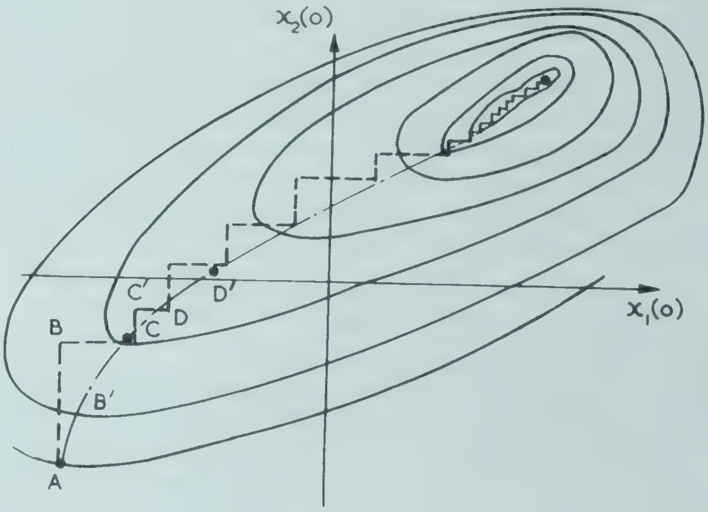


Fig. 4.—*Illustration of the maximum-gradient method*

Such demonstrations are at present lacking when the minimisation method is used for problems requiring the solution for preset conditions of sets of non-linear differential equations. Therefore the class of problems for which solutions may be sought by this method can only be determined by simultaneously working out the design of analogue computer to be used in each case. Difficulties are encountered when scaling, as in the practical realization of the minimisation method the determination of $\Delta\mu$ acquires great importance, and may be achieved only on condition that the value of μ is always below the maximum possible output voltage of the amplifier.

Design of Control Circuit

When designing the logical operation control circuit it is most important to fix the sequence in which the sought-for variables are changed. Two alternative ways may be used here, *i.e.* simultaneous modification of several variables or modification of each of the variables in consecutive order. In the above-mentioned procedures of scanning and minimisation each of the variables is subject to changes in consecutive order. However, in some circumstances and for a certain range of problems, the simultaneous modification of several variables may greatly speed-up finding out the preset solution, provided the direction of the modification has been correctly chosen.

Simultaneous modification

As an example of procedures employing the simultaneous modification of several variables, the method of maximum gradient (or high-speed approximation), illustrated in Fig. 4 may be cited. This figure presents the curves of equal values of μ versus the values of variables $x_{1(0)}$ and $x_{2(0)}$. It is required to determined, the values $x_{1(0)}$ and $x_{2(0)}$ at which $\mu = 0$. If the change in variables starts from point A, and the method of minimisation is employed, the shift of point A to the point $\mu = 0$ will take place along the curve ABCD. If the method of maximum gradient is adopted, the path A B′ C′ D′ of point A to $\mu = 0$, which must be in direction of maximum increment of value of μ, will be shorter. At the same time the logical operation control circuit will accomplish two functions: scanning to determine the best direction for simultaneous modification of both variables, and movement in the selected direction to give, at that time values close enough to the pre-selected values in relation to $\Delta\mu$.

The design of the logical operation control circuit is totally conditioned by the automatic operation of method of scanning adopted, the type and designation of the electrical analogue computer, and also by the stage of development of the scanning process. The capacities of the circuit designer are of great importance.

Alternative designs

For the logical operations control circuit two designs may be used. The first is essentially based on the same principles as the continuous-action control systems, while the second is analogous to control relay systems. In the first case the control circuit receives information not only on values μ_{j+1} and μ_j but also on the relation between the magnitude of the increment of the variable Δx and the magnitude of change in the value of μ. The magnitude of the variable increment is an output value of the logical operation control circuit and will be proportional to a certain coefficient of relation between $\Delta\mu$ and Δx. To ensure this proportionality the problem of variable modification is solved in several stages, *viz*, the first step is to determine the "trial solution" of the relation coefficient for $\Delta\mu$ and Δx; the required value Δx is next determined and, finally, the "useful solution" with the new value of Δx. This method is likely to be expedient for solving a number of problems, among which are those requiring statistical calculations, but it involves "superfluous" solutions and necessitates the presence of memories and computing elements in the control circuit.

In the second case the unique information ensuring the correct operation of the whole system are data on the sign of $\Delta\mu$, the logical operations control circuit may be designed to make maximum use of impulse technique.

The MH-II Analogue Computer

During the first stage of development of analysing mathematical machines, the Department of Electrical Analogue Computers of the Institute for Scientific Research in Computing Machinery designed the Model MH-II Electrical Analogue Computer with automatic scanning of solution. This machine should be regarded in the first instance as a laboratory unit; in making use of it, it will be possible to determine which methods are the most adequate for obtaining the optimum solution and the greatest operation range in designing special control analogue computer or predictor-counsellors.

The electric simulating circuit of the MH-II machine ensures electric simulation of sets of nonlinear differential equation of up to the 9th order, it contains up to 5 multiplications, 3–6 nonlinear relations and up to 10–12 sums. The machine is designed for unique-solution operational routine and also for repetitive solution at a frequency up to 100 cycles per second. The electronic functional blocks in the electrical simulating circuit of the MH-II machine have a greater frequency response than the blocks of standard electrical analogue computers.

The control circuit of the variables scanning operation provides for automatic modification of up to 6 variables, which may include up to 6 values of initial condition voltages and up to 3 values of constant coefficients.

Modification of the variables to be determined and determination of the optimum solution may be performed manually at the conclusion of any of the above-mentioned methods, semi-automatically when using the scanning method, and automatically when adopting the scanning or minimisation method.

Thanks to the great variety of automatic processes for scanning the preset solution, the exploitation features of the MH-II machine are very high. It presents great possibilities for the analysis and selection of the best methods of solution in a great variety of particular cases.

At all operational routines the control over the correct selection of sought parameters is according to values of μ and $\Delta\mu$, while the results are represented with the aid of indicating instruments and a special cathode-ray indicator.

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THE USE OF ELECTRIC ANALOGUE COMPUTERS FOR THE INVESTIGATION OF THE COLD ROLLING PROCESS

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SUMMARY

The paper describes an electric analogue computer developed by the Institute for Scientific Research in Computing Machinery for the investigation of aspects of the cold rolling process.

Introduction

Modern multi-stand cold rolling mills are very intricate aggregates both from the point of view of automatic control and of technology. Each of the stands of a cold mill is a self-contained mechanism where reduction of the strip-thickness is performed. The stand is provided with two systems for automatic control—one for the control of the main drive (rolling speed) and the other for controlling the screw-down mechanism (the roll-gap). At the same time all the stands are coupled by the strip being handled so that any change in the operating conditions in one of the stands provokes changes in all the other stands. Thus if for some cause or other there is a change in one of the stands (*e.g.* a decrease in the delivered gauge) a corresponding change will occur in the adjacent stand. As a result the torque of the main drive of the following stand will be reduced, whilst its speed will increase and this in turn will influence the behaviour of the next stand, and so on. Thus the multi-stand rolling mill should be considered as an inter-related system and in the study of the behaviour of such a system, not only should the technological characteristics and features of the individual stands (the rolling process) be taken into consideration, but also the power unit (the main-drive system and the screw-downs with their control devices).

Many problems arise in the design of new rolling mills and their solution is liable to influence greatly the exploitation features and technical characteristics of the future plant.

Bearing in mind the intricate design of the aggregates, the solution of the problems involved, even with the aid of calculating machines, presents great—and even insurmountable—difficulties. This is because the amount of computing is so large that several months of computer operation would be required for the calculation of only a few alternative solutions. It is quite clear that such a method is not feasible in designing new rolling mills. At the same time the increase in rolling speed necessitates immediate and careful investigations into the design of the rolling mill as this will ensure the correct design of the mill at a high technical level and also an adequate design of the drive and the automatic control.

With a view to this, the Department of Electric Analogue Computers at the Institute for Scientific Research in Computing Machinery developed, in 1956, a special electric analogue computer for studying the process of cold-rolling in multi-stand mills.

Choice of Analogue Computer

The development of an analogue computer rather than a digital computer for this purpose may be explained by the following considerations:

(*a*) The electric analogue computer is a considerably simpler and cheaper machine than the digital computer.

(*b*) In investigations carried out on rolling mills with the aid of computing machines it is desirable to be able to connect the computer with sections of the operating plant, *e.g.* the regulator (control unit) with the stand simulator (model). This would be difficult with a digital computer because the transfer equipment is rather intricate and the operation of a digital computer on a real time scale presents a great many difficulties.

(*c*) The accuracy of the results obtained in the course of investigations carried out with electric analogue computers is precise enough to satisfy engineering requirements.

(*d*) The electric analogue computer does not require special programming and allows the influence of various parameters to be determined in a simple form readily understandable by the technologist.

Description of Computer

For economy, the electric analogue computer only completely simulates three stands as where the rolling mill comprises more than three stands the simulation of the other stands, if required, may be effected in succession, beginning for example, with the first three stands and then proceeding with the simulation of the third, fourth, and fifth stands, and so on.

The electric analogue computer for the type MH-12 rolling mill comprises three unitype sections, each section simulating a single mill stand with its related equipment.

Each section consists of:

(1) Ten integrating d.c. amplifiers.
(2) Twenty summing d.c. amplifiers.
(3) Twelve d.c. inverting amplifiers (for changing the sign of input data).

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4. Four special units reproducing the relationship for the inertia member

$$\frac{U_{\text{output}}(P)}{U_{\text{input}}\ (P)} = K_{(p)} = \frac{K}{1 + pT}.$$

5. Two units reproducing the relationship for the forcing member, of the following form:

$$\frac{U_{\text{output}}(P)}{U_{\text{input}}\ (P)} = K_{(p)} = \frac{1 + pT_1}{1 + pT_2}.$$

6. Two multiplying units.

7. Two units for the execution of the relationship $U_{\text{input}}\, \mathrm{f}(t)$. (Variable coefficients units.)

8. One unit for reproducing the function with delayed argument of the form $U_{\text{output}} = U_{\text{input}}\,(t - \tau)$; where τ is a function of another variable, for example of the angular speed (expressed as tension).

9. Two universal nonlinear units for reproducing the following relationship:

$$U_{\text{output}} = U_{\text{output}_0} + \mathrm{d}U_{\text{input}} + \sum_{i=1}^{n} (\beta_i\, U_{\text{input}\ i} - U_{\text{input}\ i\ \text{initial}})$$
$$i = 1, 2, 3, \ldots 10.$$

Moreover, in the section are included special nonlinearities, which are pertinent to the rolling mill automatic control system, *viz.*: two nonlinear dead zone and saturation units; two nonlinear backlash units; and two nonlinear units for reproducing the hyperbolic function and the saturation curve.

The electric analogue computer is provided with two special cathode-ray oscilloscopes for the observation of the modification of variables in the course of the investigations. By means of these instruments the behaviour of variables in solving the problems may be easily watched on the afterglow screen of the cathode-ray tube.

All non-linear units with the exception of the multiplying units are built with transistor or vacuum diodes and allow the approximation of preset non-linear functions by means of constant lengths (or steps). In the design of the multiplying units the non-linear relations (quadratics) of thyrites are used. Multiplication is effected by the equation:

$$4xy = (x + y)^2 - (x - y)^2$$

To reduce costs, common d.c. amplifiers having deep negative feedbacks and no stabilisation of zero drift are used.

The delay unit is designed on the electro-mechanical principle with "memory" condensators. The modification of the delay value (τ) is achieved by means of a servo-system.

Linear approximation is used to increase the accuracy of the operation of the unit. This enables a function with a delayed argument to be obtained which is approximated by a total of 20 linear lengths rather than by constant steps.

The electric analogue computer may be used to operate either in one section or in all the three sections simultaneously. The computer may be successfully used not only for the solution of problems related to the simulating of the rolling mill, but also for the investigation of other problems connected with the development of automatic control systems.

While designing the electric analogue computer for the rolling mills two kinds of difficulties were experienced. The first and main difficulty was in obtaining mathematical relations representing the processes taking place in the rolling mill stands, its drives, and the automatic control systems. The second difficulty was encountered in developing special units (delay units, special nonlinear units, and others), and in solving problems related to the design of an analogue computer circuit, capable of satisfying the requirements of rolling most efficiently.

As a result of the research work carried out in the Central Research Institute of Engineering Technology equations were developed describing the drives of separate stands and control systems and also technological equations representing the process of metal reduction.

A motor generator unit comprising two motor amplifiers one of which being a compensated amplifier (the voltage applied to this amplifier is proportional to the current of the main motor) and the other, a differential amplifier (the voltage applied to this amplifier is equal to the error voltage, *i.e.*, the difference between the compensated amplifier voltage and the preset voltage, was adopted as the main drive. The latter controls the generator exciting voltage. Transformers with flexible feedbacks are provided in the circuits of both amplifiers.

The main drive system in conjunction with the control system is simulated in the analogue computer by a set of fourteen differential nonlinear equations, including equations of finite relations.

The operation of the stand proper (*i.e.* the technological relations) are represented in the analogue computer by a set of six non-linear equations. To simplify the system and to increase the accuracy of the analogue computer, the technological equations are not formed for the values proper but for their deviations. The description of the set of equations for the screwdowns which may also be simulated in the analogue computer are given in Reference 1.

A preliminary simulation in a separate circuit was made for the set of equations representing the main drives and the technological processes taking place in the stand and the interstand connections. Investigations carried out with this circuit showed that the adopted set of equations gives a reasonable representation of the processes occurring in the rolling mill.

Special units which are not available in currently manufactured analogue computers were also developed during these researches.

Use of the Computer

Investigations were subsequently carried out to determine the main parameters of the system which exercise considerable influence on the behaviour of the rolling mill. As a result of solving some particular problems it was shown that the equations for the rolling mill main drive may be approximately replaced by the delay term plus the inertia term of the first order.

In particular, such a substitution allows the determination of the main features and structure of the computing device which is a component of the control system and is designed to ensure a better control of the gauge of the strip.

The investigations carried out with the electric analogue computor have demonstrated that by using this device it is possible to reduce the deviation of strip gauge previously encountered by several times.

Moreover, operational means for starting and stopping the mill—which were considered as most typical from the point of view of transient operational conditions—were also studied by means of the analogue computer. At the present time the computer is being used for carrying out further investigations in the field of rolling processes.

Acknowledgment

Acknowledgment is made to Prof. A. A. Feldbaum, who took a leading part in the work reported here.

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USE OF ELECTRICAL ANALOGUES IN THE STUDY OF THE DYNAMIC BEHAVIOUR AND CONTROL OF DISTILLATION COLUMNS

By J. E. RIJNSDORP* and A. MAARLEVELD*

SUMMARY

The authors start from an existing theory describing the dynamic behaviour of distillation columns which has been checked by experiments on columns of different size.

Electrical analogues were constructed on the basis of this theory and used in combination with an electronic analogue computer to study column control. This simulation procedure gives good insight in the phenomena involved, and produces results in a much shorter time than other methods.

The most general analogue takes into account changes in pressure, tray hold-up, and pressure-drop across the trays. It is based on binary distillation and on a linearization of column behaviour. The greater part of the analogue consists of passive elements: resistors, capacitors, and some coils. About 25 operational amplifiers are also used.

The paper discusses the theory of dynamic behaviour and the construction of the analogue. Results of measurements are dealt with for the case of a simple analogue where the pressure was assumed to be constant and the pressure-drop across the trays was neglected. Finally, experiments on the most general analogue show that the coupling between temperature and pressure control can have a bad effect on control quality.

Introduction

There are several theoretical studies of the dynamics of continuous distillation columns in the literature.[1–5] In most of these pressure and tray hold-up are assumed to be constant. The dynamic behaviour of the vapour and the liquid flow is consequently the same on all trays in the stripping and in the rectifying section. As a result, these studies mainly give the low frequency part of the dynamics. This is useful in describing the responses to variations in feed composition or to slow variations in other external variables and in finding the approach towards equilibrium during start-up.

On the other hand, the high frequency part of the dynamics is important for the study of column control, because the resonance zones of pressure and temperature control are often in this region. It goes without saying that pressure control can only be analysed if the assumption of constant pressure is dropped. The variations in tray hold-up should also be considered, because they can have a large influence on temperature control.

With these considerations in mind Voetter and Houtappel[6] have developed a theory of dynamic behaviour of continuous distillation columns. Frequently dropping of some assumptions necessitates the making of others to keep the theory manageable. An important new assumption made by Voetter is that of linearity. Strictly speaking, the theory therefore applies only to infinitesimal variations in the column variables, although experiments have shown that it is still reasonably accurate for fairly large variations.

The main part of the theory deals with the fractional distillation of binary mixtures. The approach for the case of ternary mixtures has also been studied.

The assumptions underlying the theory† have been checked by experiments with a laboratory-scale column.[6] A digital computer has been used to calculate the dynamic behaviour of a pilot-scale column and of a refinery-scale column whose responses had also been measured. In both cases the agreement between theory and experiment was satisfactory after the correct numerical values of some static column-parameters had been deduced from the measurements.

The next step, taken in the present paper, is to apply the theory to the study of column control. A digital computer for this purpose has the disadvantage that time-consuming iteration cycles are necessary to find the adjustment of the various controller actions. It can be expected that the problem is more likely to be successful with an analogue computer approach.

Results can be obtained within a fraction of a second by using a large time-scale transformation factor between the actual column and its analogue. The effect of changing a parameter is also immediately apparent. Finally, the analogue often aids in getting an insight into the actual physical behaviour of the system.

Rose, Williams, and their co-workers[7–9] used an analogue computer for the study of column-control systems. However, they introduced the dynamics of the vapour flows as a cascade of first order responses, which is not realistic. It is more satisfactory to compare the vapour flows to the voltages appearing in a resistance capacitance ladder network, as will be shown later. Voetter and Houtappel's theory leads to more realistic analogues, and the results can be applied with more confidence.

This theory is discussed together with the principles of the analogue simulation and, results obtained with actual analogues are given.

Basic Equations for Description of Column Behaviour

In this section the basic equations of a tray are discussed. They can be used for various studies of column behaviour and in principle they also apply to the reboiler, the condenser, and the bottom and top accumulator.

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† See Appendix III.

Fig. 1 shows the variables at tray *i* of a distillation column. The vapour and liquid hold-ups are each assumed to be perfectly mixed.

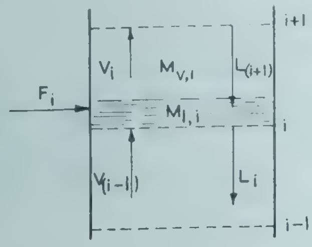

Fig. 1.—*Variables at tray i*

The material flow balance is given by:

$$\frac{dM_{l,i}}{dt} + \frac{dM_{v,i}}{dt} = L_{(i+1)} - L_i + V_{(i-1)} - V_i + F_i \quad . \qquad (1)$$

where $M_{l,i}$ is the liquid hold-up of tray *i*

$M_{v,i}$ is the vapour hold-up of tray *i*

L_i is the liquid flow from tray *i*

V_i is the vapour flow from tray *i*

F_i is the feed to tray *i*

The partial material flow balance for a component is given by an equation of the form:

$$\frac{d(M_{l,i}X_i)}{dt} + \frac{d(M_{v,i}Y_i)}{dt} = L_{(i+1)}X_{(i+1)} - L_iX_{(i)} + V_{(i-1)}Y_{(i-1)} - V_iY_i + F_iZ_i \quad . \quad . \quad . \qquad (2)$$

where X_i is the concentration of the component in the liquid on tray *i*

Y_i is the concentration of the component in the vapour on tray *i*

Z_i is the concentration of the component in the feed to tray *i*.

The energy flow balance is given by:

$$\frac{d(M_{l,i}H_{l,i})}{dt} + \frac{d(M_{v,i}H_{v,i})}{dt} - V_i^* \frac{dP_i}{dt} = L_{(i+1)}H_{l,(i+1)} - L_iH_{l,i} + V_{(i-1)}H_{v,(i-1)} - V_iH_{v,i} + F_iH_{f,i} - Q_i \quad . \qquad (3)$$

where $H_{l,i}$ is the specific enthalpy of the liquid on tray *i*

$H_{v,i}$ is the specific enthalpy of the vapour on tray *i*

$H_{f,i}$ is the specific enthalpy of the feed to tray *i*

V_i^* is the volume of tray *i*

P_i is the pressure on tray *i*

Q_i is the rate of heat loss of tray *i* to the outside of the column.

In equation (3) the term corresponding to the heat capacity of the tray itself has been neglected.‡ The term with V_i^* originates from the use of specific enthalpies instead of specific internal energies in the other terms of the left-hand side. It can generally be neglected. For larger size columns with sufficient insulation, Q_i is also negligible.

For the analogue approach, it is more convenient to write equations (2) and (3) in a different way. Subtraction of equation (1) multiplied by the value of X_i from equation (2) yields:

$$M_{l,i}\frac{dX_i}{dt} + M_{v,i}\frac{dY_i}{dt} + (Y_i - X_i)\frac{dM_{v,i}}{dt} = L_{(i+1)}(X_{(i+1)} - X_i) + V_{(i-1)}(Y_{(i-1)} - Y_i)$$
$$+ (V_{(i-1)} - V_i)(Y_i - X_i) + F_i(Z_i - X_i) \quad . \qquad (4)$$

Similarly, equation (3) can be approximated by:

$$M_{l,i}\frac{dH_{l,i}}{dt} + M_{v,i}\frac{dH_{v,i}}{dt} + (H_{v,i} - H_{l,i})\frac{dM_{v,i}}{dt} = L_{(i+1)}(H_{l,(i+1)} - H_{l,i}) + V_{(i-1)}(H_{v,(i-1)} - H_{v,i})$$
$$+ (V_{(i-1)} - V_i)(H_{v,i} - H_{l,i}) + F_i(H_{f,i} - H_{l,i}) \quad . \qquad (5)$$

For complete determination of the tray variables, five other equations are necessary. Their form depends on further assumptions about tray hold-up and the characteristics of the mixture on the tray.

Principles of Analogue Construction

Variations in pressure on the trays, and in pressure drop across the trays must be accounted for in order to arrive at a realistic description of the dynamics of pressure control. For simplicity the pressure drop across a tray is assumed to be small compared to the absolute pressure on the tray. This assumption is unrealistic in the case of vacuum distillation.

The feed to the column is assumed to be a binary mixture; thus the partial mass balance (equation (4) needs only be written for the light component.

The complete analogue can be split into a number of networks, which will be described in succession. As a rule only the trays of the stripping section will be dealt with. The analysis of other parts of the column goes along similar lines.

‡ In the analogue it can, in the first approximation, be combined with the heat capacity of the tray liquid.

The pressure network

The pressure network follows from the energy flow balance given by equation (5). It is convenient to express all flows in terms of moles per unit time; the specific enthalpies are then expressed in terms of energy per mole. This is normal practice in the Lewis and Sorel method and in the McCabe and Thiele diagram.[10] It leads to the so-called assumption of equal molal vapour and liquid flow. The specific enthalpies of liquid and vapour are then approximated by:[11]

$$H_{l,\,i} = \mathrm{f}(P_i) - aX_i \qquad (6)$$

$$H_{v,\,i} = \mathrm{f}(P_i) - aY_i + H_{e,\,i} \qquad (7)$$

where a is a constant,

$H_{e,\,i}$ is the heat of evaporation on tray i; it is considered to be a function of P_i only.

The vapour hold-up is assumed to be a function of the pressure only. Its derivative with respect to time can be written in the form:

$$\frac{\mathrm{d}(M_{v,\,i})}{\mathrm{d}t} = \frac{\mathrm{d}M_{v,\,i}}{\mathrm{d}P_i}\frac{\mathrm{d}P_i}{\mathrm{d}t} \qquad (8)$$

Substitution of equations (6)–(8) into equation (5), and addition of equation (4) multiplied by a, and leaving out the feed terms, yields:

$$H_{e,\,i}B_i\frac{\mathrm{d}P_i}{\mathrm{d}t} = L_{(i+1)}\{\mathrm{f}(P_{(i+1)}) - \mathrm{f}(P_i)\} + V_{(i-1)}\{\mathrm{f}(P_{i-1}) + H_{e,\,(i-1)} - \mathrm{f}(P_i) - H_{e,\,i}\} + (V_{(i-1)} - V_i)\,H_{e,\,i} \qquad (9)$$

where

$$B_i = (M_{l,\,i}/H_{e,\,i})\left(\frac{\partial H_{l,\,i}}{\partial P_i}\right)_{X_i} + (M_{v,\,i}/H_{e,\,i})\left(\frac{\partial H_{v,\,i}}{\partial P_i}\right)_{Y_i} + \mathrm{d}M_{v,\,i}/\mathrm{d}P_i \qquad (9a)$$

In consequence of the assumption of equal molal vapour and liquid flow, the effect on the static behaviour of the first two terms on the right-hand side can be neglected. It will be shown in Appendix I that this is also a reasonable assumption for the dynamic behaviour.

Linearization of equation (9) now yields:

$$B_i\frac{\mathrm{d}p_i}{\mathrm{d}t} = (v_{(i-1)} - v_i) \qquad (10)$$

where capital letters now indicate the average value and small letters small variations about the average value.

For bubble-cap trays the vapour flow, to a good approximation, is only related to the pressure drop across the tray. For small variations one can write:

$$v_i = (\beta V/\Delta P)(p_i - p_{(i+1)}) \qquad (11)$$

where β is a dimensionless constant,

V is the average value of the vapour flow in the stripping section and

ΔP is the average value of the pressure drop per tray in the stripping section.

Introduction of equation (11) into equation (10) yields:

$$B_i\frac{\mathrm{d}p_i}{\mathrm{d}t} = (\beta V/\Delta P)(p_{(i-1)} - p_i) + (\beta V/\Delta P)(p_{(i+1)} - p_i) \qquad (12)$$

Exactly the same relation holds for the electrical network shown in Fig. 2. The right-hand terms of equation (12) represent electric currents flowing to capacitor B_i. A similar network can be constructed for tray $(i + 1)$ (see Fig. 2). The two networks can be linked together to form one as shown by Fig. 3.

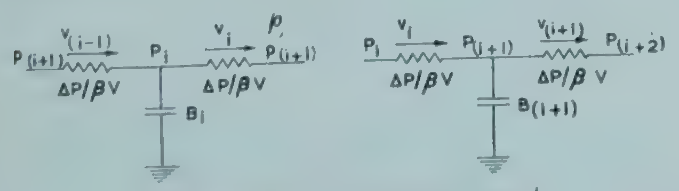

Fig. 2.—Sections of pressure network

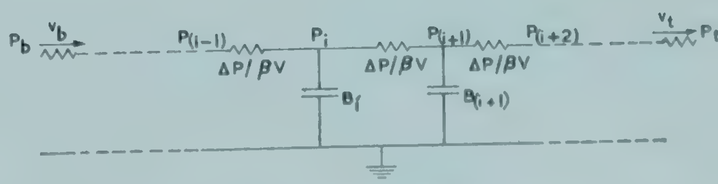

Fig. 3.—Pressure network

It is easy to extend this linking together until all trays of the column have been included. The result is called the pressure network, because its voltages represent pressures in the column. Further the currents flowing through the resistances represent the vapour flows in the column.

The reboiler network

The reboiler network provides the voltage and the current going to the bottom end of the pressure network. For a simple type of reboiler[12] with perfect mixing on both sides of the tubes, it is a conventional heat transfer analogue (see Fig. 4). Its voltages represent variations in temperatures, and its currents variations in heat flows.

To connect the reboiler network to the pressure network, it is necessary to convert from variations in temperature (t_b) to variations in pressure (p_b). This is done by adding a term corresponding to x_b (see Fig. 4), since:

$$t_b = \left(\frac{\partial T_b}{\partial P_b}\right)_{X_b} p_b + \left(\frac{\partial T_b}{\partial X_b}\right)_{P_b} x_b \quad . \quad . \quad (13)$$

This equation is a linearization of the relationship between pressure, temperature, and composition.

By a simple impedance transformation, the currents and the voltages in the reboiler analogue can be made to have the dimensions of vapour flow and pressure respectively, instead of those of heat flow and temperature. The connection with the pressure network can then be made directly, and a a good approximation to actual behaviour obtained.

A corresponding network can be used for the condenser.

The vapour-flow network

In the pressure network, currents represent variations in vapour flows. For the concentration network however, it is convenient to have the vapour flow variations represented by voltages.

This can be realized by a conversion of the pressure network, based on the formula:

$$(\Delta P/\beta V)\frac{\mathrm{d}v_i}{\mathrm{d}t} = (v_{(i-1)} - v_i)/B_i + (v_{(i+1)} - v_i)/B_{(i+1)} \quad . \quad (14)$$

This equation follows from equation (10) for tray ($i + 1$), subtracted from equation (10) for tray i, and from equation (11). It can be simulated by a network similar to that shown in Fig. 2, but with the values of capacitances and resistances interchanged (compared equation (14) with equation (12). The network can be extended to include all trays of the column. At the bottom it should be connected to a voltage representing variations in vapour flow in the reboiler. This voltage can best be obtained from the relatively large voltage drop across R_b in Fig. 4, because good accuracy is then guaranteed.

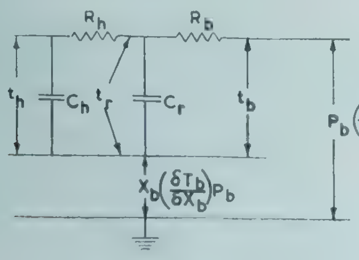

Fig. 4.—*Reboiler network*

A similar arrangement can be made at the other end of the vapour flow network.

The liquid flow network

The liquid flow network follows from the material flow balance equation (1). Linearization of this equation, and use of equations (8) and (10) yields:

$$\frac{\mathrm{d}m_{l,\,i}}{\mathrm{d}t} - (B_i - \mathrm{d}M_{v,\,i}/\mathrm{d}P_i)\frac{\mathrm{d}p_i}{\mathrm{d}t} = l_{(i+1)} - l_i \quad . \quad (15)$$

The variation of the liquid hold-up is assumed to be determined by:

$$m_{l,\,i} = \tau_l l_i + \tau_v v_{i-1} \quad . \quad . \quad . \quad (16)$$

where τ_l τ_v are constants with the dimension of time.

Using equation (16), and putting $\tau_v = 0$ we can approximate equation (15) by:

$$\tau_l \frac{\mathrm{d}\lambda_i}{\mathrm{d}t} = \lambda_{(i+1)} - \lambda_i \quad . \quad . \quad . \quad (17)$$

where

$$\lambda_i = l_i - p_i(B_i - \mathrm{d}M_{v,\,i}/\mathrm{d}P_i)/\tau_l \quad . \quad . \quad (18)$$

Here it has been assumed that consecutive trays are approximately equal.

In equation (17) a small term proportional to v_i has been neglected (see also Appendix I). Consequently the result has become very simple: the fictitious liquid flow variations λ_i are related to each other by a cascade of first-order responses.

In Appendix II it is shown that the network shown in Fig. 5 gives the correct behaviour. This network only consists of passive elements: coils, capacitors, and resistors. It is more reliable and less expensive than a cascade of resistance and capacitance sections separated by cathode followers (see Fig. 6).

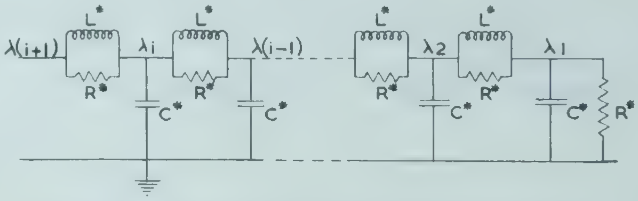

Fig. 5.—*Liquid flow network with passive elements*

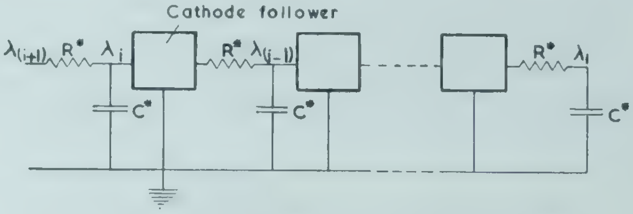

Fig. 6.—*Liquid flow network with buffer amplifiers*

At the top, the liquid flow network should be connected to a voltage representing the variation of the reflux flow and corrected for the variation of the pressure in the column top according to equation (18).

The concentration network

The concentration network follows from the partial material flow balance equation (4). Before this balance can be applied, it is necessary to have an equation relating variations of the vapour composition to other variables. For simplicity the following equation will be used:

$$y_i = \varepsilon x_i \quad . \quad . \quad . \quad . \quad (19)$$

where ε is a dimensionless constant.

This implies that the part of the thermodynamic equilibrium line in the McCabe and Thiele diagram pertaining to the stripping section is a straight line. Moreover, it implies 100% Murphree tray efficiency.

Linearization of equation (4), and use of equations (8), (10), (18), and (19) yields:

$$\begin{aligned} M_i \frac{\mathrm{d}x_i}{\mathrm{d}t} &= L(x_{(i+1)} - x_i) + \varepsilon\, V(x_{(i-1)} - x_i) \\ &\quad + \lambda_{(i+1)}(X_{(i+1)} - X_i) + v_{(i-1)}(Y_{(i-1)} - Y_i) \\ &\quad + (Y_i - X_i)(B_i - \mathrm{d}M_{v,\,i}/\mathrm{d}P_i)\frac{\mathrm{d}p_i}{\mathrm{d}t} \\ &\quad + (X_{(i+1)} - X_i)(B_i - \mathrm{d}M_{v,\,i}/\mathrm{d}P_i)p_{(i+1)}/\tau_i \qquad (20) \end{aligned}$$

where $M_i = M_{l,\,i} + \varepsilon\, M_{v,\,i}$.

This equation can be represented by the electrical network shown in Fig. 7. It is quite similar to that shown in Fig. 2, with the addition of four currents proportional to $\lambda_{(i+1)}$, $v_{(i-1)}$, $\mathrm{d}p_i/\mathrm{d}t$, and $p_{(i+1)}$, respectively.

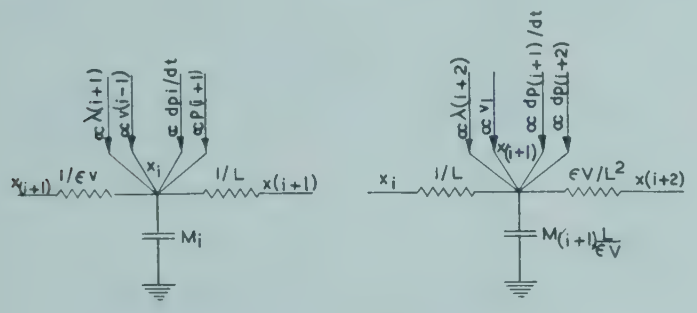

Fig. 7.—*Sections of concentration network*

The network can be extended in much the same way as the pressure network. The only complication is the fact that in general $\varepsilon\, V \neq L$. However, an impedance transformation with the factor $L/\varepsilon\, V$ of the network for tray $(i + 1)$ solves the problem (see Fig. 7), and makes extension possible.

Often $\varepsilon V = L$ is a good approximation to actual column behaviour. The concentration network then becomes symmetrical as does the pressure network.

The complete analogue

Fig. 8 shows the block diagram of the complete analogue. The concentration network operates on a low voltage level, while all other networks operate on a high voltage level. This choice allows of a simple realization of the currents proportional to $\lambda_{(i+1)}$, $v_{(i-1)}$, $\mathrm{d}p_i/\mathrm{d}t$, and p_{i+1} flowing to x_i in the concentration network (see Fig. 7): x_i is connected to $N\lambda_{(i+1)}$, $Nv_{(i-1)}$, and $Np_{(i+1)}$ *via* large resistances, and to Np_i *via* a small capacitor. Here N equals the ratio of the voltage levels.

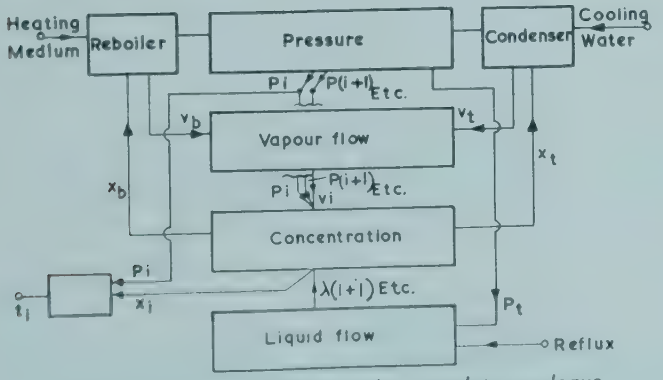

Fig. 8.—*Block diagram of the complete analogue*

When N is large, the actual currents are practically equal to the desired ones, and the load on the liquid flow, the vapour flow, and the pressure networks is small.

The variations of the tray temperatures can be determined from linear superposition of pressure and composition variations (see eq. 13). Analogues of controllers are easily constructed and connected to the column analogue. In this way, the dynamics of control can be studied on the analogue computer.

Measurements and Results

The two following examples illustrate the type of information obtained with the help of these analogues. The analogues were used in conjunction with the electronic analogue computer described by Idzerda.[3]

Analogue for constant pressure and zero pressure drop

The analogue can be greatly simplified when the pressure is assumed to be constant and the same throughout the column. Only the liquid flow and the concentration network then remain.

This simple type of analogue has been constructed for an atmospheric column with 32 theoretical trays. The feed enters at tray 16. In both sections of the column $\varepsilon V \simeq L$. Although the analogue represents a complicated set of mathematical equations, it is reliable and inexpensive because it consists mainly of passive components.

Several column responses have been studied with the aid of the analogue. Fig. 9 shows the responses of the composi-

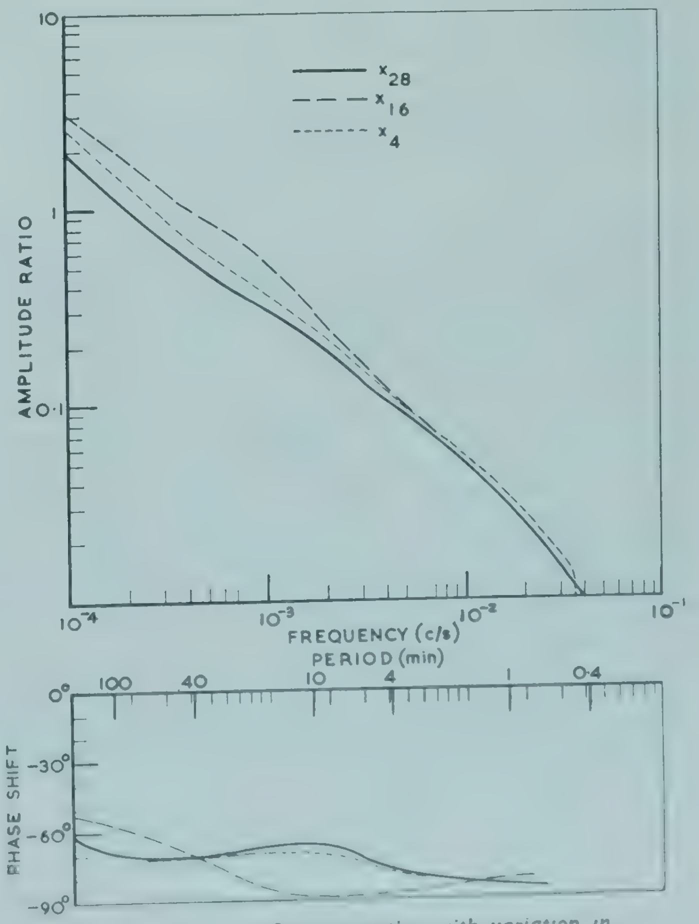

Fig. 9.—*Change of concentration with variation in vapour flow*

tions on trays 4, 16, and 28 to sinusoidal variations of the vapour flow.* They appear to be of first-order, because the phase lag does not increase beyond 90°.

Fig. 10 shows the response of x on trays 4, 16, and 18 to variations of the reflux flow. The lower the tray the greater the phase lag. This is apparently due to the increasing phase lag of the liquid flow for each lower tray. It is interesting to subtract this phase lag from the responses of Fig. 10, and thus plot the responses of the concentrations with respect to the variations of the local liquid flow. This has been done in Fig. 11. It can be seen that the results are of first-order as in the case of the responses to variations of the vapour flow.

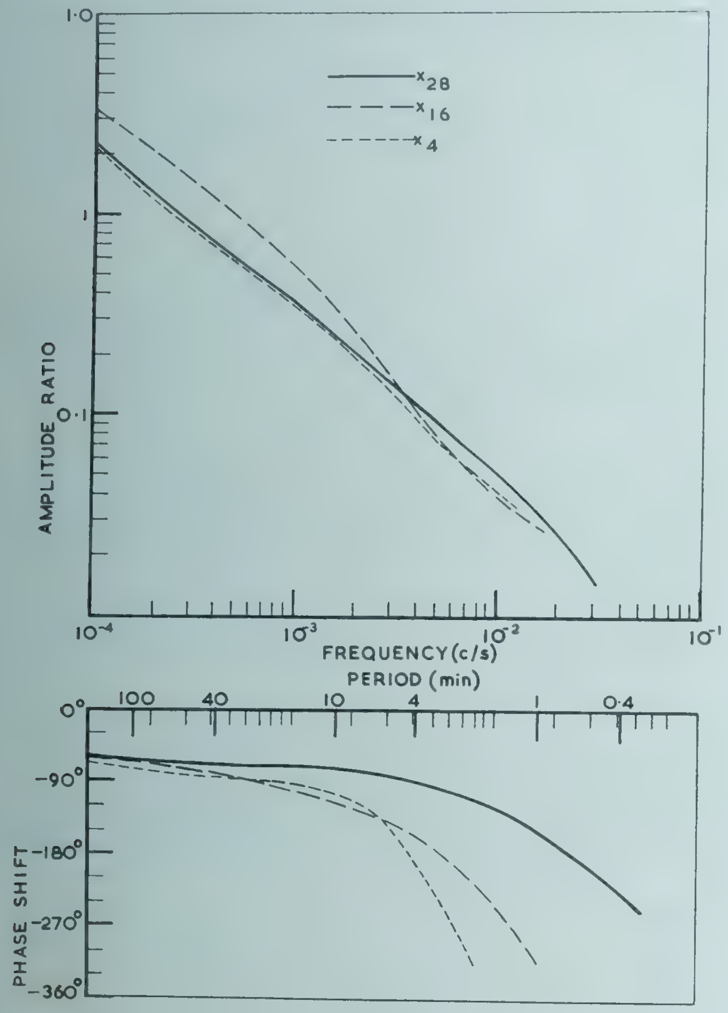

Fig. 10.—*Change of concentration with variation in reflux flow*

This suggests a simplification which can be used for approximate studies of column dynamics and control. The response of the composition on a tray equals the responses of the algebraic sum of vapour and liquid flows on that tray, followed by a first-order transfer function (see Fig. 12). The validity of this approximation has been checked under various conditions with satisfactory results.

Fig. 13 shows the response of x on trays 4, 16, and 28 to variations in feed composition. The response on the feed tray has a small phase lag, while the responses near the ends of the column have large ones. The latter are larger than those shown in Fig. 10.

This is related to the difference of the fundamental tray time constants: $M_l/L = 8{\cdot}4$ s and $\tau_l = 2{\cdot}7$ s.

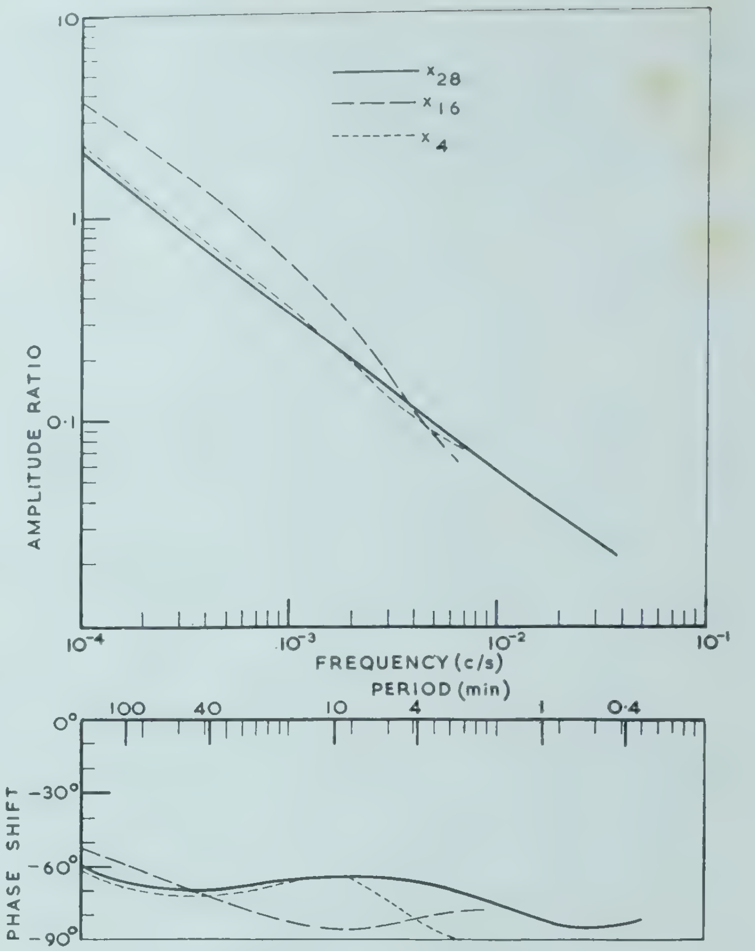

Fig. 11.—*Change of concentration with variation in local liquid flow*

Analogue for variable pressure and non-zero pressure drop

The complete analogue has been constructed for a column with 64 actual trays. Use has been made of the simplification discussed in the preceding section and shown in Fig. 12.

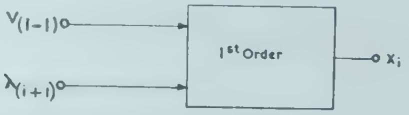

Fig. 12.—*Approximate simulation of the concentration on tray i*

This circumvents the low voltage level of the concentration network and therefore reduces problems due to drift of operational amplifiers.

Fig. 14 shows the response of the pressure on trays 8, 32, and 56 to variations of the heat input to the reboiler. The phase lag clearly increases the further the tray is from the bottom. The same is true for the responses of the vapour flows (see Fig. 15).

The analogue has also been used to study the coupling between temperature and pressure control. The coupling is found to be rather strong when temperature is controlled by the heating medium to the reboiler, and pressure is controlled by the cooling medium to the condenser.[14, 15] In particular, adjustment of controller actions is troublesome and control quality is unfavourably affected, as can be deduced from the results in Table I.

The simultaneous occurrence of two resonance periods when both controllers are in operation is noteworthy. The

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* When the pressure and the feed are constant, the variations of the vapour flow are the same throughout the column (see equation (10).

TABLE I.—*Effect on Control Quality*

Temperature control	Pressure control	Resonance period of the temperature control	Resonance period of the pressure control
Operative	Non-operative	6 min	—
Non-operative	Operative	—	50 s
Operative	Operative	45 min 3 min	18 min 1 min

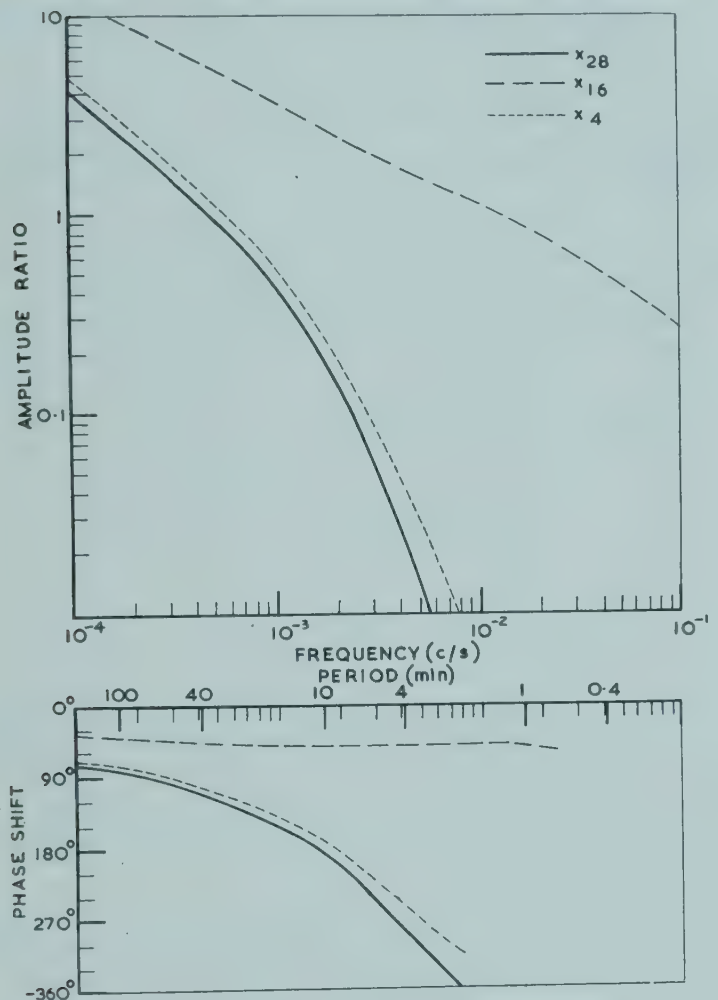

Fig. 13.—*Change of concentration with variation in feed concentration*

strength of the resonances strongly depends on the adjustment of the actions of the controllers.

It can be concluded that the coupling weakens the effect of control over a rather large range of frequencies.

Acknowledgment

Several people cooperated in the analogue studies: in particular acknowledgment is made to Mr. R. M. F. Houtappel and Mr. R. P. Offereins who carried out a large part of the work involved.

Symbols Used

For the column variables, capital letters indicate instantaneous (before linearization) or average values (after linearization). Small letters indicate variations about the average value.

a = constant (J/mole).
B = see equation (9a) (mole m²/N).
C = heat capacity (J/° C).

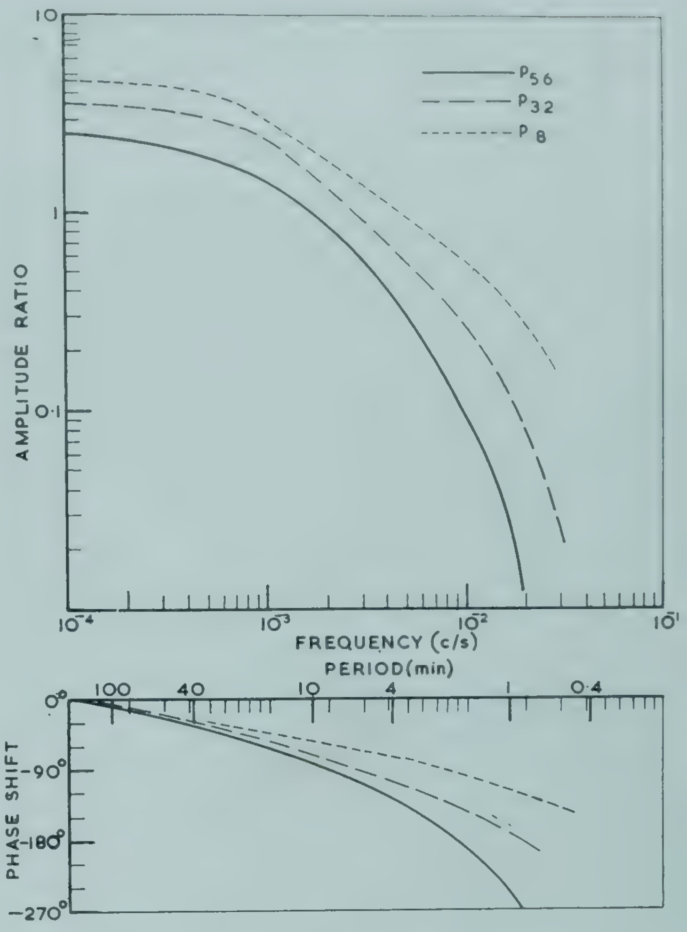

Fig. 14.—*Change of pressure with variation in heating*

C^* = capacitance (F).
C_h = heat capacity of the heating medium in the reboiler (J/°C).
C_r = heat capacity of the reboiler (J/°C).
F = feed flow (mole/s).
H = specific enthalpy (J/mole).
L = liquid flow (mole/s).
L^* = inductance (H).
M = tray hold-up (mole).
N = voltage level factor.
P = pressure (N/m²).
ΔP = average value of the pressure drop across a tray (N/m²).
P_b = variation in bottom pressure
Q = rate of heat loss (joule/s).
R = film resistance (°C/J).
R_b = film resistance on the column side (°C/J).
R^* = resistance (ohm).
R_h = film resistance on the heating medium side (°C/J).
T = temperature (° C).
t = time (s).
t_b = variation in temperature of column bottom (°C).
V = vapour flow (mole/s).
V^* = tray volume (m³).
X = concentration of a component in the liquid.
x_b = variation in bottom composition.
Y = concentration of a component in the vapour.
Z = concentration of a component in the feed.
β = dimensionless constant.
ε = dimensionless constant.
λ = fictitious liquid flow variation (mole/s).
τ_l = reflux time constant (s).

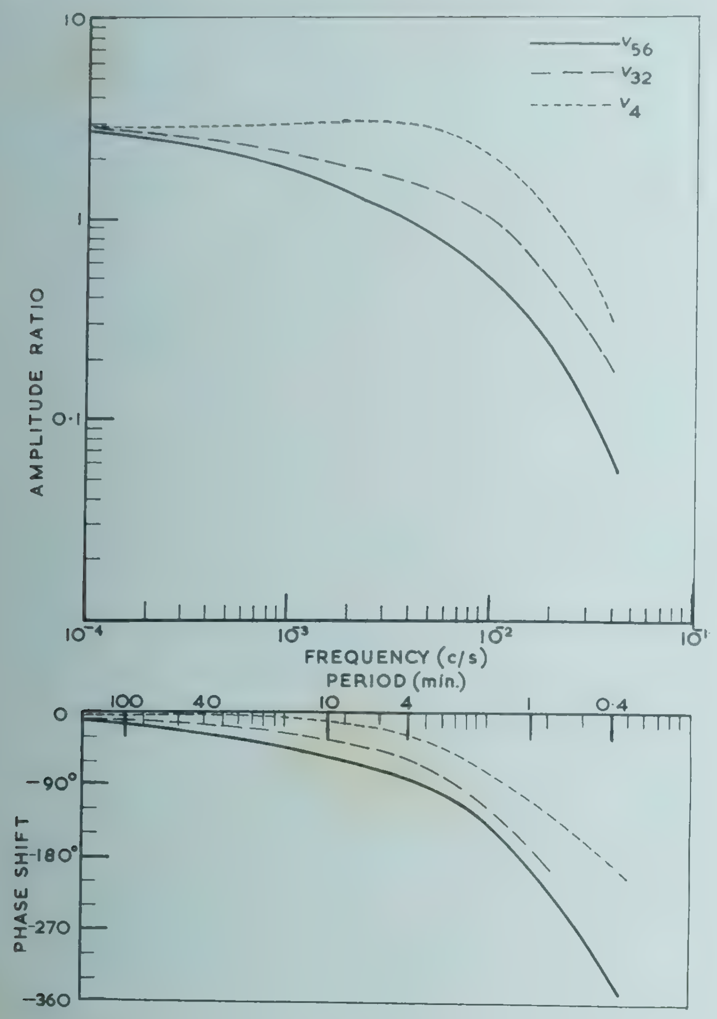

Fig. 15.—*Change of vapour flow with variation in heating*

Subscripts

b = column bottom.
e = evaporation.
h = heating medium to the reboiler.
i = tray number.
l = liquid.
r = reboiler tubes.
v = vapour.

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Appendix I

Neglecting terms of the energy flow and of the material flow Balance

Some of the terms appearing in equation (9) have been omitted in equation (10). It will be shown that this is a reasonable assumption for the dynamic behaviour.

When the pressure drop across the trays is relatively small, equation (9) can be written in the form (after linearization):

$$\begin{aligned} H_{e,\,i}B_i\frac{\mathrm{d}p_i}{\mathrm{d}t} &= -L\left(\frac{\partial H_{l,\,i}}{\partial P_i}\right)_{X_i}(p_i - p_{(i+1)}) \\ &\quad + V\left(\frac{\partial H_{v,\,i}}{\partial P_i}\right)_{Y_i}(p_{(i-1)} - p_i) \\ &\quad - l_{(i+1)}\left(\frac{\partial H_{l,\,i}}{\partial P_i}\right)_{X_i}\Delta P \\ &\quad + v_{(i-1)}\left(\frac{\partial H_{v,\,i}}{\partial P_i}\right)_{Y_i}\Delta P + (v_{(i-1)} - v_i)\,H_{e,\,i} \quad . \end{aligned} \tag{21}$$

For hydrocarbon liquid or vapour mixtures at the bubble or dew point respectively:

$$\frac{P}{H_{e,\,i}}\left(\frac{\partial H_{l,\,i}}{\partial P_i}\right)_{X_i} \sim \frac{P}{H_{e,\,i}}\left(\frac{\partial H_{v,\,i}}{\partial P_i}\right)_{Y_i} \sim 0{\cdot}25 \quad . \tag{22}$$

Using this equation and equation (11) we find:

$$\begin{aligned} B_i\frac{\mathrm{d}p_i}{\mathrm{d}t} &= v_{(i-1)}\left\{1 + \frac{\Delta P}{4P}\left(1 + \frac{1}{\beta}\right)\right\} \\ &\quad - v_i\left\{1 + \frac{L}{\beta V}\cdot\frac{\Delta P}{4P}\right\} - l_{(i+1)}\frac{\Delta P}{4P} \quad . \end{aligned} \tag{23}$$

For bubble-cap trays under 70% load, β is of the order of 1–2. It can be concluded that the effect of all terms containing $\Delta P/P$ can be neglected, as long as $\Delta P/P$ is small. This is not true for vacuum distillation.

In equation (17) the following term has been neglected:

$$(p_{(i+1)} - p_i)(B_i - \mathrm{d}M_{v,\,i}/\mathrm{d}P_i)/\tau_l \quad . \tag{24}$$

Here it has been assumed that $B_{(i+1)} \sim B_i$ and

$$\mathrm{d}M_{v(i+1)}/\mathrm{d}P_{(i+1)} \sim \mathrm{d}M_{v,\,i}/\mathrm{d}P_i$$

Using equations (9a), (11), and (22) we find for this term:

$$-v_i\left\{\frac{1}{4\beta}\cdot\frac{\Delta P}{P}(M_{l,\,i} + M_{v,\,i})/V\right\}/\tau_l \quad . \tag{25}$$

When $\Delta P/P$ is small, the factor between braces is small compared to the time constant of the liquid flow network.

Thus it is reasonable to neglect the term given in formula (25).

Appendix II

Proof of the correctness of the liquid flow network of Fig. 5

The last section of the network of Fig. 5 consists of a series connection of resistance R^* in parallel with self-inductance L^*, and resistance R^* in parallel with capacitance C^*.

If $L^* = R^{*2}C^*$, it can be shown that the input impedance of this section is equal to R^*, irrespective of the frequency. The load on the preceding section therefore is also equal to R^*, thus the behaviour of the latter section is identical to that of the former. Evidently this applies to all sections of the network.

The transfer function of each network section is that of a first-order system, thus the transfer function of the complete network is that of a cascade of first-order systems. The elements should be chosen such that $L^*/R^* = R^*C^* = \tau_l$.

Appendix III

Main assumptions made in the theory

The main assumptions made in developing the theory are:

Linear behaviour,
Binary mixture,
Theoretical trays,
Equal molal overflow,
dY/dX constant for a column section,
$\Delta P/P \ll 1$.

DISCUSSION OF PAPERS PRESENTED AT THE FOURTH SESSION

Mr. R. L. **Day** said that Rijnsdorp and Maarleveld had assumed that the vapour/liquid equilibrium diagram for the system could be approximated by two straight lines. Secondly, they had assumed that the plates were perfect stages; in other words, had a Murphree plate efficiency of 100%.

Mr. Day's laboratory had done work on the same subject: a very similar mathematical model was used, but the solutions were obtained by digital computation. Using digital computation, it was unnecessary to restrict the system by those two assumptions and for one particular system investigated it would not have been reasonable to apply either assumption.

Were those two assumptions, therefore, essential to the design of that type of electrical analogue?

Finally, he wanted to draw attention to an assumption implied in the paper and also, to his knowledge, in all other papers so far published on the dynamic behaviour of distillation columns. It was assumed that the liquid on the plate was perfectly mixed. For large columns it was well known that that was not true. Had the authors considered that factor during the course of their work?

Mr. **Rijnsdorp** replied that the assumptions made as to vapour/liquid equilibrium and theoretical trays were not absolutely necessary. An analogue could be constructed without them. For instance, an analogue could be used where the vapour/liquid equilibrium curve was approximated by a broken line with a break point for each tray. In the concentration network that involved a resistance at every tray across the capacitors representing the tray holdups. But unluckily those resistances were negative, and negative resistances could not be bought in a shop. They had to be made artificially by means of amplifiers.

A simple analogue had been built to investigate this modification. The vapour-liquid equilibrium curve was approximated by a broken line with six segments and the frequency response was compared to that measured previously. The differences of the phase-shift were found to be within the experimental error, while the amplitude ratios were within 50%, which was a reasonable margin of error considering other discrepancies between the analogue and the real column.

In that respect it should be stressed that a high degree of accuracy was not required for control studies. There were so many unknown factors in control behaviour; for instance, the exact nature of the disturbances was not known; and the requirements precisely could not always be specified accurately. Limited accuracy was therefore not a great disadvantage.

Referring to Mr. Day's question about the mixing on the tray, Mr. Rijnsdorp said that assumptions could be made about the mechanism, but chemical engineers did not agree about its true nature. There were various ways of dealing with non-ideal behaviour; vapour by-passing, liquid by-passing, mass transfer coefficients, and the like; and one did not know how to do it in a sound way. On one occasion liquid by-passing on the tray was tried out in the analogue, but it did not seem to make very much difference.

Dr. W. L. **Wilkinson** said that a recent paper by Rose and his co-workers[1] had considered a five-plate distillation column and derived an electrical analogue by which the dynamic behaviour could be studied. In doing so, they found it necessary to use a great deal of equipment and that suggested that the simulation of a realistic column with a comparatively large number of plates would require a prohibitively large analogue computer. The paper was rather disturbing to many people since it appeared that the analogue computer was of limited usefulness in this field.

On the other hand, Rijnsdorp and Maarleveld had shown that by adopting a different approach it was possible to simulate a column with a large number of plates with a modest amount of equipment. That was a reassuring and welcome contribution.

Dr. Wilkinson thought that many of the approximations and simplifying assumptions would prove to be too restrictive in practice. In particular the assumption of perfect mixing on the plates was not likely to hold good for large columns and the assumption of a linear equilibrium relation would very often not be realistic.

At the beginning of their paper the authors had stated that the approximations were justified because the predictions of the simplified theory had been tested by experiments on columns of various sizes, including large columns, and the results showed satisfactory agreement. However, in order to achieve agreement between theory and experiment the values of some of the static parameters of the column had to be deduced from the results of the dynamic experiments themselves. That procedure could not be regarded as a stringent test of the theory. If the static parameters were measured independently and then used in the theory to predict the dynamic response the discrepancy between theory and experiment caused by the over-simplification of the problem would probably show up more clearly.

Mr. **Rijnsdorp** replied that there was a case where the value of a static parameter had to be estimated from the results of the experiments themselves. That was not ideal. On the other hand, it was also difficult to circumvent it because the column was continuously making product, and its normal operation should not be disturbed by special tests.

But there was one favourable circumstance. The time constants for liquid flow going down the column—which was the static parameter under discussion—could be detected directly from the response of the bottom level to variations in the reflux flow. That was a straightforward procedure because the reflux entered the top of the column, went from tray to tray, and finally changed the bottom level. The form of the response appeared to be a combination of a large number of small exponential lags just as the theory indicated—

and the value of the time constant could be found from them. In that way it was possible to be fairly sure about the other results of the experiments.

In another case the unknown static parameter had been measured directly, independent of the dynamic measurements.

Mr. R. V. Thomas remarked that as the session was concerned with the design and use of analogue computers, he felt that he should begin by regretting that more had not been said in the papers about the design considerations and the construction aspects of the analogue computers referred to. In particular there seemed to be no mention of the accuracies of the analogues in any of the papers.

Rijnsdorp had mentioned that the Shell organisation started work on the dynamics of distillation columns ten years ago and he felt that many people would be interested to know roughly how many technologist-years had gone into that major project.

Drift troubles were mentioned in the paper in connection with the concentration network. Looking at that network, Mr. Thomas thought that another trouble which could arise was that unless $\varepsilon V/L$ was approximately unity, a very wide range of impedance levels must be necessary in the network. For example, if large changes of slope occurred along the equilibrium diagram, $\varepsilon V/L$ would seem to be rather large. Had any troubles been experienced owing to a wide range of impedance levels?

The block diagram of the complete analogue, Fig. 8, did not show any provision for the injection of feed disturbances. This was presumably just an oversight. Perhaps the authors would indicate how it was done.

It was said in the paper that the simplified concentration analogue used a first-order lag approximation. He found some difficulty in seeing how a transfer lag could be fitted to the attenuation and phase diagrams of Figs. 9 to 11 without appreciable errors of either attenuation or phase.

Mr. Rijnsdorp replied that the accuracy of the analogue computation was not very great, because great accuracy was not necessary for control work. For the most part components with 1% tolerance were used.

The analogue representation was correct for hydrocarbon mixtures which behaved more or less ideally, and for the case where the vapour/liquid equilibrium line was not very curved. Moreover the main application of the work would be for sharp fractionation where control was critical. Consequently $\varepsilon V/L$ was always near to one, and the range of impedances going though the concentration network was not large.

Feed disturbances in rate, composition, and enthalpy had not been shown on the diagram, in order to keep it simple. In point of fact they had been introduced into the feed tray sections of the passive networks and the effects of control had been studied.

The simplification of the concentration analogue mainly gave errors in the low frequency region where control was always very effective. The approximation for higher frequencies, which were important for the control study, was much better.

Mr. W. N. Jenkins drew attention to the statement in the paper by Kopai-Gora that non-linear equations were developed representing the behaviour of the stands. He asked whether any progress had been made on the derivation by experiment or theory of the equations for the later stands of the tinplate mill.

Secondly, if deviations were considered rather than absolute values, as stated on page 133, could not the deviations be made small enough that an assumption of linearity could be made?

Mr. Kopai-Gora replied that the equations and formulae which formed the foundation of the electric analogue computer development were obtained as the result of the theoretical study of the phenomena taking place in the rolling mill stands, the drivers, and the metal in rolling processes, as well as of several experiments made on operating rolling mills and devices. In particular, several factors were determined by processing of certain practical data. Except for these, the accuracy of the set of equations representing the process occurring in the rolling mill was checked by the transient response of the real rolling mill. The electric analogue computer made it possible to obtain the transient response under the most disturbing effects up to the simulation of the rolling of separate strips, as the computer had been designed for the simulation of the process for starting and stopping the rolling mill.

The electric analogue computer simulated the set of equations in deviations which made it possible, in certain cases, to replace non-linear dependencies (at small deviations) by linear ones. However, the complete linealization was not achieved because it could lead to large errors, especially in cases of simulation of the processes for starting and stopping the rolling mill.

The main purpose of the simulation of the deviations was not to simplify the set of equations but to improve the accuracy by means of increasing the scale correlations.

Dr. H. H. Rosenbrock said that when—about four years ago—he and his colleagues began to look into the dynamic behaviour of distillation columns, they debated for a long time whether to use analogue computers or digital computers. They found, as Dr. Wilkinson had pointed out, that previous work with analogue computers had needed very large amounts of equipment. They therefore decided, partly on those grounds and partly because of other difficulties, to use a digital computer for the main part of the work.

Four years ago, they were not completely in possession of all the facts. One of the facts was that Rijnsdorp and Maarleveld would present, four years later, a very excellent and very simple means for using ananalogue computer.

His company had used a digital computer to solve the differential equations, similar assumptions being made to those made in the paper. The equations were solved by a stepwise procedure at successive time intervals, so that starting from one condition of the column one ended up with the column in its steady state, corresponding to the operating conditions. The reflux ratio, the boil-up, and so on were put in as data at the beginning.

The first computer run was usually made in the same way as the column would be started up. For example, the column would be filled with liquid having the same composition as the feed, and the approach of the column to its steady state was computed. This gave the start-up time, and the equations at that stage were not linearised.

When the steady rate was achieved, a small disturbance was put in (change of feed rate or composition for example, or change of reflux ratio, or boil-up). One needed to study a number of such changes and because they were small the behaviour could usually be assumed linear over the small range of the disturbance. The calculations showed how the composition on each plate changed as the column approached a new steady state. At that stage, a choice was made of the point at which to control the column—on which plate to measure the temperature, or vapour pressure, or whatever parameter might be used.

The step response computed for that plate was imitated by means of a transfer function, and this was used as a part of the control loop in an analogue computer. One obtained, for example, the transfer function between a change of reflux and a change in temperature on the given plate, or change of boil-up and temperature, or change in feed composition, and so on. In that way, the blocks could be simulated in the analogue computer. The controls were added, and that gave a complete system set up for analogue computation.

One saw how the system behaved and if one wanted to find out how it behaved under any other operating conditions—because its behaviour could be quite different under different operating conditions—one had to go back to the beginning and start with a fresh digital computation. The amount of work involved was quite large. One could spend a couple of hours on digital computing and a day or two days on the analogue machine for each operating condition.

Because a great amount of effort had to go into it, the method was not very useful for design work at that stage. The simple analogue presented by the authors was therefore very interesting, and he would like to put the following questions to them.

First of all, did pressure variation from top to bottom of the column make much difference? In his own case, it had so far been neglected. The effect could be included in the digital computer programme, but that would complicate matters and made the solution more time-consuming. Would its neglect lead to difficulty and if so in what circumstances? The phase lag found by the authors did not seem to be due primarily to the compressibility of the vapour, which only entered in equation 9(a). It seemed to be due rather to the fact that if the pressure in the column was increasing then the temperature on each plate would be rising. That could only occur if each plate in turn condensed a certain amount of vapour and returned it as liquid to the bottom of the column towards the top and the pressure-drop across the plates would consequently change. Was that the correct interpretation? If so, one would expect that changes in the pressure difference between points in the column would not in themselves be very important except in vacuum conditions. The changes in vapour flow rate throughout the column might be more important. Changes in the general pressure in the column would also be important, but could presumably be dealt with by a simpler calculation and a simpler analogue.

The various methods so far available for dealing with the problem each had their difficulties and each would run into trouble, he thought, at some stage. The difficulties would be different according to the different methods used.

In return for the information for which he had asked about the limitations of his own work, he would like to describe circumstances which he thought might show up limitations in the authors' method of linearising. He had recently analysed a column in which the top and bottom products were both nearly pure. The column was very much over-designed and was producing almost a complete split of the materials going in. The composition curve within the column is given in Fig. 1.

With 50A and 50B entering, A was 49·99 at the top and B was 0·01.

At the bottom A was 0·01 and B was 49·99.

(These are not the actual figures, but illustrate the effect.)

In those conditions, if the amount of the top product was changed from a total of 50 to a total of 51, it was obvious that 1·0 at least must be B, so that the product purity at the top went down to at best 1·0 of B and 49 of A. In fact, there would be a little more B and a little less A.

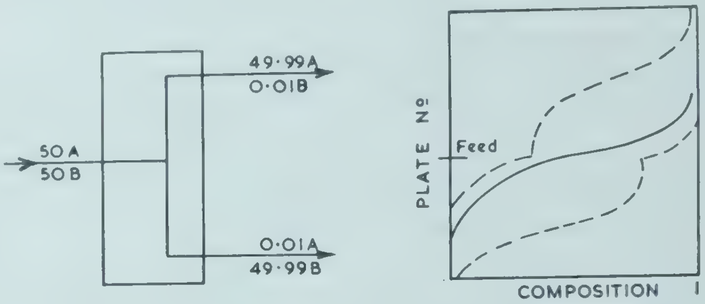


Fig. 1.—*Composition curve within the column*

The composition curve therefore had to move up the column. Conversely, if the amount of top product was decreased to 49, then at least 1·0 of A must come out at the bottom, so that the composition curve drifted all the way down the column.

He was not suggesting that the equations could not be linearised in that situation. But if they were linearised, consideration must be given to the changes in flowrate at the top and bottom for which the linearisation held, and they were so small that they were meaningless in practice. That was a situation where the authors might find there was some difficulty with the type of linearisation they had adopted.

Linearisation might also cause difficulty when starting from a known steady state. The steady state had to be compatible with the computer, but the steady state represented by the analogue computer was not necessarily the same as the steady state of the column. Did the authors find any trouble because of that?

He believed that the authors had neglected the heat balance halfway through their analysis to the extent of assuming equal molal vapour and liquid flows. Did they feel there were circumstances in which that assumption was important? In his own work, he had so far taken varying liquid and vapour flows where appropriate at the steady state, but had neglected any change of flows during a transient. The intention was to write the effect into the programme but that had never been done. He did no feel that it was very important for small transient changes. Did the authors?

How many plates could the authors deal with in their analogue? In his case, he had had to deal with up to 350 plates for heavy water distillation. Would that make the computer too large?

Again, how did one change the parameters in the analogue? Was it simply a matter of turning a knob, or was it necessary to unsolder resistors and replace them?

Binary systems were rare in practice. One would like to deal with multi-component systems. Was there any possibility that the analogue presented by the authors could be enlarged and generalised to deal with multi-component systems?

In that connection, he would like to comment on a point made by Mr. Day on theoretical plates. So far, when one went away fom binary to multi-component systems, there seemed to be no way of dealing with anything but theoretical plates. The attitude adopted in his case was that if one had to use theoretical plates with multi-component systems one might just as well do so with the binary system.

In connection with accuracy, there was one difficulty with large columns used, for example, for heavy water distillation.

It was not really a problem of accuracy but rather one of measurement. If one had a column producing heavy water, the feed went in near the top and the concentration was one part in 7000 of deuterium in the hydrogen. That was the naturally occurring proportion, and going out of the top was the depleted water from which the deuterium had been stripped. This had a concentration of, say, one in 70 000 of deuterium. At the bottom, if the column were rather large, one might have something like 50% of deuterium. The problem was not that one could not simulate the problem accurately enough. In fact, one probably did not know the plate efficiencies within 20%, so the accuracy of the representation was probably quite good enough.

But in the analogue one would get voltages which represented the various proportions from 0·5 to 0·999 986. How did one measure them? There seemed to be no way of measuring voltages to such accuracy. The chemist could measure with much higher accuracy than the electrical engineer. Did the authors know of any possible way of dealing with that particular problem?

J. E. RIJNSDORP replied that the authors had found—to their surprise—with regard to the importance of pressure variations in the column, that pressure variations and pressure-drop variations were not only important in columns operating under low pressure, but also in atmospheric columns. The choice of the locations of the detecting elements of temperature and pressure control were especially much influenced. Different locations gave different results because of the pressure-drop variations in the column. Generally, the phase-lag due to the pressure drop variations was determined more by the condensation and vaporisation on the tray than by compression of the vapour hold-up.

Linearisation where there was extreme purity at the top and bottom was an extreme simplification, but mainly from the static point of view. He had the feeling, based on the experience obtained with the analogue, that the dynamic response for higher frequencies was not so sensitive to those non-linearities, because the concentrations at the top and bottom only affected the concentration at some tray after a rather long time: thus the effects of extreme product purities was only felt in the static and low frequency region.

The incorporation of the steady state in an analogue was simple, because only variations of the variables were represented. Thus the steady state corresponded to zero voltage throughout.

One could try to put the heat balance on the trays into an analogue, but it would lead to much complication. A simpler solution was to apply the method of designing distillation columns whereby one assumed fictitious molecular weights of the components in such a way that one can use the assumption of equal molal overflow. That could be found in Ref. 11 of the paper.

Mr. Rijnsdorp thought that almost an infinite number of plates could be dealt with. Each plate would probably take with it a few extra resistors and capacitors. However, by using network methods one could lump the complete network and represent several trays by means of one network section. Thus the number of components can be kept on a reasonable level.

In connection with changing the parameters, it would have been possible to make an analogue with variable resistors and capacitors but that was discarded for economical reasons. Fixed resistors and capacitors were put in, which had to be unsoldered and replaced by others when the analogue had to be modified. The extra effort of soldering was not very great, because there were so many measurements for each situation.

A special analogue for ternary mixtures had been built to study the responses of tray temperatures to variations in vapour and liquid flow. The results show that the assumption illustrated by Fig. 12 of the paper was in most cases also valid for ternary mixtures. By extrapolating, one could expect the same result for multi-component mixtures.

As to how to measure a column with a very high amount of top product and a low amount of bottom product, the case was very special and extreme. Perhaps something could be done in the analogue by making a voltage transformation to keep on a reasonable level.

Mr. R. VICHNEVETSKY commented on optimization which was relevant to the paper by Witenberg, and to the papers by Coutie and by Kalman, Lapidus, and Shapiro. Optimisation of chemical processes had been of increasing interest in the last few years, and it was interesting to note that it was not only in that domain that the question had been raised. A problem of the same nature arose in the field of the use of computers.

When it was desired to study a process by computation methods, the first step was to establish a suitable mathematical model. It consisted of two parts, the first a mathematical formulation, the second, the allocation of numerical values for the parameters of that formulation.

The first—choosing some mathematical formulation—was largely dependent on the "skill of the operator".

The second—discovering the most suitable numerical values for the parameters—was exactly the same problem as that of optimising a chemical process. After the error function between the process response and the simulated response had been expressed, the parameters would be optimized in order to minimize that error function.

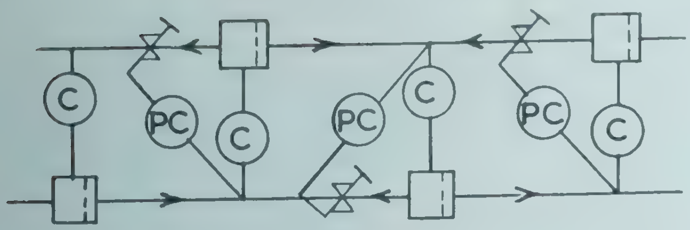
Problems of that class had been solved many times on computers. In particular, analogue computers had been shown to be very well suited to them.

A great deal of practical experience had been obtained in the use of analogue computers in optimisation procedures, while the experience gained in optimisation of chemical processes was much smaller. It was to be expected that the experience of the analogue users would be used in all that still remained to be done in optimisation of chemical processes.

Mr. W. LAWS remarked on the value of the electronic analogue model, particularly with regard to the control of pressure-flow relationships of a gaseous diffusion cascade consisting of a large number of similar stages. Each stage could, for example, consist of a compressor which had to maintain a pressure-differential across a membrane sheet and means for controlling the amount of material in the stage. Part of such a plant is shown in Fig. 2.

It was, for example, convenient to place a control valve in one of the lines. If automatic control valves with time lags were used it would be convenient to control the aperture of the valve in response to the variations of an absolute pressure say, of the compressor inlet on each stage. Particularly because of the time lags introduced into each control system, the analytical treatment became very difficult.

It was, of course, possible by digital computation to get precise values once the cascade equation was set up. But in order to get a more practical "feel" of the control problem, it was decided at U.K.A.E.A., Capenhurst, to investigate the



C = Compressor
PC = Process controller.
Dotted lines represent membrane sheets

Fig. 2.—*Part of a diffusion cascade*

possibilities of using analogue techniques. Rather than make a mathematical study by those techniques, it was decided to make an approach which was at least relatively independent of the admittedly approximate analytical work which was in progress. Electronic models were therefore made as accurately as possible for both steady-state and transient conditions.

In the analogue, absolute pressure was represented by voltage with respect to earth potential and mass flow was represented by current both for steady-stage and transient conditions. The compressor was represented by a special form of feedback amplifier in the sense that input currents were always equal.

The restriction to flow provided by the membrane sheet was represented by a resistor and non-linear resistances were generated by operational amplifiers to represent, for example, the pressure-flow behaviour of the valve aperture; and a simulated form of the process controller was used. In order to complete the picture, stage volume was simulated by partially distributed capacitors, one side of each being connected to earth. The simulator might be regarded as a sheet with the various simulated elements across it, which was supported by the terminals, waste, feed and product, and also by the plant volume capacitors. The model approach had been used relatively independently of the analytical work and it was interesting to note that the operators of the simulator, who incidentally were electronic engineers, regarded it not as a computing machine but as a plant.

Personally, Mr. Laws thought that the electrical analogue model approach was perhaps very valuable (although it might not be very accurate) in getting the operational and design staff to obtain a "feel" for the problem in a relatively independent manner.

Mr. R. V. Thomas wrote:

The equipment referred to in Witenberg's paper is of great interest as it is an example of a combination of analogue and digital techniques—the use of analogue techniques to simulate a process and the use of digital techniques to facilitate the manipulation of the simulator—that may become of importance in the future. This combination has obvious applications outside the field of manufacturing process control, in particular, in the development of mathematical models. To mention but one, analogue computers have been used to investigate the steady-state profiles of distributed chemical reactors.

A complication experienced is that the boundary conditions are often such that the settings for some of the initial conditions have to be found by trial and error,[2] the criterion being that, at the end of the run on the analogue computer, the solution should satisfy the end boundary conditions. This process is difficult to perform manually when several degrees of freedom are involved. If, in addition, parameter settings had to be found to satisfy some other criteria, manual search would become very difficult. A combination of analogue and digital equipment on the lines described by Witenberg could be of great value for this type of mathematical investigation. It is a pity that Witenberg has not given more information about the Model MH-II Electrical Analogue Computer, especially the units labelled 1 to 4 and 6 to 12 in the block diagram of Fig. 1 and about the errors of the analogue machine, unit 5 and the influence of these errors on the performance of the overall system.

Mr. Witenburg wrote in reply:

It is known that errors of the analogue computing machine are determined by several factors which include a type of the set of the differential equations and errors of the separate units. It is necessary to suppose that electrical analogue circuit errors of the MH-II computer should be of the same order as those of the group of analogue machines of the average class accuracy. The errors of the analogue machine might influence the performance of the overall system in two ways: systematic errors which would lead to errors in the parameter values which were to be determined, and random errors which would increase the time of scanning.

Dr. R. Edgeworth Johnstone asked Gillings after the meeting what provision had been made in the computing procedure for estimation of the actual costs to be entered for each annual period. He would be further interested to know whether any statistical treatment of the deviations of cost estimates from actual costs was envisaged, and whether the computing method could deal adequately with this aspect or show any special advantages with respect to it.

Dr. Gillings wrote in reply:

The computing method as described deals only with the stage of transforming forecasted amounts for the diverse types of cost incurred in building the plant and operating it over its expected life. As a result of this, profitability of a process plant installation can be estimated on an objective basis in alternative ways, some of which are outlined in the legend to Fig. 2, and account can be taken of any variations in operating conditions or environment which can be forecast, in a manner which is not readily possible by established estimating methods. The preparation of the forecast costs which constitute the input data has to be regarded as completed, and as a step prior to the estimation as actually described. An additional, and quite feasible step would be to integrate the profitability estimation with any computing methods applied in the technical stages of plant specification and estimation. While, as remarked in the paper and its presentation, analogue computing has advantages at exploratory stages, it is evident that the comprehensive estimation could be carried out as an extension to work for technical data on digital machines.

Substantial questions are raised in considering the treatment of variability and indeterminacy in the estimation of costs and realisations entered for each year, and subsequently in the results of the economic estimation. As is remarked briefly in the paper, a partly quantitative approach to estimating the influence of such variability is quite feasible. It would further more be possible in principle to represent the influence of variability of specific cost items, and their combined effect, with any specifiable level of precision. But this might prove to be merely formal, rather than useful, if the real uncertainties in derivation or in application of the data are dominant. The specific treatment by the analogue computation of costs relating to future years may be valuable in emphasising the importance of such uncertainty in data about events and operation still distant. Moreover the facilities for rapid exploration of a range of possible situations would significantly aid the selection of courses of action least influenced by any uncertainties which tend to impair profitability.

References

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[2] Beutler, J. A., and Roberts, J. B. *Chem. Engng Progr.*, 1956, **52**, 69F.

SESSION 5

THE USE OF COMPUTER TECHNIQUES IN LARGE AND SMALL COMPANIES

Chairman: **Mr. J. F. Coales, O.B.E., M.A., M.I.E.E., F.Inst.P.**
President of The Society of Instrument Technology

THE ORGANISATION OF A COMPUTER-AIDED DESIGN DEPARTMENT

By R. W. H. SARGENT, Ph.D., D.I.C., A.C.G.I., A.M.I.Chem.E.*

SUMMARY

The paper first briefly outlines the general characteristics of computers as they affect the formulation of problems. An illustration of programming procedure is presented. The need for standardisation in programme technique and the presentation of data and results is discussed.

The different types of problem handled by a design department are then considered, together with their characteristics and the required methods of treatment.

This forms a basis for consideration of the general organisation of the department, with the types and grades of personnel required for each activity, and the means employed to ensure maximum efficiency in the utilisation of the computer.

Introduction

This paper is concerned with the organisation of a relatively small design department, having free access to a computer on the same premises. This may appear to be a Utopian situation, but the department need not be the sole user of the machine: it may have access to it only for limited periods each day.

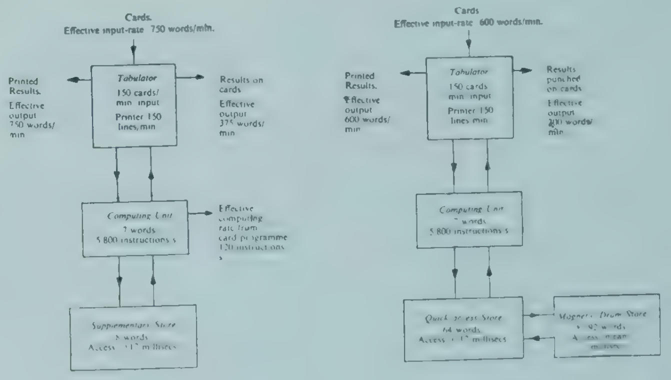
The department envisaged would deal with all technical aspects of design and construction, including costing of equipment, although the basic data for the latter would be provided by other departments. It would be completely autonomous as far as computer operation and programming are concerned, but is not considered as providing a computer service to all branches of the company.

Whilst the general considerations discussed in the paper would apply to a department of any size, the detailed discussion of organisation is based on a department employing five qualified chemical engineers, including the head of the department, with nine assistants of lower grades, and a secretary and typist. Such a department could be expected to handle up to 60 major plant designs in a year, with a somewhat larger number of minor plants and items of equipment. A modest development programme could also be dealt with.

General Characteristics of Computers

Before considering an appropriate organisation it will be necessary to outline some of the properties of computers and the way in which problems are presented and solved. It is not the intention to give details of computer construction, nor of programming techniques; those appropriate to low-temperature plant design, using a very simple card-programmed computer, have already been described by the author,[1] and other applications are adequately covered in the other papers of this symposium. Card-programmed computers may now be considered as obsolescent, but all the techniques and considerations involved in programming for them can be directly applied to stored-programme computers with magnetic-drum storage. Fig. 1 shows the essential features of the two types of computer. The access times and capacities indicated refer of course to specific computers, but they are typical values. An excellent summary of the characteristics of British computers appeared recently.[2] The problems of access and storage of data and results are common to both types, but on different time-scales: the drum-store takes the place of storage on cards.

Apart from the increase in speed, the stored-programme computer presents the enormous advantage of making any part of the programme accessible to the computer—for modification if necessary—at any time during the calculation. This makes it possible to treat problems much more completely on the machine, with less dissection of the programme and less intervention by the designer.


Cards.
Effective input-rate 750 words/min.
Printed Results.
Effective output 750 words/min
Tabulator
150 cards/min input
Printer 150 lines/min
Results on cards
Effective output 375 words/min
Computing Unit
7 words
5 800 instructions/s
Effective computing rate from card programme 120 instructions/s
Supplementary Store
Access
Cards
Effective input-rate 600 words/min.
Printed Results.
Effective output 600 words/min
Tabulator
150 cards/min input
Printer 150 lines/min
Results punched on cards
Effective output 300 words/min
Computing Unit
words
5 800 instructions/s
Quick Access Store
64 words
Access
Magnetic Drum Store

Card-Programmed Computer (F3M)

Stored-programme Computer (F3B-AET)

Note: 1 word = 48 binary digits,
or 12 decimal digits,
or 3 programme instructions.

Fig. 1.—*Diagrammatic layout of computers*

In order to obtain the maximum advantage from this, even more adaptation of the department's organisation is required than in the case of the card-programmed computer. Whilst another large part of the routine design is taken from the designer's shoulders, the problem of programming is more complex, and involves a correspondingly greater proportion of logical planning as distinct from numerical computation.

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In the author's view, this places the problem of programming even more in the field of the chemical engineer than in that of the mathematician, although the latter's services would still be useful on questions of numerical techniques for the solution of individual stages of the problem. However, where staff is limited, it makes for greater efficacy to train a chemical engineer in the relevant branches of numerical analysis rather than make a mathematician assimilate the theory and design techniques associated with each problem to be programmed.

Programming

These points are well illustrated by the process-design of a distillation column for separating a multicomponent mixture, and the study of this problem will also serve to show the steps involved in establishing a computer programme.

A general method of carrying out a plate-to-plate calculation of a column section for specified end-conditions has been given previously,[1] but this is only a part of the complete problem, for which the successive steps may be written:

1. Estimate product-compositions compatible with required purities and recoveries and overall material balance.
2. Fix heat input to reboiler.
3. Calculate end-conditions for rectifying and stripping sections, using overall heat-balance.
4. Plate-to-plate calculation of rectifying section.
5. Plate-to-plate calculation of stripping section.
6. Find optimum feed-plate location, based on variation of concentration of key-components.
7. Recalculate concentrations of light components in bottom product and concentrations of heavy components in top product, from match of feed-plate compositions.
8. Correct heavy components in bottom product, and light components in top product to conform with overall material balance.
9. Repeat from step 2 until solution converges.

Each step here involves a complex calculation, which may in turn be broken down into more elementary steps—a process which may be continued until each step is sufficiently simple for direct transcription into machine-code.

A complete outline-programme, or "organigram", for the distillation column is given in Table I. The steps given above are generalised to include the possibility of secondary feeds and withdrawal of side-streams. Break-downs of two of the steps are illustrated, for which purpose it is assumed that enthalpies are functions of temperature and pressure only.

This particular organigram represents the simplest logical sequence for carrying out the complete design, and as such is only the first stage in programming. It must of course be completed with an input sequence to feed all the initial data into the machine, and an output sequence for printing the desired results. At this stage it should be tested on the machine unless considerable prior knowledge of the computation has been obtained in other ways.

Apart from eliminating errors, this testing provides checks on rates of convergence of trial-and-error calculations, and relative times of calculation of the individual steps. Numerical methods may need alteration in the light of these results, and it may also be desirable to modify the logical design of the programme. This may involve simply a rearrangement of the order of execution, or it may extend to elaborate cross-linking between the steps, the object being to obtain the fastest calculation rather than the simplest.

In the initial stages, for example, different numerical methods might be more suitable than those used in the final stages. Again, it might be permissible to use empirical relations, or simplified design methods until a certain degree of precision is obtained, and in the same way some steps need not be repeated at each cycle of the iteration, but only if certain parameters change by more than a predetermined amount.

Thus in the case of the distillation column, the simplified equations[1] for the plate-to-plate calculations are used initially, whilst the heat-input to the reboiler, which is fixed by a determination of minimum-reflux, is repeated only if the calculated product distribution is grossly different from that estimated. The method of correction of product compositions from the feed-plate composition match may be greatly simplified as convergence is approached.

Standardisation

Because of this possibility of reshuffling of individual steps, and of the subsequent integration of programmes into still larger schemes, it is essential that each part of the programme has a standardised entry and exit sequence, and that it conforms with a systematic addressing scheme. This also applies to methods of storing data and results; apart from a standardised form of presentation so that standard sub-programmes can be used to extract data for a given set of conditions, it is desirable, where possible from space considerations, always to store a given set of data in the same place in the machine. This applies particularly to physical data, such as enthalpies, which are used in many different calculations. Time on input routines is then saved, and only one set of cards, or tape, is required for the data.

This standardisation of presentation should be carried through all the work of the department, and efficiency is improved by using standard specification sheets, not only for items of equipment, where they can be printed directly by the computer in most cases, but also for every design procedure whether this is carried out by the computer or not.

In this way it is ensured that all the data required for a solution are available—nothing wastes more time than the interruption of a calculation in order to search for a missing factor—and further, that the data are presented in consistent units. The standard form of presentation also makes for greater speed and fewer mistakes in punching cards or tape for input to the computer. Finally, it is much simpler to classify, check, or refer to a design dossier composed of standard specification and calculation sheets.

To minimise waste of time in recopying figures, with the attendant risk of copying-errors, sheets should be carefully thought out, and wherever possible should serve both for specification and calculation when the latter is done by the designer. Examples of sheets of this type have been given.[1]

Classification of Problems

The work carried out by the department can be broadly classified into three groups:

1. Collection and correlation of basic data.
2. Development of design techniques.
3. Design calculations.

The last group can further be divided as follows:

(*a*) Complex problems.
(*b*) Sequences of relatively short calculations.
(*c*) Isolated and non-routine problems.

Of these, the design of a distillation column represented by the programme of Table I is typical of sub-group (*a*); other examples are the hydrodynamic design of column-plates, and the design of a heat exchanger. The latter (for which the preliminary outline-programme is given in Table II), is a good example of the necessity for integration of the process design and mechanical design in order to obtain a self-consistent solution. This programme was elaborated in the same way as that for the distillation column in order to accelerate execution: it lends itself admirably to the application of evolutionary optimising techniques to obtain an exchanger of minimum cost for the specified duty. There is a relatively large number of adjustable parameters, such as tube-diameter, baffle-spacing, and segmental baffle-section, which are only loosely determined by process or mechanical requirements, and must ultimately be fixed to give minimum cost. Since these factors are strongly interacting it is essential to employ some sort of statistical procedure or evolutionary technique to obtain the optimum combination.

TABLE II.—*Preliminary Organigram for Heat Exchanger Design*

STAGE I

Jumps			Mod. ins
	1	Fix velocity for each stream.	
↑	2	Prepare heat-content *vs.* temperature tables for each stream.	
	3	Calculate no. of tubes for each pass.	
↑	4	Fix tube-sheet thickness.	
	5	Determine tube-sheet and end-box layout.	
	6	Determine shell and tube-sheet diameter, baffle-bracers, baffle-spacing, *etc.*	
	7	Do stress analysis of shell, tube-sheets and end-boxes.	
	8	Compare results with allowable stresses, repeat from 4 if exceeded.	
	9	Calculate shell-side HT coefft. *vs.* temperature table.	
	10	Calculate HT coefft. *vs.* temp. tables for each tube-pass.	
	11	Calculate pressure-drop per unit length for each stream.	
↑	12	Determine temperature distribution for one increment of length.	
	13	Sum length and pressure-drops.	
	14	Compare calculated and desired outlet temp. for key-stream—repeat from 12 if unequal.	
	15	Compare calculated and desired outlet temps. for other streams and adjust by-pass flows where appropriate.	
	16	Correct calculated pressure-drops for adjusted flows.	
	17	Compare calculated and allowable pressure-drops.	
	18	Adjust stream velocities where necessary to conform.	
	19	Compare with assumed velocities and repeat from 2 if unequal.	
	20	Determine weight and cost of exchanger.	

Whilst this sort of optimisation is probably unnecessarily refined for the majority of design problems, it is often justified by the extent of the overall savings obtained, and depends essentially on the proportion of the total cost of plant entailed and the speed of computation.

The solution of the complex problems of sub-group (*a*) often involve several hours of continuous computer time, the time of preparation (card or tape-punching, input and output routines) being small compared with the time of computation. It makes for much greater flexibility of organisation if special provision is made for interrupting a calculation, and extracting and punching all the intermediate data and programme modifications onto cards ready for feeding-in and continuing the calculation later. This is particularly true if the computer is used independently by several different departments during the day, although, of course, one would endeavour to avoid such interruptions if possible.

Apart from major design problems of this type, there is a variety of shorter problems, some of which occur many times in the design of a plant. Typical of these are the calculation of dew-points and boiling-points of multicomponent mixtures, flash-equilibrium determinations, and the calculation of heat transfer coefficients from empirical correlations. Because of the frequency of their occurrence it is relatively easy to organise the overall plant design so as to submit groups of these problems together for computation, and if several plants are being designed simultaneously, the similar problems for each plant can be grouped with those of other plants and treated concurrently.

These sequences form sub-group (*b*) above, the problems involved being characterised by the fact that the preparation-time forms an appreciable proportion of the total time of solution. The grouping is essential in order to benefit by the computer speed, since the preparation-time is thereby reduced. For example, much of the basic data is common to all problems, and the input and output routines can be grouped together; the time for card or tape punching is also reduced.

It often happens, particularly in the early stages of plant design, that this type of problem occurs singly, separated from the others of similar type by a complex problem of type (*a*) which depends on its result. In such a case it is usually quicker for the designer to perform the calculations himself rather than wait to have it done by the machine. The reasons are not quite the same for a stored-programme computer as those pointed out by the author for the card-programmed computer.[1] In the latter case, the preparation-time is usually much larger than the computing-time, and can be cut significantly by grouping problems; in the present case, preparation rarely exceeds 30% of the total time. The stored-programme computer therefore calculates appreciably faster than the designer, even for a single problem, but, since it is working continuously in a well-planned department, it is unlikely to be available at just the moment that the designer requires it, and a waiting-period is almost inevitable.

These isolated problems solved by the designer himself form the third sub-group (*c*) above. Also in this category can be placed all those qualitative problems of plant or flow-sheet arrangement, and of assessment of alternative solutions, as well as special design problems solved by non-routine methods.

Of the other two main groups, development of design techniques involves a certain amount of theoretical work and numerical analysis, but is mainly a question of programming and has already been fully discussed.

The collection and correlation of basic data is a composite problem, and overflows to a certain extent into other departments. Once the correlation-programmes are established the correlation itself is a question of routine computer operation together with the filing and classification involved in replacing superseded data and keeping appropriate records. The collection of data is largely a question of librarianship, for which a specialised department usually exists. But the main difficulty is to obtain someone with sufficient technical qualifications to recognise from abstracts, or a cursory glance at an article, the relevance of the work or data to the require-

TABLE I.—*Organigram for Design of Distillation Column*

Stage I		Stage II		Stage III	
1	Estimate product compositions.				
2	Fix heat-input to reboiler.				
3	Calculate end-conditions from overall heat balance.				
4	Calculate top-condenser.				
5	Introduce secondary-feed if appropriate.				
6	Take off side-stream if appropriate.	1	Test key-component concentration.		
		2	Jump if not suitable for side-stream.		
		3	Test whether liquid or vapour required, and modify succeeding instructions accordingly.		
		4	Calculate and store qty. and composition of side-stream.		
		5	Modify molar upcurrents of column-section.		
		6	Calculate and store temperature and heat-content of side-stream.		
		7	Modify thermal upcurrent of column-section.		
7	Calculate top-section plate.	1	Estimate temperature of plate $n + 1$.		
		2	Obtain partial enthalpies h_n and H_{n+1}.	1	Round off temp. to nearest table entry, and calculate interpolation factor.
				2	Calculate address in table from rounded temp.
				3	Interpolate between successive entries in table, for all components.
		3	Calculate qty. of liquid from plate "*n*" (*cf.* eqn. 19. Ref. 1).		
		4	Calculate qty. and compn. of vapour from plate "$n + 1$".		
		5	Calculate dewpoint of vapour from plate "$n + 1$".	1	Obtain relative volatilities at estimated temp.
				2	Calculate reference equilibrium constant.
				3	Obtain temperature from this.
				4	Repeat if different from estimated temp.
		6	Compare dewpoint temp. with estimated temp.		
		7	Repeat from 2 if unequal.		
8	Compare key-component concentration with value for feed-plate.				
9	Repeat from 5 if unequal.				
10	Calculate reboiler.				
11	Introduce secondary feed appropriate.				
12	Take off side-stream if appropriate.				
13	Calculate bottom-section plate.				
14	Test for optimum feed plate location.				
15	Repeat from 11 if test negative.				
16	Recalculate distribution of light components from match at feed-plate.				
17	Recalculate distribution of heavy components.				
18	Compare with estimated products and repeat to convergence from 2.				

ments of the design department, and who is at the same time content to limit himself to this.

Quite independently of this, such limited use of a person with the requisite qualifications would be wasteful, and he could not be expected to maintain his efficacy if he were not working actively in the field at the same time. The best compromise seems to be to employ a chemical engineer responsible for developing correlation techniques as the major part of his activities, who can also supervise and assist the librarian in collecting data and check the results of the correlations. He would also serve as liaison officer with the measurements laboratory. However, it is desirable that he should from time to time participate in the other functions of the department, so as to be able to assess the suitability of the forms of correlation. From the other point of view, he should not be the only person familiar with the routine of collecting and correlating data, so that this aspect of the work is not halted at times of illness or holiday.

Departmental Organisation

Now that we have considered the computer and programming techniques and the different types of problem arising in the day to day work of the department, we are in a position to discuss the personnel required. There is a surprising variety both in the kinds and the complexity of the problems involved, so that there is scope for a corresponding variety in the training and grades of the personnel.

Apart from the head of the department and the administrative side of the organisation, involving secretary, typists, and filing-clerk, the technical personnel can be divided into three categories, the number of each for a typical department being given in parentheses, as follows:

(1) Design engineers (4).
(2) Assistant designers (6).
(3) Computer operators (3).

The computer operators

The computer operators should be capable of punching cards or tape and of operating the computer, using any established programme or routine test procedure. Usually they will also do a limited amount of typing, mainly involving transcription of results.

The assistant designers

The assistant designers cover a wider range of activity, the qualification normally required being advanced level G.C.E., or its equivalent. Broadly, they should be capable of handling any problem which can be set down as a formalised procedure, or expressed precisely in mathematical terms, whether this involves numerical solution of the problem, or setting up the corresponding programme for the computer. Thus, they should be able to transcribe an organigram into machine-code, and establish the corresponding numerical check, and programme-testing procedure, or even establish the later stages of the organigram itself, where the operations involved in each step are explicitly formulated, such as in the third and succeeding stages of the distillation-column example given in Table I.

In addition, they should be able to solve numerically the routine design problems, so as to deal with the isolated problems discussed under sub-group (3*c*), making their own preliminary estimates for trial-and-error calculations where necessary, and to be able to tackle special problems under the guidance of a design engineer.

The design engineers

A design engineer would take full responsibility for the complete design of a plant, based on the specification of the customer; he would maintain liaison with other service departments, such as project engineering, draughting and construction, costing, technical sales, and commercial departments, and should be capable of discussing technical points with the customer or subcontractors. It is his job to establish the process flowsheet of the plant, choose between possible alternative solutions, and resolve any problems arising in the design. In general he would do little actual computation, but rather break down the general design of the plant into a logical sequence and formulate each step in such a way as to enable an assistant designer to carry out the detailed work. A single plant design will not therefore occupy his whole time, and he may be able to handle two or three plants simultaneously, and still leave a proportion of his time for development of design techniques and computer programming, or the supervision of data collecting and correlation.

A design engineer must obviously be a fully qualified chemical engineer, with experience in plant construction, operation, and commissioning as well as design experience. He must also be trained in computer programming and numerical analysis.

General

Whilst it would be possible to allocate a designer as assistant to each design engineer, it makes for greater flexibility to maintain the assistant designers as a pool, to be allocated to different engineers according to the varying requirements, whether these are for routine plant-design or for programming and testing. The same considerations apply in a general way to the computer operators, who are not allocated to any specific type of duty.

This system provides for maximum variety in the work for all grades, and so maintains interest, and makes for improved efficiency. It also tends to equalise the members of a given grade, and gives less scope for petty jealousies.

In practice of course, the grades are not so perfectly clear-cut, and there is ample scope for progression. Thus a computer operator becomes, with experience, capable of testing parts of a new programme, without the aid of an assistant designer, and eventually may be able to transcribe organigrams into machine code. Computer operators can also carry out some of the simpler numerical computations, but they can rarely rise to the full grade of assistant designers because of a basic lack of mathematical training, being recruited mainly from secondary modern schools, or having at most Ordinary level G.C.E.

Assistant designers make steady progress with experience, and can gradually be entrusted with more complex problems; in some cases they can make preliminary estimates of a technical nature, such as of heat-transfer coefficients, or tube-sizes for heat exchangers, *etc*. Eventually they can be entrusted with complete independent sections of plant to design, or even repeat a complete plant design, following an established process and flowsheet, but for slightly different feed and product specifications.

Draughtsmen

No mention has been made of draughtsmen in the above organisation, which may seem surprising. The fact is that the comprehensive computer programmes used have virtually eliminated the need for design on the drawing-board, and it has been found possible to dispense with draughtsmen in the design department, except for one to draw process flowsheets, plot curves, *etc*. In the final stages of design however, there is close liaison between the design department and the construction engineering department who produce the detailed working drawings.

Organisation of Computer-time

The sequence of calculations for the whole department must be so organised as to keep the computer running continuously during each period of availability. Since the demand for computation is irregular, and depends on the progress of the individual plant designs, the planning to attain this end presents in itself a complex problem.

Fortunately, it is usually possible to predict the occurrence of large problems of subgroup (*a*) a day in advance, and often obtain a rough estimate of the number of calculations forming subgroup (*b*) at the same time. Thus, an overall daily plan for computer utilisation can be drawn up at the end of the preceding day, and estimates of the time of availability of the results given to the design engineers. This makes it possible at the same time to allocate the assistant designers to the appropriate design engineers, and distribute the work of preparation and machine-operation amongst the computer operators.

This planning is of course the duty of the head of the department, usually with the co-operation of a computer operator to assess the preparation and computation times. Often the planning is a pure formality, but occasionally it involves decisions on the order of priority of the different projects, and arbitration between design engineers with rival claims on the services of assistant designers.

During the daily execution of the programme it is left to the initiative of the computer operator in charge of the machine to make minor alterations to improve the continuity, and to incorporate extra short calculations, either by addition to the sequences of subgroup (*b*) already decided, or by including those of type (*c*) at each changeover of the machine. The average waiting time between demand (in the form of a specification sheet) and availability of the result for these additional calculations is about three-quarters of an hour, the maximum time being of course the length of the longest programme of type (*a*), but the maintenance of this standard depends on efficient overall planning for each day.

As already pointed out, the demand for routine plant-design computations is very irregular, and there are inevitably gaps in the planned programme, either through temporary lulls on the design side, or through unforeseen difficulties. These gaps can always be filled by the long-term projects for data correlation or development of new programmes. Since, however, the length of the gaps is a measure of the efficiency of planning, both in the organisation of computer-time and in the general arrangement of each plant design, there is a risk of halting long-term improvement by over-efficient organisation. To obviate this, it has been found desirable to allocate 15% of the total computer time to general studies, in addition to gap-filling.

Concluding Remarks

The most interesting aspect in the management of such a design department is the extent to which personnel with only a very general educational background can be used to deal with quite complex problems which on the surface seem to require training in chemical engineering. This, however, puts a correspondingly greater onus on the trained engineers of the department, and a new type of chemical engineer emerges. It is far more difficult to translate the complex of knowledge gained in plant experience into computer programmes than to use it directly in plant design, and it becomes more than ever necessary to interpret phenomena in terms of first principles which can then be reapplied. In the same way, the form of many of the current theories of chemical engineering processes is dictated by the necessity of making the mathematics manageable; the computer programme is a much more powerful tool than analytical mathematics, so that it is often necessary to go back beyond the analytical theory to the basic physical principles involved, to obtain maximum advantage from the computer. Only a trained chemical engineer can do this, and the more knowledge he has of the computer and the logic of its programming system and of numerical techniques in general, the more satisfactory will be the result.

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PROCESS INVESTIGATION AND PLANT DESIGN IN LARGE AND SMALL COMPANIES

By S. T. LUNT, B.A.*

SUMMARY

This paper outlines some possible organisations, discusses the costs of computer services, and suggests some ways in which the smaller units of industry might be served. Each company will have its own particular pattern of production processes in terms of the range and quantity of products manufactured at a given site and within a given department. It would, therefore, be impossible—as well as misleading—to recommend any given organisation but it is hoped that the paper will serve as a useful basis for discussion.

Introduction

Other papers presented at this symposium have described some techniques and equipment which may be used to improve the performance of existing processes, and also in the various stages of research, development, and design of new ones. Many of the investigation techniques involve a knowledge of chemistry, physics, instrumentation, engineering, mathematics, and computer technology.

Large companies are adopting different organisations to bring the various skills together; small companies may feel that the cost of an investigation is too great or that it requires the services of scientists with more advanced training and with a broader background than are employed by them.

Process (Investigation) Departments on Production Plants

Production plants in Great Britain and in U.S.A. are frequently organised along the lines indicated in Fig. 1. It is assumed that a number of separate but related processes are located at a works.

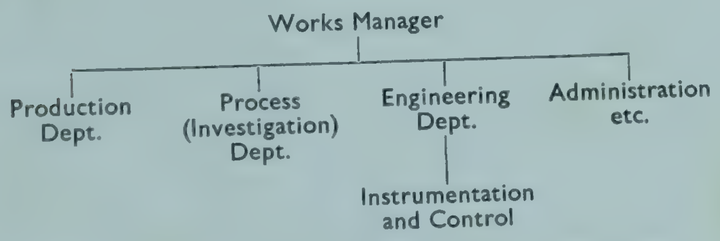

Fig. 1.—*Organisation of production plant in Great Britain and the U.S.A.*

The Process (Investigation) Department is sometimes termed a Process Department, a Technical Department, or a Technical (Service) Department.

Responsibilities of departments

The purposes of the various departments are given below.

PRODUCTION DEPARTMENT

The Production Department has responsibility for the day-to-day running of the process, the achievement of production targets, and the organisation of process labour.

PROCESS (INVESTIGATION) DEPARTMENT

The Process (Investigation) Department is responsible for the investigation of processes and the maintenance and improvement of technical efficiencies. It may carry out laboratory and semi-technical scale experiments as part of this work. Some works do not have a Process Department responsible to the works manager. In these cases technical assistance can be requested from a headquarters technical department, but with such an arrangement it may be difficult to obtain an adequately close and integrated co-operation.

In works having a Production Department and a Process Department it may be possible to employ as plant managers in the Production Department men who have less detailed knowledge of chemistry, but who are good organisers and may have qualifications in some other field.

ENGINEERING DEPARTMENT

The Engineering Department deals with the maintenance of plant, with installation, and with modifications to plant. In America the term "Engineering Department" is often used to designate a chemical engineering group whereas in Great Britain an Engineering Department would be staffed mainly by mechnical engineers.

The process and engineering groups are combined in a Process Engineering Department in some American organisations. In this case the emphasis is definitely on chemical engineering.

INSTRUMENTATION AND CONTROL SECTION

In almost all British companies, instrumentation and control has been regarded as a part of the (Mechanical) Engineering Department. The reasons for this are mainly historical. Many years ago instruments began to be introduced on a significant scale for the measurement and later the control of the operating conditions. The equipment often required considerable maintenance, and as it was largely

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mechanical this work was naturally allocated to the Engineering Department.

Instrumentation is now beginning to be regarded as an integral part of the process instead of being assumed to be an attached service. The field is also broadening by the introduction of data handling equipment, and of complex control systems possibly using computers. A tendency is developing, particularly in America, to divide the responsibilities which are at present carried out by a typical instrument manager into two distinct parts: (*a*) process control, data handling etc., and (*b*) instrument maintenance and installation. This tendency can be visualised as logically leading to the organisation outlined in Fig. 2.

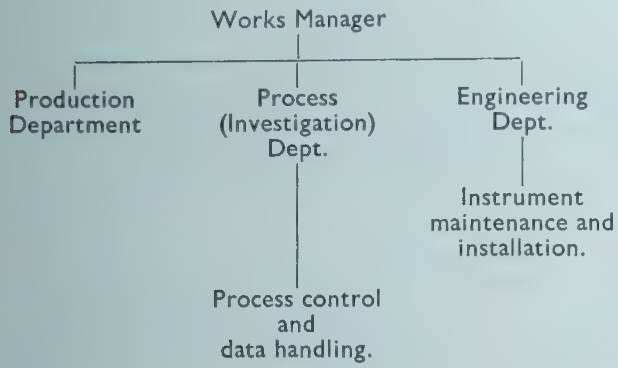
Works Manager
Production Department
Process (Investigation) Dept.
Engineering Dept.
Instrument maintenance and installation.
Process control and data handling.

Fig. 2.—*Revised organisation of production plant*

In this, an instrument manager would be responsible for the maintenance and installation of the equipment, and the instrument technicians would be responsible to him. He should be a good organiser, teacher, and leader of labour, but the control engineer would be a member of the Process Department and should be a good scientist with an interest in the process. A high standard can be demanded of both, and they can have complementary status.

Some oil companies operate with an organisation closely resembling that of Fig. 2, and it has been instituted at a new works with most encouraging results. On the other hand, a case could be quoted where personal friction developed and the system was abandoned.

Experience has also shown that Process Departments located on a works are often so preoccupied with the technical aspects of immediate importance that they do not tackle problems on a sufficiently fundamental basis. This is probably because the time and equipment required for laboratory experiments are too formidable. In at least one company in America an independent group has been set up to undertake a study of the techniques of process investigation and the fields of application of computers and advanced instrumentation.

Inter-works organisation

In the oil and chemical industries similar operations are often carried on at different sites. An independent, inter-works group could well serve a number of these sites by acting as a consultant service to the Process (Investigation) Department in the various works. Such a group should include a chemist with a background in the appropriate field of manufacture, a mathematician with an interest in statistics and a knowledge of numerical analysis, a control engineer with knowledge of advanced instrumentation and data reduction equipment, a chemical engineer, and a computer application technologist. Other skills may be added in particular cases, or one member may be competent in more than one field.

More use has been made of computers and advanced instrumentation in process investigation and plant design in America than in Great Britain, and it is perhaps significant that such development work has almost always been initiated by a headquarters group rather than by a works-based Technical Department.

In some companies, work in this field is carried out by the headquarters research departments. In such cases it is imperative that the long-term speculative research work should not be sacrificed for the quick returns which may be gleaned from short-term development projects. If it is essential to combine both aspects of work in one department, there should be distinctive titles for the sections dealing with each.

Digital and Analogue Computers

The techniques of process investigation and plant design involve the increasing use of both analogue and digital computers. In each case a large investment is involved by any company which decides to purchase one of adequate capacity.

The implications of installing such machines will be discussed separately.

Analogue computers

Analogue computers have been associated with studies of control problems and digital computers with engineering design, statistics, and the determination of quantitative chemical kinetic data. Recently a trend has been developing, particularly in U.S.A., towards the use of analogue computers for the latter application. The setting up of computing centres equipped with large accurate analogue machines which are convenient to use has played a major role in this development. From the author's limited experience, it is estimated that an analogue machine suitable for a large proportion of the problems likely to be met in the chemical industry would require a capacity of about 200 amplifiers together with appropriate ancillary equipment. American experience has confirmed that this size of machine, or a slightly larger one, will be required, and in the chemical industry emphasis must be placed on the provision of adequate non-linear equipment such as function generators, multipliers, *etc.* These machines are usually installed in the form of two or three identical units which can be used independently on separate smaller problems, or as one large machine for the more comprehensive investigations. Such equipment may cost £50 000–£100 000, or even higher if more extensive facilities are to be provided. Application engineers are invariably attached to the machine for day-to-day operation such as the supervision of maintenance and for the provision of assistance to potential users. A ratio of one engineer for every 35 amplifiers gives an approximate establishment. Maintenance and administrative staff are also required.

Fortunately it is not always necessary to purchase machines; many manufacturers operate a computing service. Charges for the use of such machines depend on the size of the problem, the time which is spent on the machine, and the amount of time devoted to the problem by the application engineers. It is almost impossible to give a close estimate of the cost of using machines in this way, but in the early stages of such work, whilst time is being used to some extent in feeling one's way, charges of £250 to £1000 or even more can be expected. These figures do not include the time spent by the user in preparing the data before visiting the computing centre. It is recommended that companies proposing to carry out such work for the first time should visit several of these centres and discuss their proposed project. It may well be that advice will be received on the preparation of the problem which will save considerable time, and therefore money.

It is often found with analogue computers, particularly when they are used on control problems, that a more complete understanding of the process is obtained and that this in turn reveals some alternative system, *e.g.* a rearrangement of control loops, which gives improved operation. In such cases it may also be found that the final solution appears to be obvious when viewed and explained with the benefits of hindsight. In these circumstances the value of the analogue computer may be queried on the grounds that the solution would have been foreseen if sufficient thought had been given to the problem in the first place. Nevertheless, at least one company with a large machine is preparing to extend it, because they are confident that without the machine some of the final results would not have been obtained as quickly, if at all.

Digital computers

Many companies have purchased computers principally for clerical and commercial work. Reports to justify the purchase of such machines may well mention that they "could be used for technical calculations". But the requirements of the machines for the two purposes differ in some important respects. Whilst it is true that either type of machine can be used for both fields of application, the use of a commercial machine for scientific work has presented some avoidable obstacles in technical computation. It is perhaps significant that many American companies with large commercial-type machines are replacing them by or supplementing them with large scientific-type machines. There is also the probability that the financial returns to companies in the process industries will be far greater from the use of scientific computers in the technical field than from the use of commercial machines for clerical and other work.

A development has taken place during the last three years which is of the greatest importance to users when preparing programmes for digital computers. The writing of programmes used to be a tedious labour for which specially trained programmers—usually mathematicians—were generally employed. This arrangement has been known to lead to errors when the programmer did not understand the whole background of the problem, and it invariably caused delays. But interpretive coding systems have been developed to enable users to prepare their own problems using a logical system which closely resembles conventional algebra. Two such coding systems are known as the "Autocode", prepared by Brooker of Manchester University for the "Mercury" computer,[1] and "Fortran", devised by I.B.M. for their 704 machine.[2] It has been shown that chemists, engineers, physicists etc., can be taught "Autocode" within a week, after which they have been able to prepare programmes for some of their own projects. The more practice they obtain the more proficient they become, and they are able to see more clearly the possibilities of undertaking more extensive problems.

The choice of a particular computer which was purchased recently for calculations in the field of process development and plant design was determined not only by the speed and size of the machine but also by the availability of an efficient coding system for use with it.

Organisation of Digital Computer Laboratory

The proposed organisation for the use of a large computer for scientific calculations is outlined in Fig. 3.

Routine computation

The "Mercury" is a general purpose computer which can be used for calculations in several fields. Once a programme has been written it must be punched on to a paper (ultimately magnetic) tape which is fed to the machine. This work is carried out by machine operators, usually women. With experience these operators learn to detect some of the slips which tend to be made by inexperienced people when writing their first few programmes.

Results of calculations are produced by the machine on paper tape and must be fed into a printer to present the answers in the required form. This process will also be controlled by the machine operators.

The responsibility for the correctness of a programme or for wastage of machine time due to a faulty programme must rest with the originator of the programme.

Errors are frequently found during the initial stages of developing new programmes. Cases will often arise when a machine operator cannot be expected to take the responsibility for, say, terminating a programme which is appreciably exceeding the estimated machine time. A senior person should therefore be available at all times when untried programmes are being run.

Mathematical development

The "Mercury" is a comparatively new computer; a large library of sub-routines for performing standard mathematical operations will not be available initially.

It is the accepted practice amongst computer users to share such programmes and thus some effort must be devoted to this work; this will be undertaken by the three mathematicians. Naturally, the programmes to be developed will be those particularly suitable to meet the anticipated requirements of the particular company.

It is important that the members of this group should be able to foresee these future requirements. Some help should be obtained in this work from the members of the Application

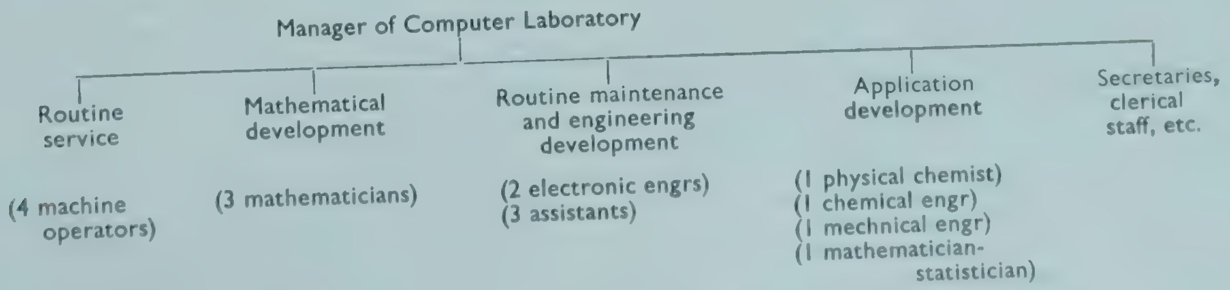

Fig. 3.—*Proposed organisation for a large computer for scientific calculations*

Development Section (discussed below). In all this work the members of this group will be called upon to exploit and develop new and advanced mathematical techniques and develop subroutines on the machine for using them.

Maintenance and engineering development

ROUTINE MAINTENANCE

It is necessary to organise the scheduled testing and maintenance of the equipment to detect faulty units before their breakdown causes trouble. Special searching tests are required to locate weak components which can then be replaced and the faulty unit repaired later. Experience may indicate that additional testing and simulation equipment are required.

Not all faults will be located during maintenance periods in this way; hence maintenance staff must be available whenever the machine is operating.

ENGINEERING DEVELOPMENT

Operational experience with the computer will suggest that engineering modifications will be required from time to time.

The electronic engineers responsible for this work will be able to use the maintenance assistants for the construction of units when they are not engaged in maintenance work.

Application development

The use of computers within a company will be established most effectively by sending out specialists from the computing centre to individual works and to research and technical departments. The specialists would be skilled in various fields, *e.g.* chemical, mechanical, and electrical engineers who would visit design departments, and chemists who would visit those responsible for drafting basic design data, etc. Initially this service could be started with, say, 4 graduates, a chemical engineer, a physical chemist, a mechanical engineer, and a mathematician having an interest in statistics. The members of this team must retain close contact with the group responsible for mathematical development so that techniques and programmes will be available when required.

Cost of computer

Digital computers for applications in these fields are produced by several manufacturers, and cost between £40 000 and £100 000. It may be significant that almost all users in U.S.A. are replacing smaller machines by large ones which are usually rented rather than purchased. Companies are probably more willing to change from a rented model than they would be to purchase outright the larger computer if this meant that a heavy capital investment were to be rendered obsolete.

Users in Great Britain can hire time at computing centres operated by the manufacturers and by several of the universities. Machine time is charged at various rates between about £30 and £100 per hour. It may require from a few seconds to several minutes to feed a programme into a machine. Computation may take a few seconds for, say, the calculation of high-pressure flanges, or up to an hour or more to simulate a year's operation of a multi-stage chemical process. To these charges must be added the cost of the time of staff spent in the writing of programmes. This may take from one day to several weeks depending on the complexity of the problem and the extent to which it is intended to make a general purpose programme suitable for a wide range of applications. In all cases it is probable that the time for preparation and testing of the programme will be substantially reduced by using interpretive coding systems.

As was suggested for the analogue computer, companies preparing to use digital computer techniques for the first time should discuss their proposed applications with a number of computing centres. It may be found that programmes have already been prepared for the solution of some particular problem on one or more machines, or the members of the computing centre may be able to suggest alternative approaches and other profitable applications.

Large financial returns have been claimed by several companies from the use of digital computers in U.S.A.—savings of millions of dollars in capital expenditure by better design, higher throughputs from existing plants, and improved conversion efficiencies.

Analogue and Digital Installations

In future, large process companies will be installing both analogue and digital computers—several companies in U.S.A. already have both, sometimes in a central computing department. For some computation one type of machine is obviously the more suitable; for many problems either could be used. In some instances it is possible to use the analogue computer to investigate quickly the relative merits of a wide range of alternative hypotheses; when this range has been sufficiently narrowed the final solution may demand the accuracy of a digital computer. Conversely, digital machines have been used to calculate steady-state operating conditions before investigating the controllability of a system on an analogue computer.

Process Development in Smaller Companies

In view of the high capital cost of the general purpose analogue and digital computers, only the larger companies will be able to purchase them. Such companies will also probably have sufficient technical staff to use them efficiently. Two other cases should be considered:

(*a*) companies which cannot afford to purchase computers or to rent them for their sole use, but which have a substantial number of scientifically trained staff available for work in this field. These are likely to be the medium-size companies,

(*b*) companies which employ very few, or no such staff—the small companies.

Medium-size companies

Time on computers can be hired at a number of computing centres, where advice will also be given on the preparation of the data. It will be assumed that the staff of such medium-size companies will be able to collect such data as is available from plant and laboratory measurements. But it may be found necessary to instal more equipment to obtain additional critical data. Quality measuring equipment could be particularly valuable but this, together with the arrangements required to feed a sample of the process fluid to the instrument, will often be expensive—from £250–£1500 installed cost.

It may be difficult to establish a convincing financial case for the purchase of such instrumentation. Samples could be analysed in a laboratory without such equipment, but as the time required for each analysis increases so the number of samples taken will probably decrease and an imperfect picture of the true operating conditions at a given point will be obtained. The installation of quality measuring instruments, even for a period of a few days or weeks, would be much more satisfactory.

To meet this demand, manufacturers might be able to introduce a scheme for hiring such equipment, analogous to the hiring of large computers.

Several obstacles can be foreseen—equipment to be used in such applications would often be installed on operating plant and could receive some severe treatment due to accidental mishandling or being sprayed with corrosive fluids. But this would also bring instrument manufacturers into the most intimate touch with operating conditions on process plants and this experience in turn could help the manufacturers to improve their designs.

In some of the smaller of the medium size companies it may be desirable to call in process development groups from consultant or contracting firms who are initiating services in this field. Research Associations, set up to serve particular industries, could also play a part in such work. One problem arises when outside assistance is obtained, *i.e.* process information must be given to the investigating group, and if this group studies a similar process elsewhere at a later date, knowledge gained from the first application must be of value. Any improvements developed in conjunction with the first company might be protected by patents. During a recent visit to America it was noted that several companies were inviting specialist groups to study some of their processes with a view to devising special equipment, even though this involved the disclosure of a good deal of basic process information.

Small companies

Where companies have very few scientifically trained staff it is probable that the process could be improved substantially without the use of computers. The first stage towards any improvement is the obtaining of a more complete understanding of the process, and the measurement of operating conditions is the first essential in this work. The installation of simple flow, temperature, level, and pressure indicators may reveal that operating conditions are very different from, or much more variable than, those which had been assumed. The management of such companies are advised to call in the manufacturers of this basic instrumentation whose staff are available to advise on the fields of application of their equipment. Instrument manufacturers in turn should consider whether they are themselves taking adequate initiative in their approach to small companies with a view to demonstrating to them the potential value of instrumentation. The employment of more chemical engineers as technical representatives of instrument manufacturers could be of considerable advantage both to the instrument companies and their potential customers.

Conclusions

The installation of instrumentation and the use of computers can help to improve the efficiency of many processes. When a company does not feel competent to select the instruments or to use computing facilities, it can usually obtain any necessary guidance.

Process companies should also consider whether their organisations are such that they will make the most rapid strides in the development of their processes.

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DISCUSSION OF PAPERS PRESENTED AT THE FIFTH SESSION

Mr. B. W. Balls said that it had been rightly suggested that the manufacturers of instruments should be represented in the discussion and his primary rôle was to speak as a representative of a manufacturer. He had also to speak as a chemical engineer because of his membership of The Institution of Chemical Engineers which, incidentally, had borne the heat and burden of the day in organising the conference. He had also to speak as a member of the Society of Instrument Technology. It was interesting to observe that approximately 65% of the people who had registered for the conference were chemical engineers. However, one could not avoid speaking personally and it was sometimes difficult to divorce one's affiliations on such an occasion.

Two interesting papers had been presented in the fifth session. That by Sargent was concerned with an organisational matter and showed how computers could be used most effectively to tackle the many problems of plant design.

Many of the things said by Lunt were sound common sense—the condensed experience of a number of years of hard work in the organisation to which Lunt belonged.

At the end of his paper Lunt commented particularly on the rôle of the instrument manufacturer. In the early days of process control there was always a very close liaison between the manufacturer and the user, with excellent results. One heard such deprecating terms as "ironmongery" and "nuts and bolts" and yesterday Dr. Westcott had talked about "black boxes". It would be a pity if anything happened to promote the "black box theory" but Dr. Westcott was correct in suggesting that there could be a tendency for that to occur. There was a danger of the user-industries—probably unconsciously—forcing the manufacturers into a position where they would become mere purveyors of equipment and in that way something valuable would be lost.

Responsible manufacturers in the instrument industry were deeply concerned with the problem of providing staff of sufficient technical education to assist the user with his process problems. It might well be that a user company could expand sufficiently to be relatively independent of the services offered by the manufacturer, but in industry as a whole there was still a tremendous amount of co-operative work to be done.

There were the related problems of quality and cost. With regard to quality, the chemical engineer had a training especially suitable for the work of control engineering in the field of application. That training taught him to study processes in the way Lunt had advocated in his paper. The chemical engineer thought in terms of dynamics of a process and in terms of unit operations, both of which were important in the field of control engineering. He was concerned with corrosion and behaviour of materials and, because of his design experience, he had an understanding of the overall process requirements. In connection with cost, an old English proverb stated that the man who paid the piper called the tune. For the tune to be rewarding the piper required high skill and was in a position to demand that the price should be sufficient to cover his expenses as well as his skill.

Lunt had done well to draw the attention of the smaller company to substantial improvements which could be made without the use of computors. It was frequently the case that users would request expensive analytical instruments, without first making use of available equipment at much lower cost which nevertheless would solve 90% of their process problems. A typical example could be provided by a paper machine, where the two important analytical measurements were basis weight and moisture content of the paper. However, variations in the weight could be reduced substantially by careful attention to control of stock preparation and the flow of stock to the machine, and a high degree of control of moisture in the paper could be obtained by very simple regulation of the steam supply to the dryers. The curve showing results achieved plotted against cost rose steeply, but then tailed off asymptotically towards the 100% achievement line and it was important to see that the initial gains were made at lowest costs before embarking upon more elaborate schemes where costs could be expected to increase logarithmically.

In the field of analytical control, the suggestion by Lunt to make equipment available on hire was a good one since, for reasonable work to be done, the manufacturer should be in the position to cover some of his engineering expenses, particularly in cases where it was doubtful if a reasonable result could be achieved. No user should be averse to paying a hire charge for expensive equipment and high quality service on the understanding that eventually purchase of the installation, if successful, would lead to considerable benefit to his process economics.

Wide experience in the handling of process control problems revealed that many of them derived from measurement difficulties. Progress in many industries had been delayed by the inability to provide suitable and reliable measurement techniques whereas the associated control problems were very simple. Typical examples were the control of flow of acid phosphate rock in the fertiliser industry and the control of flow of wood pulp in the paper industry.

To summarise, future progress lay in the direction of increasing technical co-operation between user and manufacturer of equipment, of an increasing awareness of the need to meet the costs involved, and an increasing appreciation of the need for simple automatic control loop. Real progress could not be made in the field of computor control before progress had been made on those lines.

Mr. E. W. R. Little said that he must disagree with Mr. Balls when he suggested that the chemical engineer was the best man to apply controls to a process. Traditionally, that was incontrovertible, but, traditionally, while processes had no doubt been considered as dynamic they were considered in a dynamically steady state; the increasing prominence of control engineering was due to consideration of deviations from the steady state—there was no doubt that such deviations were not the primary consideration of the chemical engineer.

It was for that reason that instrument engineers, as the people who had had to struggle with controls for years, had been the first to consider the wider implications of control as part of a process. The instrument engineer and the chemical engineer tended to get at cross purposes with each other when, having done the work and having designed a process, the chemical engineer relaxed and put on the architectural decorations such as instruments which measured and controlled in the way he thought the items which should be measured and controlled.

Mr. Little found it very difficult to disagree with Lunt in any way. What Lunt had said was perfectly obvious. That obviousness and simplicity came from a profound understanding and very clear thought about the subject. It was quite clear that Lunt's thesis was correct and that the application of instruments to a plant was a process development job which had to be done within the process development team. He made the plea that the control engineer, who was largely still the instrument engineer because he was the man who had developed the subject, should be included in the development team through a conscious decision of management that he was a necessary member.

He saw difficulties in an ordinary works of having a control engineer and instrument maintenance engineer as co-equals and having complementary responsibilities. It would more often pay for the control engineer (no matter what his background so long as he had an adequate training to deal with the subject), who would be called in on any process development problem, to be given an administrative link directly with the engineering and maintenance units in the factory. Any piece of equipment was best designed when the maintenance problems were before the designer for every minute of his working day; that could be done best by a close connection between the two sides of instrumentation and control. The answer might well be to have a qualified engineer primarily for process development and with a cross link to a foreman in charge of maintenance activities.

He hoped that Lunt would comment as to whether he felt on consideration that 200 amplifiers was really adequate for an analogue machine for general purpose control problems. Mr. Little thought that it had probably limited the scope.

Mr. D. L. **Cooper** said that in Lunt's paper the main emphasis had been placed on problems of the organisation of departmental control in large plants. A paper which described an actual exercise in design or instrumentation could usually be defended, if all else failed, by an appeal to the fact that "It would work in practice". A paper of the kind which Lunt had presented had to be defended on purely logical grounds.

He thought that most of them would agree that small firms had been reluctant to take up process investigation and control and that they had been rather neglected by firms associated with control engineering. It had occurred to him during the symposium that control engineering was biased towards large-scale plant in general and towards heavy chemical engineering in particular. As he was neither a control nor a chemical engineer he might have gained the wrong impression, but a glance at the firms and interests represented at the symposium seemed to bear out his view.

The problem of medium and small-size firms could be stated quite simply. It was finance. Lunt had rightly pointed to the difficulties of calling in consulting or contracting firms. It might be that one firm paid for the experience-gathering time of the consulting team and consequently subsidised indirectly another firm who made use of that experience later.

He thought that the latter probably gave scope for the research associations, for if the initiative in any investigation came from within the research association the results would be available to all members. If it were necessary for the initial work to be carried out at the premises of a member firm the fact that that firm got any benefits first was a compensation for having been a guinea pig.

In some cases the work might be carried out entirely within the association. There were numerous examples of both types of development in the reports of most research associations.

One difficulty which he could see in the formation within research associations of teams which could deal with problems which required expensive equipment was that of finance. Only the largest research association, as far as he knew, had a digital computer, for example. Most research associations had incomes of less than £100 000 a year which had to pay for all research, both fundamental and applied. It might be that there was a case for a common computer service, both digital and analogue, which could serve all research associations on a co-operative basis.

Another difficulty was the problem of communication between scientist and management. That was a problem which increased as the firm got smaller. He did not know whether that was because the manager of a small firm was less familiar with technical jargon or more ready to admit that he was. It was a serious problem. He did not know many managers of small firms who would get excited if one put forward a scheme for the optimisation of the dynamic performance of his plant with a view to obtaining a minimum cost function. He would probably say that all he wanted to do was to find out how to make more profits.

Prof. C. J. D. M. **Verhagen** said that he had found many useful suggestions about the organisation of a department for the design of plant in the two papers. However, one point had not been discussed and that was the usefulness and desirability in future of having facilities for coupling a computer directly on line with the measuring instruments of, for example, a pilot-plant, so as to get information about the dynamic behaviour, characteristics, optimal control, and so on. In other parts of measuring technique, in wind-tunnel research, for example, there was such direct coupling on line.

He could remember at least one paper which had referred to a chemical plant in the neighbourhood of Niagara Falls and which had been connected by telephone line to a general computer unit in Philadelphia to determine the behaviour of the plant, as to yield, material balance, etc.

In future, for supervisory control or for process optimisation, it might be practicable occasionally to couple the plant with the computer centre in order to evaluate the best situation.

That led him to ask the authors whether they thought that it would be reasonable, at least in the far future, to include facilities in the organisation of a central computer department for coupling computers with measuring instruments. On the other hand, what were their views about the type of computer which they would have to buy or rent for such a purpose? For example, there would have to be computers with interrupt programme features and so on. Or did the authors think that it would always be better to produce data on punched tape, for instance, and to send that tape to the computer centre and afterwards to work out the data when the computer had time to make the necessary evaluation?

Mr. G. M. E. **Williams** asked if Sargent would define what he meant by a major design. He had calculated that on the average Sargent's five chemical engineers could spend about 163 man-hours per job, and his assistants 294 man-hours, which gave four man-weeks and seven and a half man-weeks respectively: he found it hard to determine from that what kind of major job it was of which such a staff could dispose in so short a time.

It would also have been useful if Sargent had put more emphasis on the use of a computing machine as a service to an organisation as a whole for other kinds of work as well as that described in his paper in order to get more effective utilisation from it.

The concluding remarks of Sargent's paper were most important as he had given an example of the delegation of responsibility in the technical field. Many people took too pedantic a view about the possibility of doing that. His suggestion was very good.

Lunt unfortunately seemed a little prone to what had been once described to the speaker as the occupational disease of going in for organisational charts. In recent years he had been many such charts. Having such a chart did not effect any practical result as people still had to be made to do the jobs specified in them and it was the task of management to see that they did.

He regarded the reference in Lunt's chart to admintration as inadequate. He thought that that was unfortunate, because the more automatic a process became in the sense of automatic control theory the stiffer it became. In the Oxford Dictionary the word "automatic" was defined as "self acting" and "not accompanied by volition or consciousness" and "not characterised by active intelligence". That meant that as the plant became more sophisticated, stiffer, strapped with more interconnections and feed-back paths, the organisational work had to be better and better in order to achieve higher utilisation to justify the capital investment. He wanted incidentally to pick up Lunt's comment that the economic justification of equipment before purchase was an obstacle. It was not an obstacle but common sense.

In his chart Lunt had referred to the engineering department, which was also called the works or plant engineers department and other titles. Mr. Williams' generation was now aged 35 to 45 years and where it served in such posts it needed some attention to bring its thinking up to date and he did not think that with the current rate of advance in and state of development of technology they would make any real progress until they had tackled that problem.

He felt that carrying out an investigation for a major change was probably better done by an individual nominated by the board of a company or the equivalent organ elsewhere, with the responsibility and backing of the board behind him. He would co-ordinate work and seek advice, but he would hold all the strings. When one had committees, charged with a major project which affected several branches of a company, the members generally fell between the chairs on which they were sitting.

Mr. Balls had talked of chemical engineers and Mr. Little had talked about control engineers, but Mr. Williams took the view recently expressed elsewhere by the chairman that the best type of man for a special job was the round man. The best kind of peg to fit a hole of any shape was a round peg.

What was needed for the medium and small companies was the much greater development of standardisation of sampling points, thermometer pockets, and so on, so that those would become of comparatively low cost. It would then be economic to install such points *ad lib* so that, to use the phrase of electronic engineers, instrumentation could be jacked in wherever it was wanted. They also needed portable equipment, as Lunt had rightly said, and cheap data recorders which could be operated without being connected to the mains or some other service. The speaker had done some work on such a recorder recently and it provided an answer to the problem.

He was very glad to hear Mr. Cooper, a member of a research association, draw attention to the advisory services offered by the associations. There were many such free services which could be used and many which did not cost very much if approached in the right way and there were already one or two examples of remarkable improvements arising form very small outlay.

Dr. C. M. WILSON said that Lunt had given the impression that he could formulate a clear distinction between the scientific and commercial applications of computers. It had been their experience that this was extremely difficult to do and much of the so-called commercial work done on computers today was extremely mathematical.

Sargent had recommended that a central computing unit should provide a service to the individual departments within a firm and had suggested that comparatively unskilled people could operate the machine. He only partially agree with that philosophy of use of the computer. It was highly desirable with production work on computers where a programme had been developed to operate in that way, but where a programme was still being developed the person who wrote the programme should develop it.

Lunt had indicated that the great value of standardisation of programmes was that they were interchangeable and had later said that subsequent engineering modifications to the computers might be desirable and advantageous. Such modifications had to be thought about very carefully, for if computers were changed after they had been in operation for eighteen months or so it might be found that the programmes were no longer interchangeable. It might have been that Lunt merely meant a modification to improve reliability without in any way changing the function of the computer, in which case all was well. However, Lunt had not made that clear and he would like the point clarified.

Lunt had suggested that the number of sub-routines available in connection with Mercury was rather small. He suspected that by virtue of necessity the paper had been written some time ago. He wanted to make it clear that that was not the case, although, naturally, in two or three years' time the library of subroutines would be much larger.

He assured Prof. Verhagen that a lot of work was being done in connection with on-line process control.

Dr. D. G. OWEN said that he wanted to refer particularly to the opening paragraphs of Lunt's paper, where he had made a very clear case for bringing in a team of people which elsewhere would be called an operational research group, or sometimes a work study group. He found it almost unbelievable that in the remainder of his paper about the organisation of the use of computers for design and the choice of instruments, and the question of whether they were desirable or economic and would serve a useful purpose to management, Lunt managed not to mention the fact that the techniques involved were normally considered to be in the region of operational research, or work study. In many other industries the work would be considered as requiring co-ordination with many other management problems.

He wanted, facetiously, to ask Lunt whether his remarks meant that Imperial Chemical Industries Ltd. would reject work study as well.

Mr. F. P. STAINTHORPE said that he wanted to take up the question of a source of data recording and operating personnel for the study of plant dynamics. In the colleges they had a problem of placing students in useful work during vacations so that the work could be combined with study. By collaborating with various firms, they were able to send study teams to a number of projects and to produce useful result very quickly.

In 1959, for example, the Manchester College of Science and Technology would be sending out teams to work on packed distillation columns, all of which would be different and handling different systems so that they would fairly rapidly fill in a certain amount of the missing data.

The work was being done on pilot columns—in a sense analogues of full-scale units—and in all cases the pilot columns were in existence but not being used. They had already been installed and had done their useful work and were waiting to be used again. These projects did not cost the firm or anyone else very much money. The only difficulty lay in the planning; for adequate preparation it needed six to nine months to arrange for the study teams to go out.

The CHAIRMAN commented that Sargent had said that he wanted to provide for the extraction of data from the process of calculation and he had also said that it was desirable always to store a given set of data in the same place in the machine. As Mr. Coales understood it, that was most undesirable with drum storage but card storage was particularly suitable for the purpose, so that although Sargent said that card programming was obsolescent it seemed that there might be a case for using cards in his applications.

Mr. LUNT replied that he agreed with those speakers who had suggested, directly or indirectly, that the financial aspects of the subject should be investigated further. He said that there must be many cases where process development work and instrument installation was not undertaken because the costs of such work could be foreseen though no results could be guaranteed at that time. When the costs were high (say thousands of pounds), the arguments against starting such work might be too great. But the returns from the successful outcome of that work might offset the cost in a very short time. Perhaps one of the big differences between the American and the British approaches was that the former adopted a more venturesome policy and the profits which resulted from successful experiments were far greater than losses incurred by the failures.

Mr. Lunt suggested to Mr. Little that the control engineer and the instrument engineer should share an office. By such means each would be aware of the other's problem, but neither function need be regarded as superior to the other. With regard to the size of an analogue computer installation, in the section where Mr. Lunt had been working they had had a 10–20 amplifier unit, but a machine of that size was little more than a toy. On the other hand up to 300 amplifier units had been used in America and represented a complex plant. Mr. Lunt felt that a 200 amplifier machine would enable users to tackle many realistic problems. When they had mastered the potentialities of a machine of that size they could either consider the procedure of a more extensive one or take the problems to computing centres set up to handle such work.

Dr. Owen was probably aware that Mr. Lunt's company had been, and still was, a major pioneer of the effective use of work study in Great Britain. He expected this work to continue. In the past, work study had helped to achieve spectacular improvements in manual operations, but attention was now being directed towards process considerations.

Dr. Wilson queried the type of engineering development which might be carried out on the digital computer. Mr. Lunt stated that they were intended to provide additional facilities such as magnetic tape input and output, cathode ray presentation etc.; they would not affect the programming of such a machine. At some stage it might be possible to link it directly to an operating process, as Prof. Verhagen mentioned. But Mr. Lunt felt that for some time to come the most effective way of dealing with an existing process would be to collect data on punched card or tape and process it afterwards to determine the characteristics of that process on a large general purpose computer.

Dr. SARGENT replied to the Chairman that what he had in mind was that if one needed to use a table of enthalpies, say, for a variety of programmes, it was desirable that it should always have the same address for the same temperature. Then one could have one standard subroutine which would be able to look up the enthalpies for a given temperature and interpolate in the table.

There was an extension which one could make which was very simple and which obviated the point in question. One could arrange programmes so that the table was still in standard sequential order on the drum but so that the starting point on the drum was adjustable and alterations could be made in the routine in accordance with the address for the first entry. One could move the position on the drum by altering the first address and one could arrange for that address to be given in the input routine when putting the table into the machine in the first place.

When he had referred to major plant design he had meant the emphasis to be on the design and not on the size of plant. For example, a 200 ton oxygen plant would be a minor plant design whereas a plant using only 500 cubic feet a minute of a relatively complex gas, involving two or three columns, and perhaps with a variable feed-load or feed-composition, would be a major plant design. The words "major" and "minor" depended on the amount of work to be done on the design and not on the size of the plant. With that reservation, the sort of design of which he had been thinking was a plant for separating hydrocarbon gas mixtures such as were obtained from cracking fuel oil for example, and producing fractions such as ethylene, ethane, propylene, propane, and C_4's.

Such a plant would involve something like four distillation columns with four multi-stream exchangers, perhaps five or six simple two-fluid exchangers, separate vessels, and so on. He thought that that was typical of the sort of application. There were other plants of similar types which came within the category of major plant design. He was grateful to Mr. Williams for having pointed out the throughput of his department, of which they were rather proud.

He had placed a lot of emphasis in the paper on the fact that their department was autonomous and that it could deal with the whole problem and did not in any way try to divide the work between chemical engineers and mathematicians. They did not try to run their department as a computer service. They had found it better for each department to deal with its own problems and programme and use the machine independently. Thus the accounts department dealt with most commercial problems, and there was an independent operational research team to deal with the larger economic problems for the company as a whole. Inter-departmental agreement on the allocation of machine-time to the individual departments was organised on the same lines as the group allocation within the technical department described in the paper.

CLOSING OF SYMPOSIUM

THE FUTURE

by Sir Harold Hartley, G.C.V.O., C.B.E., M.C., D.C.L., F.R.S., M.I.Chem.E.
Past President of The Institution of Chemical Engineers, Past President of the
Society of Instrument Technology.

THE FUTURE

By Sir Harold Hartley, G.C.V.O., C.B.E., M.C., D.C.L., M.I.Chem.E., F.R.S.

Sir Harold Hartley said:

I have been asked to wind up this symposium by speaking to you of The Future. Now there is only one thing relevant to this occasion that seems to me quite certain about the future: with the increasing complexity of every operation the material welfare of mankind in the years that lie ahead will depend more and more on the combined operations of specialists—provided, of course, that they are engaged in peaceful tasks. That for me is the significance of this gathering and there could be no better omen for the future. Looking back at similar meetings in the past to discuss these problems of process development, there was the meeting of the instrument technologists in Cambridge in 1956 to discuss the dynamic characteristics of process plants; in the previous year they had joined with the chemical engineers to discuss the use of control equipment in the process industries, and now, when the use of computers has become almost a matter of routine, we have with us, most appropriately, the British Computer Society, to complete the present trinity. We enjoy, too, the advantage of the presence of many distinguished workers from abroad. and I am sure you would like me to express our thanks to them for joining in our discussions.

Many of you must think it odd, as I do myself, that you should have asked the oldest member of this symposium to speak about the future. Perhaps the idea behind it was that as my chances of survival are the smallest of any of you I might be less inhibited by the fear that I should live to see my predictions falsified. But I am not a specialist, I can only speak of the matters you have been discussing as one who has watched the game from the touch-line, with, I hope, occasional encouraging words to the players.

What has interested me particularly in your papers has been their emphasis on control. This was the direct concern of most of them and was implicit in all, showing its significance in the optimization of modern processes both mechanical and chemical. Other aspects of plant design which were relevant to the title of this meeting received little attention except from Mr. Gillings in the final papers. This emphasis and the sophisticated quality of the systems you discussed, confirmed my conviction that control engineering has reached the stage when it should be recognized as a primary technology. This has a dual significance. First control engineering must have its own teaching course. Then the control engineer must be a member of the design team from the outset: control must not be regarded as something to be added at a late stage of design.

Control engineering has its own philosophy and disciplines. It is concerned with the control of moving things, whatever they may be, electrons in a communication circuit, molecules in a chemical reaction, parts of a machine, guided missiles, or satellites. It is concerned with the stability of systems from ships to nuclear reactors. Mr. Coales, in his Presidential Address to the Society of Instrument Technology only a week before this symposium pointed out how the progress of control engineering will be dependent on the teaching facilities for technologists, technicians, and craftsmen, and the urgent problems that are involved. I hope you will all read his address.

There was a common objective running through all your papers—production by the most economical means. The obvious goal if we are ever to meet the needs of the world's growing population, with their demands for higher standards, in addition to the need for each country to keep level in the keen race of international competition. In all this rapid development we must keep in mind the economic aspect which was emphasized in Mr. Coutie's opening paper and by Mr. Gillings.

Since 1945 we have been living in a period of rapid progress during which the trend towards the continuous operation of processes of all kinds has demanded increasingly automatic control, without which indeed many of them would have been impossible. What have been the major fields of progress that have contributed to this rapid development? In spite of the fact that empiricism and the intuition of experience have undoubtedly played a great part in the early stages, and they still do, one must put theory first. It is true that the three term controller, the basis of so much control, was developed empirically to enable controls to match the characteristics of the plant. However in the future, with the more complex problems we have to meet, control theory will become more and more indispensable.

Fields of Development

The theory of control

Clerk Maxwell was the pioneer in his paper "On Governors" in 1868 in which he outlined the theory of the stability of controlled systems. Then came a long gap until the subject began to acquire momentum with the study of servomechanisms. The last war gave it an immense stimulus when mathematicians, like Norbert Wiener, were concerned with the problems of artillery control, guided missiles, and aircraft interceptions. It quickly became a happy hunting ground for mathematicians particularly in Russia where a massive effort was devoted to the theory of control in all its aspects. The result is that more rapid progress has been made in the theory than in the practice of control, and the control engineer must be well acquainted with certain branches of mathematics with which other engineers are not much concerned, such as statistical theory and probability, the Laplace transformation, methods for dealing with non-linear differential equations, and numerical methods which can be used with electronic computers. All this strengthens my conviction that the control engineer of the future will need special training to equip him for his tasks.

One point is important for the future. We must make sure that the gap between theory and practice does not grow too large. I want to make an appeal for more combined operations between mathematicians and control engineers in the next stage. Some of the joint work to be done may be unspectacular but it is essential to progress.

Electronic techniques

Another field of rapid progress during the war was in electronic techniques with the development of radar and navigational aids, guided missiles, and the innumerable ways in which electronics were found to prove an indispensable aid. New inventions such as the magnetron and later the transistor were powerful tools while the speed, the accuracy, and the versatility of electronic devices revolutionized communication systems. They have found many applications in control engineering and Mr. Redding's paper raises the question of the rival advantages of an all electric control system as compared with the pneumatic controller which has been so invaluable in the past. However, that, I know, is a controversial subject, in which I must tread very warily.

Computers

Undoubtedly the most striking application of electronics has been in the development of digital and analogue computers, which has made many new things possible with the vast difference in speed of action of the computer and the human brain. The ubiquity of their applications in the control field is seen in the papers we have discussed. At present they serve different purposes, the digital with its versatility and accuracy as a general purpose computer, and the analogue for the quick exploration of a problem, and as a simulator and a controller. Even the simplest controller is doing the work of a computer. Development of the digital machine is proceeding fast in the direction of increased speed, more fast storage, and smaller size of the basic computer. In three years' time speeds may be ten times greater and an increase of 100-fold is foreseen in the future. At the same time transistors are diminishing the power consumption and ferrite cores as memory units are diminishing the size. Quite new techniques are on the horizon especially those based on electrical properties at very low temperatures and magnetic properties of very thin films, both of which may increase the speed and lessen the cost of fast storage. There will be also great improvements in the logical uses of digital computers, of which the economy of autocoding is one example. All that will open up many fresh possibilities which we are just beginning to understand. For example, a central computer might follow the operations of a number of different plants simultaneously for practical purposes, and could be interrupted automatically at intervals to show their exact condition. Increase in speed, fast storage, and improved logic will give the digital computer many new and far reaching applications.

The special purpose analogue computer, tailored to the size and nature of the problem has been a most valuable instrument in simulating the mathematical form of an operation and in exploring its possibilities in the design stage, with the advantage of continuous visual presentation, until the optimization of the parameters justifies an accurate calculation by a digital machine. For complex problems which require a large, expensive analogue machine, it may well be that with the higher speeds of digital machines and their improved logic in the future, it will be economical to use them both for the exploratory stage and in final calculations. For special purposes and in controllers, the smaller analogue computers will find ever increasing uses and their techniques will be improved also. For example their accuracy has been increased greatly in recent years. Professor Gabor, in his recent inaugural lecture, described the new Wilby multiplier which consists of a small soft-iron armature between two electromagnets, one of which is excited by the sum of two currents and the other by the difference. The resulting force on the armature which is measured by a piezocrystal is proportional to the product of the two currents. This new device will simplify and speed up the present multiplying techniques of analogue machines. Then there is the digital differential analyser which combines the digital and analogue principles.

We are only in the early stages of computer design and techniques, although we have moved a long way since the foundation of the subject were laid ten years ago. The difference in the time scale between their action and the human senses is so vast that it isn't easy to comprehend all their possibilities in the future. Their speed and flexibility should make them the biggest single factor in process development in the future. The utility of the rapidly increasing number of computers will, however, depend on there being enough people trained to use them and to service them. Not every mathematician need be an expert in computers, but in future every mathematician should be taught to understand their potentialities so that he will know when to turn to them for help, and this applies equally to scientists and engineers. In addition many more technicians and crafsmen must be trained to service them to guarantee the high availability which is needed to justify their cost.

We are moving fast into a period which will make much greater demands on the mathematical knowledge and the mathematical awareness of the entire community and this can only be met by a great improvement in the general quality of our mathematical teaching.

Measurement

Now the practical value of all this progress in control theory, in electronics, in computers and control mechanisms is dependent on measurement, and that aspect, with the exception of the digital tachometer, has been rather taken for granted in this symposium. It is basic for the future and today control techniques are more advanced than measurement. This is particularly true of quality analysis, involving the determination of the concentration of one or more chemical constituents. However, rapid progress has been made in recent years in applying new techniques which can be applied on the plant thus avoiding the long delays of former days of sample analysis in the laboratory. In addition to the great advantages of gas chromatography, a wide range of physical measurements have been adapted to this purpose:— mass spectrometry, nuclear magnetic resonance, magnetic susceptibility, dielectric constant, infra-red and ultra-violet gas analysers, microwave spectrometry, the sonic analyser, radiometric techniques, the square wave and other forms of the polarograph, and even K capture X-rays. The improvement of measuring techniques, their accuracy and sensitiveness, should have the highest priority, as they are at present the limiting factor in the development of control. And this applies to other variables besides chemical composition. The necessary speed will vary with the time constant of the operation but in many processes a quick continuous indication is essential. Mr. Kalman and his colleagues make the shrewd comment that "there are cases where the dynamics of the measuring elements pose more severe problems than the dynamics of the process."

Control of Mechanical Systems

After this brief sketch of these four fields of development and their import for the future, I come now to the progress that has been made in the design and control of process plants which is illustrated in the papers you have discussed. There is in general a striking contrast between the mechanical systems and those involving chemical reactions. In the former the dynamic characteristics of the moving parts are known, their interaction and their stability can be expressed in mathematical form which allows of their investigation at the design stage by analogue computers, until a stage is reached when the more accurate solution afforded by a digital computer is justified. Similarly in the design stage the action of the appropriate controls can be investigated with a simulator. This was illustrated in the paper dealing with the control of axial flow compressors in which an analogue machine was used to simulate both the operating conditions and the controls. In another paper the accurate coordination of multiple shaft speeds was secured by a digital tachometer linked to a series of analogue controllers. The tandem rolling mill for the production of thin sheet steel presents a more complex problem as in addition to the interrelation of the speeds of the rolls governing the tension and the effect of the screw-down motors, there is an incomplete knowledge of the characteristics of the physical changes of the metal involved in the process. So that in addition to the analogue computer studies of the control problem the actual process is being investigated on a model scale. This shows the difficulties that may arise when physical changes are involved in a mechanical production process.

All four papers concerned with mechanical operations dealt with the use of computers in control problems, and there is, in addition, the wide range of other applications of computers in the optimization of mechanical and process plant design as described by Mr. Sargent which will be of increasing importance in the future from the many economies they will effect.

Process Control

Turning now from mechanical to chemical processes there is first the choice between batch or closed systems and continuous open systems with the economic advantages offered by the latter if they are practicable and their larger output is justified. For both the plant design falls into two stages, first the optimization of the flow sheet in terms of steady state operation, followed by the means of control to deal with the inevitable disturbances, coupled with the additional possibility of changing the conditions of operation to meet differing requirements.

In the first stage of design there is in general a lack of studies with simulators and computers similar to those with mechanical systems, largely on account of the complexity of the kinetics of the reactions which are rarely simple, but complicated by consecutive and side reactions and often dependent on catalysis and therefore difficult to reduce to an exact mathematical form. Difficulties also arise in the establishment of the transfer functions of the plant. But even at this stage there is an increasing use of analogue computers to simulate the conditions in control loops where mainly physical processes are concerned which can be expressed mathematically as in several of the papers presented to this symposium. Messrs. Keating and Townend have described the use of a digital computer for the systematic examination of the steady state of a superfractionator in order to investigate its flexibility and to obtain quantitative information about the variables in the steady state conditions. Messrs. Rijnsdorp and Maarleveld go a step further and study the dynamic behaviour and control of distillation columns using the simplified theory due to Voetter. They show that a simple analogue computer provides a quick means of ascertaining the different responses to variations in the operating conditions.

It is obviously important to get as near the optimum as possible in the compromise which every flow sheet represents and to this laboratory studies make an essential contribution. Professor Denbigh has made an interesting investigation of the kinetics of open reaction systems showing in some cases their advantage over the batch process in providing an optimum yield at some intermediate stage, and he has also shown the advantages offered by optimum temperature sequences in tubular reactors as for example in the synthesis of ammonia, but he has not dealt with the control problems involved. Aris and Amundson have made a general study of the stability of chemical reactors and its relation to control.

Until now in the absence of exact knowledge of the dynamic characteristics of a plant at the design stage, the control loops have been designed mainly on the basis of past experience and the characteristics of the plant investigated by various methods after it has been commissioned, *e.g.* correlation analysis of the records of normal operations, though in general I believe this has not proved effective, or by the introduction of sine wave, step, or pulse inputs and the analysis of the responses. These have been directed chiefly to the study of single control loops to enable the controller to be adjusted to match the dynamic characteristics of the particular variable that they control. It is inherently improbable that such settings of the independent loops represent in aggregate optimum operating conditions. In some plants there is a variable of overriding importance which is made the master control and the settings of the other loops are adjusted automatically by reference to it. This is a first step towards integrated control.

The present system of independent loops thus represents the intermediate stage between manual adjustment of the controls and an integrated system in which they are linked together with some self-optimizing device to secure the best coordination between them. This is the next objective and it is already the focus of an attack which several of the papers have described. It must obviously proceed by steps, the first being the optimization of the conditions in each separate unit of a plant and the integration of its controls leading on to the possibility of the integrated control of the whole plant at an optimum level in relation to the demand for a variety of products of specified quality—in fact the automatic factory. But that is a long way off.

Now I come to the most difficult part of my task, to try and forecast development in this field. When I read some of your papers I feel a little like Alice in Wonderland: "Somehow it seems to fill my head with ideas—only I don't know exactly what they are . . . " I had to write this paper before I had the benefit of listening to your discussions and, like most of you I expect, they have helped to clear my mind. It would be a poor compliment to the symposium if they hadn't, so now I must make some changes in what I have written.

Future Developments

It seems to me that the principle of factorial experiment described by Box and Wilson in which systematic changes are introduced into the input variables, and the responses are analysed by various means, must be the basic approach to the optimization of an existing plant, whatever the way in which it is applied. Mr. Coutie describes its use as a method of evolutionary investigation applicable to both batch and continuous processes to determine their most economic conditions of operation to yield certain products. From the results of programmed sets of changes in the operation variables, the response surfaces of yield and purity can be calculated from which the path of approach to optimum conditions can be determined. This method obviously takes a considerable time, and only deals with the steady state. It has the great advantage that it does not depend upon a theoretical knowledge of the process behaviour but obtains information at each stage by means of an experimental variation of conditions. A computer could in principle carry out the whole procedure continuously, gradually adapting the controls until they reach optimal conditions, which they can then maintain, taking into account changes such as the deterioration of the catalyst. This has already been done experimentally by the Dow Chemical Company in their new styrene plant using a procedure which I suspect is similar to the scanning technique described by Mr. Witenberg.

When we consider the dynamic behaviour of a plant, the output which we can measure at any moment depends not only on the present values of the controlled variables but also on the way in which we have recently changed them. We thus have the problem, not only of finding the best values for the variables, but also the best way in which to change from one value to another. Control theory can then tell us how to achieve these desired changes.

The paper by Messrs. Kalman, Lapidus, and Shapiro shows how this problem of optimization might be attacked by using a computer for several purposes in cyclical order. The computer issues commands to the various controllers, it also injects a series of changes into the operation variables, follows the responses to these, and signals the next change. At the same time, it continually analyses the process measurements to determine an accurate model of the process dynamics from which the optimum control procedure could be derived. Presumably the initial series of changes are programmed and after this the computer takes charge working on an optimization routine.

Professor Gabor, in his recent inaugural lecture, gave a preview of his predictor which is an analogue computer for carrying out the dynamic control of a process in the optimal way. It is capable of adjustment according to the process which it is controlling, and could replace Block A in Fig. 1 of the paper by Messrs. Kalman, Lapidus, and Shapiro. This shows one way in which our present analogue controllers may develop—by becoming more general and by having facilities by which a digital computer can set them to the best response.

Uttley and Andrew's learning machine is another approach to process optimization in which random changes are introduced into the settings of the controllers and the output of the process is measured by suitable transducers and recorded. The positive or negative effect of each change on the output is recorded in the machine's memory, and gradually the control functions are made to approach their optimum forms. If the number of variables is not so large as to make this logical type of learning machine prohibitively complex, it may have the advantage that it can be applied to the control of processes which are not amenable to precise mathematical analysis. However, what worries me about these learning machines is that we already know a great deal about the processes we are concerned with and they might waste a lot of time learning what we know already. Do not we really need a teachable machine that we can use to fill the gaps in our knowledge? Such a machine would apply the Box and Wilson technique automatically when it had been started with the advantage of our existing knowledge.

Those are in outline some of the methods that can be applied in the next stage of plant optimization and integrated control. For some routine processes the optimum conditions of operation having once been determined there may be no case for a self optimizing system of integrated controls. But in most processes conditions may vary, *e.g.* with an ageing catalyst, with different feedstocks, or with varying end products to meet changing market conditions. In such cases there is an obvious advantage in a self optimizing system of integrated controls if it can be made economical. The symposium has shown that, although there have been some applications of small special purpose computers, and computer-like techniques, in the on-line control of plant, we are still a long way from using a large digital computer for overall control.

The main reason is economic, and here the industry is showing a wise caution in insisting on an economic case before plunging in. Some examples in the field of application of computers to "business" work have shown the importance of this. A second reason is the lack of instruments suitable for connection to digital computers. Applications such as those described by Mr. Sage and Messrs. Churchill and Hopkins show what can be done in cases where this is not a limitation.

There are obviously great prizes to be won and as this is the road to the future, effort and expenditure on research and development must be lavish while at the same time their application must be aimed at economy and the maximum simplicity and reliability on the plant to avoid disappointments and frustration. Progress will depend on the closest touch with plant operation. The value of these new methods can only be tested on the plant and they will only advance by a combined operation of the makers and designers, the plant operators and the control engineers. Fortunately it will be possible to explore these uses with peripatetic units like the present pneumatic analysers which can move from plant to plant so that their potentialities can be examined with the benefit of the closer knowledge of the plant characteristics that such investigations will reveal.

Control in Living Organisms

The discussions at this symposium have been centred on the control of man-made systems and no mention has been made of the control of the processes of living organisms which, as Denbigh has pointed out, are continuous steady-state systems. Sir Cyril Hinshelwood, in his Presidential Address to the Royal Society two years ago, spoke of the three stages through which scientific theories often pass. The first stage of gross over-simplification, reflecting partly the need for practical views and even more the enthusiastic aspirations after elegance of form. The second when the symmetry of the hypothetical systems is distorted and the neatness marred as recalcitrant facts increasingly rebel against uniformity. Lastly comes the third stage . . . " if and when it is attained, when a new order emerges, more intricately contrived, less obvious and with its parts more subtly interwoven, since it is of nature's not of man's conception . . . "

There could be no better example of this than the theory of chemical kinetics which is the basis of so much process control and we have not yet reached the third stage. The complexity of our man-made systems which we are trying to control is puny compared, for example, to the multiplicity of interlocking reactions which Melvin Calvin has shown to exist in the photosynthetic activities of plants. We know almost nothing of the cunning of nature's controls by which she regulates the process of growth, or the conversion of chemical energy into muscular effort at constant temperature, or the fixation of nitrogen at atmospheric temperature.

I will repeat again the words of Steinmetz, the genius of Schenectady, to Karl Bosch, the head of the I.G. "Bosch", he said "I know you can make indigo cheaper than God, someday you may make rubber cheaper than God, but you will never make cellulose cheaper than God." I believe that we shall always have to rely on living processes for the synthesis of those complex proteins and carbohydrates on which nature has decided in the long course of evolution that man must live. But biochemical engineering provides the opportunity of a partnership with nature with the exciting possibilities of the latest development of continuous fermentation in which the average time of residence of the micro-organisms in the reaction vessel and their concentration is determined by the rate of flow of nutrient. And here again control is the important factor. The monitoring of the supply of nutrient involves difficult problems of measurement which have yet to be solved. Perhaps the study of this new phase of an old partnership will bring us nearer to the third stage of knowledge that Sir Cyril postulates. In saying this, don't think I am ending on a pessimistic note. It's another challenge and we are just entering into a new phase of process control which is in its infancy, good progress is being made and I am confident that great advances will follow even in my own lifetime.

The large attendance and stimulating papers and discussions we have had at this symposium are a good omen for the future.

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